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Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Sample Question Paper Version 6.0

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **16** pages.

Formulae A Level Mathematics B (MEI) H640

Arithmetic series
$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series
$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series
$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \ldots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \ldots\ + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ \ldots \qquad \left(|x|<1,\ \ n \in \mathbb{R}\right)$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles
$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration
$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations
$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1} S_{xx}$ where $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

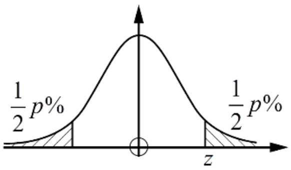
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (23 marks)

1 **In this question you must show detailed reasoning.**

Find the coordinates of the points of intersection of the curve $y = x^2 + x$ and the line $2x + y = 4$. **[5]**

2 Given that $\mathrm{f}(x) = x^3$ and $\mathrm{g}(x) = 2x^3 - 1$, describe a sequence of two transformations which maps the curve $y = \mathrm{f}(x)$ onto the curve $y = \mathrm{g}(x)$. **[4]**

3 Evaluate $\int_0^{\frac{\pi}{12}} \cos 3x \, \mathrm{d}x$, giving your answer in exact form. **[3]**

4 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x^3 - 4$ for $-1 \leq x \leq 2$.

For $\mathrm{f}^{-1}(x)$, determine

- the domain
- the range. **[5]**

5 In a particular country, 8% of the population has blue eyes. A random sample of 20 people is selected from this population.

Find the probability that exactly two of these people have blue eyes. **[2]**

6 Each day, for many years, the maximum temperature in degrees Celsius at a particular location is recorded. The maximum temperatures for days in October can be modelled by a Normal distribution. The appropriate Normal curve is shown in **Fig. 6**.

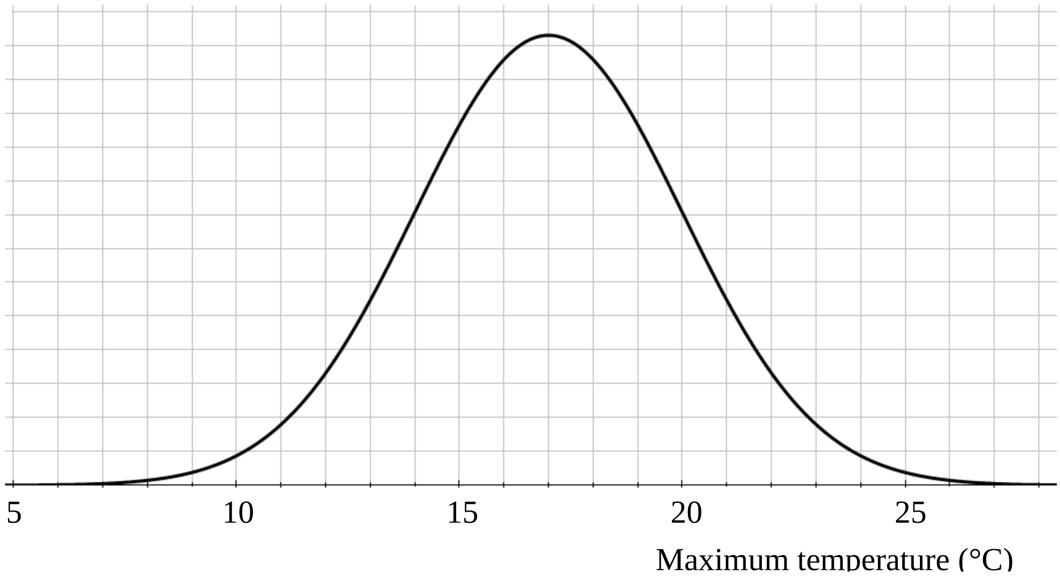


Fig. 6

(a) **(i)** Use the model to write down the mean of the maximum temperatures. **[1]**

(ii) Explain why the curve indicates that the standard deviation is approximately 3 degrees Celsius. **[1]**

Temperatures can be converted from Celsius to Fahrenheit using the formula $F = 1.8C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

(b) For maximum temperature in October in degrees Fahrenheit, estimate

- the mean
- the standard deviation. **[2]**

Answer **all** the questions.

Section B (77 marks)

7 Two events A and B are such that $\mathrm{P}(A) = 0.6$, $\mathrm{P}(B) = 0.5$ and $\mathrm{P}(A \cup B) = 0.85$. Find $\mathrm{P}(A \mid B)$. **[4]**

8 Alison selects 10 of her male friends. For each one she measures the distance between his eyes. The distances, measured in mm, are as follows:

51 57 58 59 61 64 64 65 67 68

The mean of these data is 61.4. The sample standard deviation is 5.232, correct to 3 decimal places.

One of the friends decides he does not want his measurement to be used. Alison replaces his measurement with the measurement from another male friend. This increases the mean to 62.0 and reduces the standard deviation.

Give a possible value for the measurement which has been removed and find the measurement which has replaced it. **[3]**

9 A geyser is a hot spring which erupts from time to time. For two geysers, the duration of each eruption, x minutes, and the waiting time until the next eruption, y minutes, are recorded.

(a) For a random sample of 50 eruptions of the first geyser, the correlation coefficient between x and y is 0.758.
The critical value for a 2-tailed hypothesis test for correlation at the 5% level is 0.279.
Explain whether or not there is evidence of correlation in the population of eruptions. **[2]**

The scatter diagram in **Fig. 9** shows the data from a random sample of 50 eruptions of the second geyser.

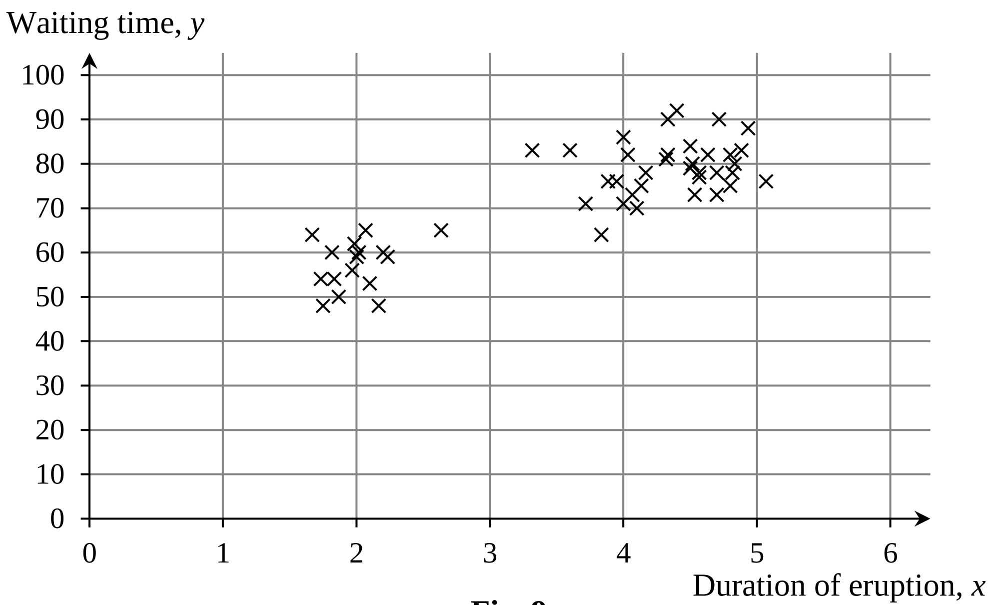


Fig. 9

(b) Stella claims the scatter diagram shows evidence of correlation between duration of eruption and waiting time. Make two comments about Stella's claim. **[2]**

10 A researcher wants to find out how many adults in a large town use the internet at least once a week. The researcher has formulated a suitable question to ask.

For each of the following methods of taking a sample of the adults in the town, give a reason why the method may be biased.

Method A: Ask people walking along a particular street between 9 am and 5 pm on one Monday.

Method B: Put the question through every letter box in the town and ask people to send back answers.

Method C: Put the question on the local council website for people to answer online. **[3]**

11 Suppose x is an irrational number, and y is a rational number, so that $y = \dfrac{m}{n}$,

where m and n are integers and $n \neq 0$.
Prove by contradiction that $x + y$ is not rational. **[4]**

12 **Fig. 12** shows the curve $2x^3 + y^3 = 5y$.

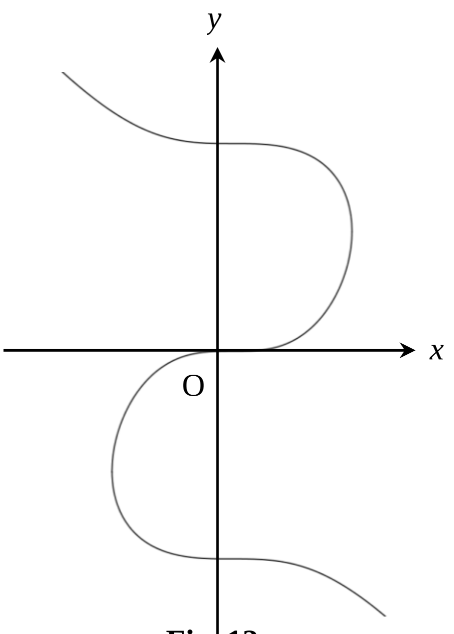


Fig. 12

(a) Find the gradient of the curve $2x^3 + y^3 = 5y$ at the point $(1, 2)$, giving your answer in exact form. **[4]**

(b) Show that all the stationary points of the curve lie on the y-axis. **[2]**

13 Evaluate $\displaystyle\int_0^1 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x$, giving your answer in the form $a + b\ln c$, where a, b and c are integers. **[6]**

14 In a chemical reaction, the mass m grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{m}{t(1+2t)} \ .$$

At time 1 minute, the mass of the chemical is 1 gram.

(a) Solve the differential equation to show that $m = \dfrac{3t}{(1+2t)}$. **[8]**

(b) Hence

(i) find the time when the mass is 1.25 grams, **[2]**

(ii) show what happens to the mass of the chemical as t becomes large. **[2]**

15 A quality control department checks the lifetimes of batteries produced by a company.

The lifetimes, x minutes, for a random sample of 80 'Superstrength' batteries are shown in the table below.

Lifetime	$160 \le x < 165$	$165 \le x < 168$	$168 \le x < 170$	$170 \le x < 172$	$172 \le x < 175$	$175 \le x < 180$
Frequency	5	14	20	21	16	4

(a) Estimate the proportion of these batteries which have a lifetime of at least 174.0 minutes. **[2]**

(b) Use the data in the table to estimate
- the sample mean,
- the sample standard deviation. **[3]**

The data in the table on the previous page are represented in the following histogram, **Fig 15**:

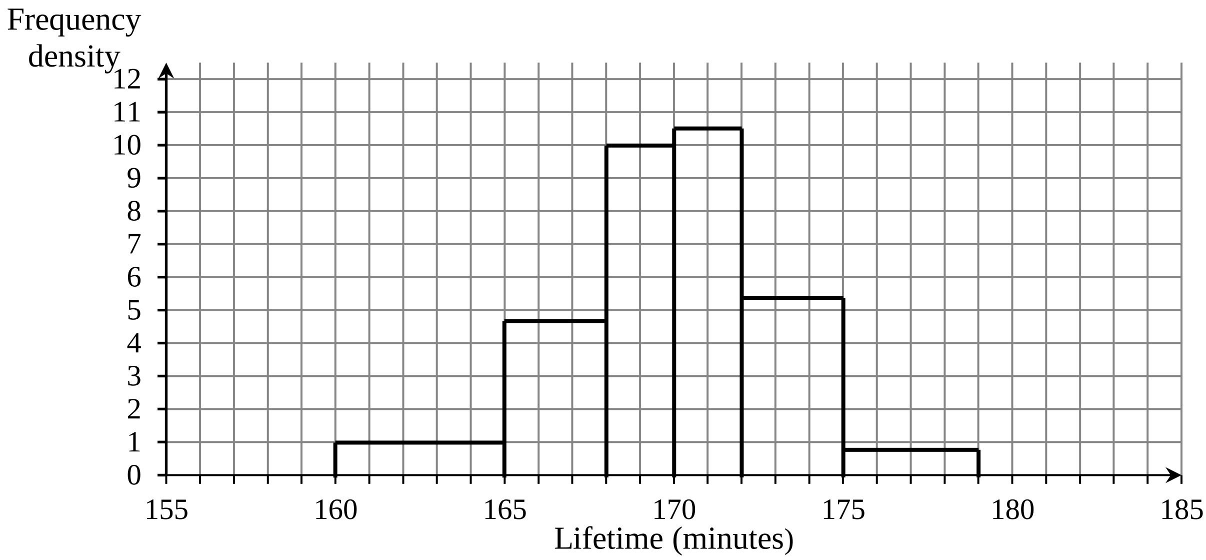


Fig. 15

A quality control manager models the data by a Normal distribution with the mean and standard deviation you calculated in part **(b)**.

(c) Comment briefly on whether the histogram supports this choice of model. **[2]**

(d) **(i)** Use this model to estimate the probability that a randomly selected battery will have a lifetime of more than 174.0 minutes.

(ii) Compare your answer with your answer to part **(a)**. **[3]**

The company also manufactures 'Ultrapower' batteries, which are stated to have a mean lifetime of 210 minutes.

(e) A random sample of 8 Ultrapower batteries is selected. The mean lifetime of these batteries is 207.3 minutes.

Carry out a hypothesis test at the 5% level to investigate whether the mean lifetime is as high as stated. You should use the following hypotheses $H_0 : \mu = 210$, $H_1 : \mu < 210$, where μ represents the population mean for Ultrapower batteries.

You should assume that the population is Normally distributed with standard deviation 3.4. **[5]**

TURN OVER FOR THE NEXT QUESTION

16 **Fig. 16.1**, **Fig. 16.2** and **Fig. 16.3** show some data about life expectancy, including some from the pre-release data set.

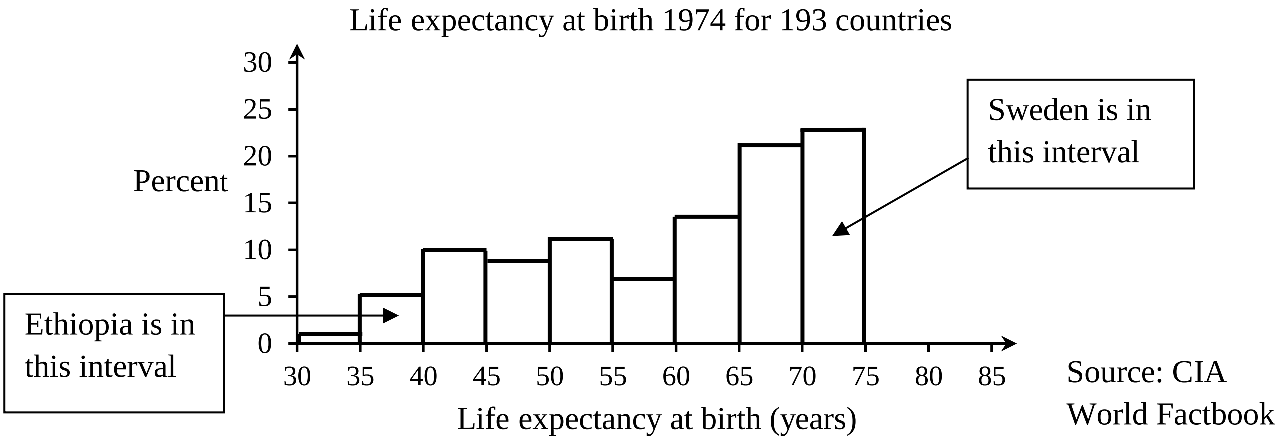


Fig. 16.1

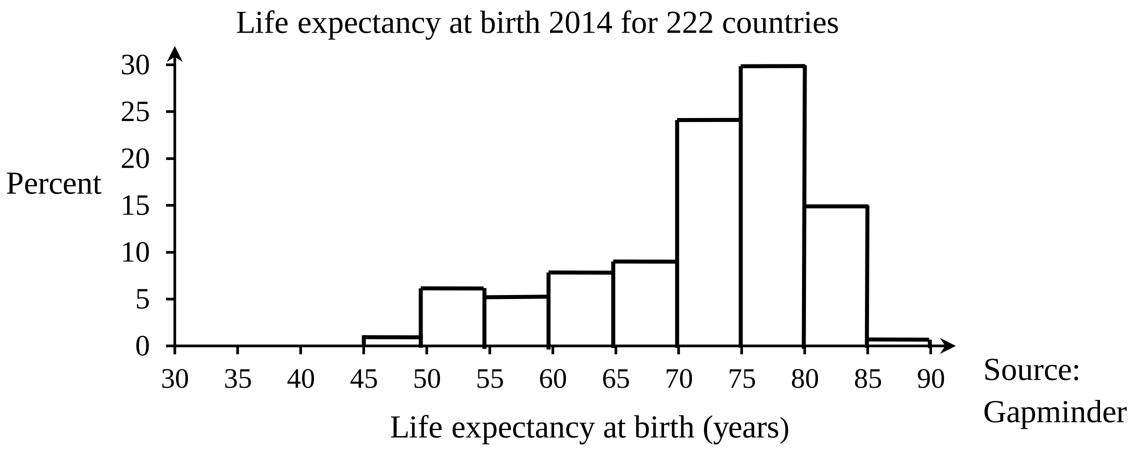
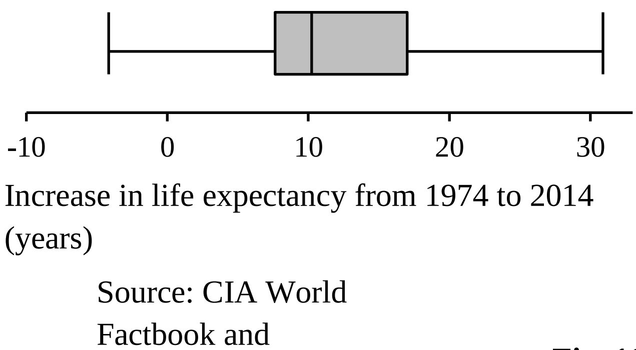


Fig. 16.2



Increase in life expectancy for 193 countries from 1974 to 2014	
Number of values	193
Minimum	- 4.618
Lower quartile	6.9576
Median	9.986
Upper quartile	15.873
Maximum	30.742

Fig. 16.3

(a) Comment on the shapes of the distributions of life expectancy at birth in 2014 and 1974. **[2]**

(b) **(i)** The minimum value shown in the box plot is negative.
What does a negative value indicate? **[1]**

(ii) What feature of **Fig 16.3** suggests that a Normal distribution would **not** be an appropriate model for increase in life expectancy from one year to another year? **[1]**

(iii) Software has been used to obtain the values in the table in **Fig. 16.3**.
Decide whether the level of accuracy is appropriate. Justify your answer. **[1]**

(iv) John claims that for half the people in the world their life expectancy has improved by 10 years or more.
Explain why **Fig. 16.3** does **not** provide conclusive evidence for John's claim. **[1]**

(c) Decide whether the maximum increase in life expectancy from 1974 to 2014 is an outlier.
Justify your answer. **[3]**

Here is some further information from the pre-release data set.

Country	Life expectancy at birth in 2014
Ethiopia	60.8
Sweden	81.9

(d) **(i)** Estimate the change in life expectancy at birth for Ethiopia between 1974 and 2014.

(ii) Estimate the change in life expectancy at birth for Sweden between 1974 and 2014.

(iii) Give **one** possible reason why the answers to parts **(i)** and **(ii)** are so different. **[4]**

Fig. 16.4 shows the relationship between life expectancy at birth in 2014 and 1974.

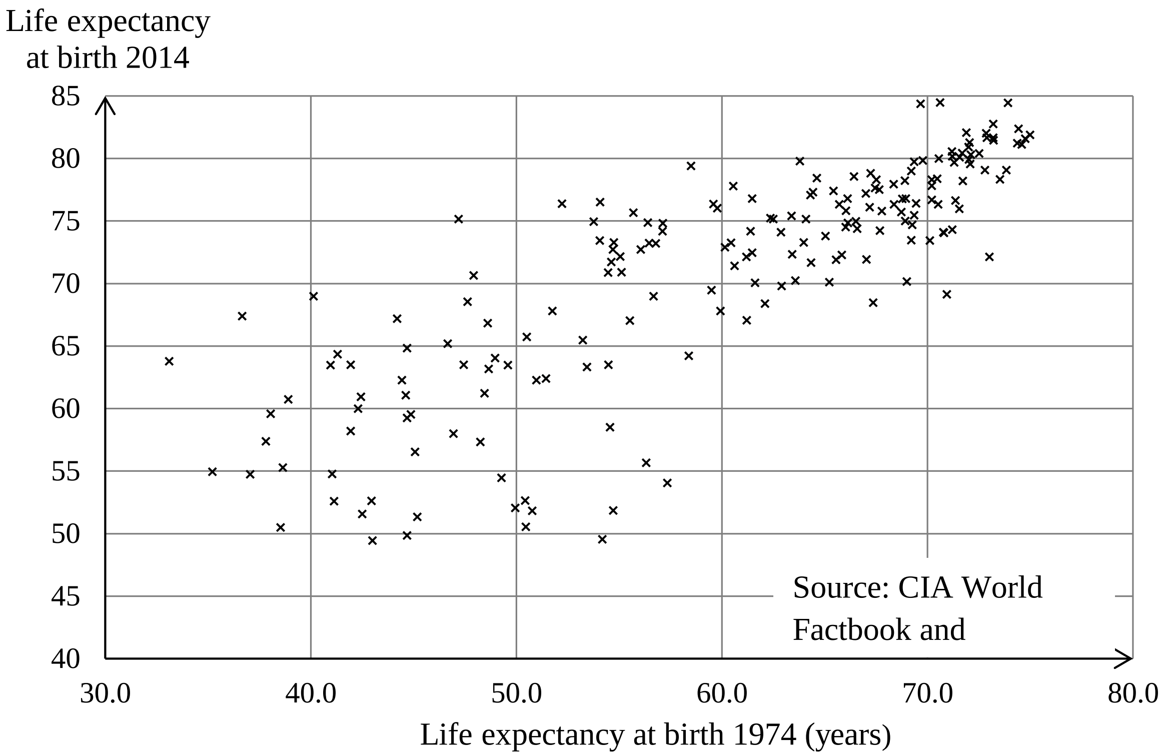


Fig. 16.4

A spreadsheet gives the following linear model for all the data in **Fig 16.4**.

(Life expectancy at birth 2014) = 30.98 + 0.67 × (Life expectancy at birth 1974)

The life expectancy at birth in 1974 for the region that now constitutes the country of South Sudan was 37.4 years. The value for this country in 2014 is not available.

(e) **(i)** Use the linear model to estimate the life expectancy at birth in 2014 for South Sudan. **[2]**

(ii) Give two reasons why your answer to part **(i)** is not likely to be an accurate estimate for the life expectancy at birth in 2014 for South Sudan.
You should refer to **both** information from **Fig 16.4 and** your knowledge of the large data set. **[2]**

(f) In how many of the countries represented in **Fig. 16.4** did life expectancy drop between 1974 and 2014? Justify your answer. **[3]**

END OF QUESTION PAPER

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…day June 20XX – Morning/Afternoon

A Level Mathematics B (MEI)
H640/02 Pure Mathematics and Statistics

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			**DR** $y = 4 - 2x$ $4 - 2x = x^2 + x$ $\Rightarrow x^2 + 3x - 4 = 0$ $\Rightarrow x = 1$ or $x = -4$ $y = 2$ or $y = 12$ (1,2) and (-4,12)	 **M1** **M1** **A1** **A1** **A1** **[5]**	 2.1 1.1 1.1 1.1 2.5	 Eliminating x or y must be seen Form a quadratic equation For final A mark, corresponding values of x and y must be expressed as coordinates from well set out correct solution	 Or $y^2 - 14y + 24 = 0$ SC1 for one pair of coordinates only
2			[1-way] stretch scale factor 2 in y-direction translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	**M1** **A1** **M1** **A1** **[4]**	1.1 1.1 1.1 1.1	 If transformations given in reverse order then **M1**, **A1**, **M1** are still available (but not final **A1**) Or −1 in y-direction	

Question			Answer	Marks	AOs	Guidance	
3			$\int_0^{\frac{\pi}{12}} \cos 3x \mathrm{d}x = \left[\frac{\sin 3x}{3}\right]_0^{\frac{\pi}{12}}$	B1	1.1	$\frac{\sin 3x}{3}$	
			$= \frac{1}{3}\left(\sin\frac{\pi}{4} - 0\right)$	M1	1.1		
			$= \frac{\sqrt{2}}{6}$ o.e.	A1	1.1	Must be in exact form	
				[3]			
4			$y = x^3 - 4$ $x \leftrightarrow y$ $x = y^3 - 4$	M1	1.1	attempt to invert	
			$\Rightarrow x + 4 = y^3$ $\Rightarrow y = \sqrt[3]{x+4}$ so $f^{-1}(x) = \sqrt[3]{x+4}$	A1	1.1	accept $y = \sqrt[3]{x+4}$ but not $x = \sqrt[3]{y+4}$	
			range of f is $-5 \le y \le 4$	M1	1.1	May be implied	
			so domain of f^{-1} is $-5 \le x \le 4$	A1	1.2	or $[-5, 4]$	
			range is $-1 \le y \le 2$	B1	1.1	or $-1 \le f^{-1}(x) \le 2$ or $[-1, 2]$	
				[5]			
5			Binomial(20, 0.08)	M1	3.3		
			P(2 blue) = 0.27[11]	A1	1.1	**BC**	
				[2]			
6	(a)	(i)	Mean = 17	B1	3.4		
		(ii)	**Either** Points of inflection are approx. 3 above and below mean so SD = approx. 3	B1	2.4	AG	
			Or Limits are approx. 9 above and below mean so SD = 9 ÷ 3 = 3	B1	2.4	AG	
				[2]			

Question			Answer	Marks	AOs	Guidance	
6	(b)		Mean in Fahrenheit = 1.8 × 17 + 32 = 62.6 SD in Fahrenheit = 1.8 × 3 = 5.4	**B1** **B1** **[2]**	**1.1** **1.1**	FT their mean	
7			$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= 0.6 + 0.5 - 0.85$ $= 0.25$ $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{0.25}{0.5}$ $= 0.5$	 **M1** **A1** **M1** **A1** **[4]**	 **3.1a** **1.1** **1.1** **1.1**		
8			Increases a value by 6 New value is closer to 62 than the old value is to 61.4 51 changes to 57 **or** 57 changes to 63 **or** 58 changes to 64	**M1** **M1** **A1** **[3]**	**3.1b** **2.2a** **2.2a**	Implied by correct answer or pair of values differing by 6 Implied by correct answer or new value closer to 62 than old value	
9	(a)		0.758 > 0.279 So there is sufficient evidence of correlation (in the population)	**M1** **A1** **[2]**	**1.1** **2.2b**	 Oe but not evidence of positive correlation.	
9	(b)		E.g. diagram shows positive correlation overall, but the data consists of two distinct clusters. E.g. neither of the two clusters show evidence of correlation	**B1** **B1** **[2]**	**2.3** **2.2b**	Accept other suitable correct comments	

Question			Answer	Marks	AOs	Guidance	
10			Method A - E.g. Will not sample people who work then/people who do not walk down that street. Method B - E.g. This will only get answers from those who want to send in an answer. Method C - E.g. This will only get answers from those who use the council website. E.g. Those who use the internet more frequently are more likely to see the question.	**B1** **B1** **B1** **[3]**	**2.4** **2.4** **2.4**		
11			Suppose $x + y$ is rational So $x+y=\frac{p}{q}$, where p and q are integers $\Rightarrow x=\frac{p}{q}-\frac{m}{n}=\frac{(pn-mq)}{qn}$ which is rational x is irrational so this is a contradiction	**B1** **B1** **B1** **B1** **[4]**	**2.1** **2.1** **3.1a** **2.4**	or stating that the difference of two fractions is rational	
12	(a)		$6x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=5\frac{\mathrm{d}y}{\mathrm{d}x}\left[\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6x^2}{5-3y^2}\right]$ when $x = 1$, $y = 2$, $6+12\frac{\mathrm{d}y}{\mathrm{d}x}=5\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{6}{7}$	**M1** **A1** **M1** **A1** **[4]**	**1.1a** **1.1** **1.1** **2.1**	implicit differentiation correct substituting $x = 1$, $y = 2$ cao	
12	(b)		$\frac{\mathrm{d}y}{\mathrm{d}x}=0$ so $6x^2=0$ $x=0$ so all stationary points lie on y-axis	**M1** **A1** **[2]**	**1.2** **2.1**	Substitute $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into their differentiated expression Completion of argument	

Question			Answer	Marks	AOs	Guidance	
13			let $u = 1+\sqrt{x}$ $\mathrm{d}u = \frac{1}{2}x^{-\frac{1}{2}}\,\mathrm{d}x$	M1	3.1a	substituting $u = 1+\sqrt{x}$ or $w = \sqrt{x}$	Evidence of method must be seen
			$\Rightarrow \mathrm{d}x = 2(u-1)\mathrm{d}u$	A1	1.1	$\mathrm{d}x = 2(u-1)\,\mathrm{d}u$ or $\mathrm{d}x = 2w\,\mathrm{d}w$	
			$\Rightarrow \int_0^1 \frac{1}{1+\sqrt{x}}\mathrm{d}x = \int_1^2 \frac{2(u-1)}{u}\mathrm{d}u$	A1	1.1	$\frac{2(u-1)}{u}(\mathrm{d}u)$ or $\frac{2w}{(w+1)}(\mathrm{d}w)$	
			$= \int_1^2 \left(2 - \frac{2}{u}\right)\mathrm{d}u$	M1	3.1a	splitting fraction or dividing to get $2 - \frac{2}{(w+1)}$ (or substituting $u = w + 1 \Rightarrow$ $\frac{2(u-1)}{u}$ and then splitting fraction)	Evidence of method must be seen
			$= \left[2u - 2\ln u\right]_1^2$	A1	1.1	or $\left[2w - 2\ln(w+1)\right]_0^1$ if still in terms of w	
			= 4 – 2ln 2 – 2 = 2 – 2ln 2 **or** 2 – ln 4	A1 **[6]**	1.1	cao	

Question			Answer	Marks	AOs	Guidance	
14	(a)		$\int \frac{\mathrm{d}m}{m} = \int \frac{\mathrm{d}t}{t(1+2t)}$	M1	1.1a	separating variables	
			$\frac{1}{t(1+2t)} \equiv \frac{A}{t} + \frac{B}{1+2t}$	M1	3.1b	using partial fractions	
			$\Rightarrow 1 \equiv A(1 + 2t) + Bt$ $t = 0 \Rightarrow A = 1$	M1	1.1	substituting values, equating coeffs or cover up	
			$t = -½ \Rightarrow\ 1 = -½B \Rightarrow B = -2$	A1A1	1.1 1.1	$A = 1$, $B = -2$	
			$\Rightarrow \quad \int \frac{\mathrm{d}m}{m} = \int \left(\frac{1}{t} - \frac{2}{1+2t}\right)\mathrm{d}t$ $\Rightarrow \quad \ln m = \ln t - \ln(1 + 2t) + c$	B1FT	2.1	FT their A, B, condone no c	
			$t = 1, m = 1 \Rightarrow c = \ln 3$	M1	1.1	evaluating constant of integration	
			$\Rightarrow \ln m = \ln\left(\frac{3t}{1+2t}\right)$	A1	2.1		
			$\Rightarrow m = \frac{3t}{(1 + 2t)}$	[8]		AG	
14	(b)	(i)	$1.25 = \frac{3t}{(1+2t)}$	M1	1.1a		
			$\Rightarrow 1.25 + 2.5t = 3t$ $\Rightarrow t = 1.25 \div 0.5 = 2.5$ minutes	A1 [2]	1.1		
14	(b)	(ii)	$m = \frac{3}{\left(\frac{1}{t} + 2\right)}$	M1	3.1b	or substituting a large value for t	
			$\to 1.5$ [grams]	A1 [2]	2.2a		

Question			Answer	Marks	AOs	Guidance	
15	**(a)**		Estimated number $= 4 + \frac{16}{3} = 9\frac{1}{3}$	**M1**	**3.1b**	for attempt at interpolation	
			$\frac{9\frac{1}{3}}{80} = 0.1166...$ so proportion is approximately 0.117	**A1**	**1.1**		
				[2]			
15	**(b)**		E.g. Midpoints	**M1**	**1.1**	evidence of valid method for estimation	
			Mean = 170	**A1**	**1.1**	**BC** Mean in the range 169-171	
			Standard deviation = 3.4	**A1**	**1.1**	**BC** SD in the range 3-3.5	
				[3]			
15	**(c)**		The histogram e.g. seems to have a rough bell shape e.g. is symmetrical (around the estimated mean) e.g. appears to have all data within 3 s.d. of the mean	**B1**	**3.5a**	for one reason	
			so this does support the manager's belief	**B1**	**3.5a**	for at least two reasons and 'supports belief'	
				[2]			
15	**(d)**	**(i)**	P(Lifetime > 174) for $N(170, 3.4^2)$	**M1**	**3.4**	oe	
			0.1197	**A1**	**1.1**	**BC** FT their mean and standard deviation	
		(ii)	Answer is very similar to estimate in part (i)	**B1**	**3.5a**		
				[3]			

Question			Answer	Marks	AOs	Guidance	
15	(e)		**Either**				
			Test statistic = $\dfrac{207.3-210}{3.4/\sqrt{8}}=-2.246$	**M1**	**3.4**	Must include √8	
			Lower 5% level 1 tailed critical value of z = −1.645	**A1**	**1.1**		
			−2.246 < -1.645 so significant	**B1**	**1.1**	For comparison leading to correct conclusion	
			or				
			Under H_0, $\overline{X} \sim \mathrm{N}\left(210, \dfrac{3.4^2}{8}\right)$	**M1**	**3.4**		
			$\mathrm{P}\left(\overline{X} \le 207.3\right) = 0.01235$	**A1**	**1.1**	**BC**	
			0.01235< 0.05 so significant	**B1**	**1.1**		
			There is sufficient evidence to reject H_0	**A1**	**2.2b**		
			There is sufficient evidence to conclude that the mean lifetime is less than 210 minutes.	**A1**	**2.4**		
				[5]			

Question			Answer	Marks	AOs	Guidance	
16	**(a)**		Comment about shape of distribution for first graph Comment about shape of distribution for second graph	**B1** **B1** **[2]**	**2.2b** **2.2b**	Comments can be combined e.g. Both distributions negatively skewed gets both marks e.g. 1974 distribution has greater spread than 2014 gets both marks	If zero scored, **SC1** for "The 2014 distribution is shifted to the right of the 1974 distribution" oe
16	**(b)**	**(i)**	Life expectancy went down [between 1974 and 2014] in [at least] one country	**B1** **[1]**	**2.2a**	**NOT** increase in life expectancy is negative	
16	**(b)**	**(ii)**	The box plot is not symmetrical.	**B1** **[1]**	**3.5b**		
16	**(b)**	**(iii)**	Not appropriate with reason	**B1** **[1]**	**2.4**	e.g. [some] values of life expectancy are estimates The values of life expectancy are not available to this level of accuracy	
16	**(b)**	**(iv)**	Comment about life expectancy at birth data for countries and not individual people	**B1** **[1]**	**2.4**		
16	**(c)**		Use of Q3 + 1.5 × (Q3 - Q1) 15.873 + 1.5(8.9154) = 29.2461 (years) The maximum value is an outlier as 30.742>29.2461.	**M1** **M1** **A1** **[3]**	**1.2** **1.1** **1.1**		

Question			Answer	Marks	AOs	Guidance	
16	**(d)**	**(i)**		**M1**	**3.1b**	Attempt to estimate change in life expectancy at birth soi.	
			approx 60.8 - 37.5= 23.3 (years)	**A1**	**1.1**	FT 'their 37.5 between 35 - 40'	
		(ii)	Change in life expectancy for Sweden approx 81.9 - 72.5 = 9.4 (years)	**A1**	**1.1**	FT 'their 72.5 between 70 - 75'	
		(iii)	E.g. Countries with a lower life expectancy in 1974 have greater opportunity to increase life expectancy in 2014.	**B1** **[4]**	**3.2a**	OR Countries with a higher life expectancy in 1974 have less opportunity to increase life expectancy in 2014.	
16	**(e)**	**(i)**	30.98 + 0.67 × 37.4 = 56.0 (years)	**M1** **A1** **[2]**	**3.4** **1.1**		
16	**(e)**	**(ii)**	E.g. Large amount of scatter at the lower values [and South Sudan is 37.4]. E.g. Not having the data value could indicate that there are problems in the country which could mean it does not follow the pattern for other countries	**B1** **B1** **[2]**	**3.5b** **3.5b**	B1 Reason inferred from Fig 16.4 B1 For knowing why data may be missing	
16	**(f)**		Correct method Clearly explained 6	**M1** **B1** **A1** **[3]**	**3.1b** **2.4** **1.1**	e.g. draw " $y = x$" on graph e.g. The value on the vertical axis must be lower than the one on the horizontal axis FT their correct method	

Question	AO1	AO2	AO3(PS)	AO3(M)	Total	LDS
1	3	2			**5**	
2	4				**4**	
3	3				**3**	
4	5				**5**	
5	1			1	**2**	
6 a		1		1	**2**	
6 b	2				**2**	
7	3		1		**4**	
8		2	1		**3**	
9 a	1	1			**2**	
9 b		2			**2**	
10		3			**3**	
11		3	1		**4**	
12 a	3	1			**4**	
12 b	1	1			**2**	
13	4		2		**6**	
14 a	5	2	1		**8**	
14 b i	2				**2**	
14 b ii		1	1		**2**	
15 a	1		1		**2**	
15 b	3				**3**	
15 c				2	**2**	
15 d	1			2	**3**	
15 e	2	2		1	**5**	
16 a		2			**2**	
16 b i		1			**1**	
16 b ii				1	**1**	
16 b iii		1			**1**	1
16 b iv		1			**1**	1
16 c	3				**3**	
16 d i	1		1		**2**	
16 d ii	1				**1**	
16 d iii			1		**1**	1
16 e i	1			1	**2**	
16 e ii				2	**2**	1
16 f	1	1	1		**3**	
Totals	**51**	**27**	**11**	**11**	**100**	LDS
						4

Summary of Updates

Date	Version	Change
October 2018	2	We’ve reviewed the look and feel of our papers through text, tone, language, images and formatting. For more information please see our assessment principles in our “Exploring our question papers” brochures on our website.
May 2022	5.3	Copyright acknowledgements updated.
January 2025	6.0	From 2025 E1/G1 marks will be annotated using M1, A1, or B1.The previous E1/G1 annotation awarding criteria has always existed as a sub-section within M (method), A (accuracy), or B (independent). Removal of E1/G1 annotation will increase marking consistency and teacher comprehension of our approach to marking.

Accredited

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Printed Answer Booklet

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:
- Question Paper H640/02 (inserted)

You may use:
- a scientific or graphical calculator

First name	
Last name	

Centre number						**Candidate number**				

INSTRUCTIONS

- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **16** pages.

Section A (23 marks)

1	

PLEASE DO NOT WRITE IN THIS SPACE

2	
3	

4

5	
6 (a)(i)	
6 (a)(ii)	
6 (b)	

Section B (77 marks)

7	

8	
9 (a)	
9(b)	

10 (a)	
10 (b)	
10 (c)	
11	

12 (a)

12 (b)

13

14 (a)

(answer space continued on next page)

14 (a)	(continued)
14 (b)(i)	
14 (b)(ii)	

15 (a)	
15 (b)	

15 (c)	
15 (d) (i)	
15 (d) (ii)	

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15 (e)	

16 (a)	
16 (b) (i)	
16 (b) (ii)	
16 (b) (iii)	
16 (b) (iv)	

16 (c)	
16(d) (i)	
16(d) (ii)	
16(d) (iii)	

16 (e) (i)	
16 (e) (ii)	

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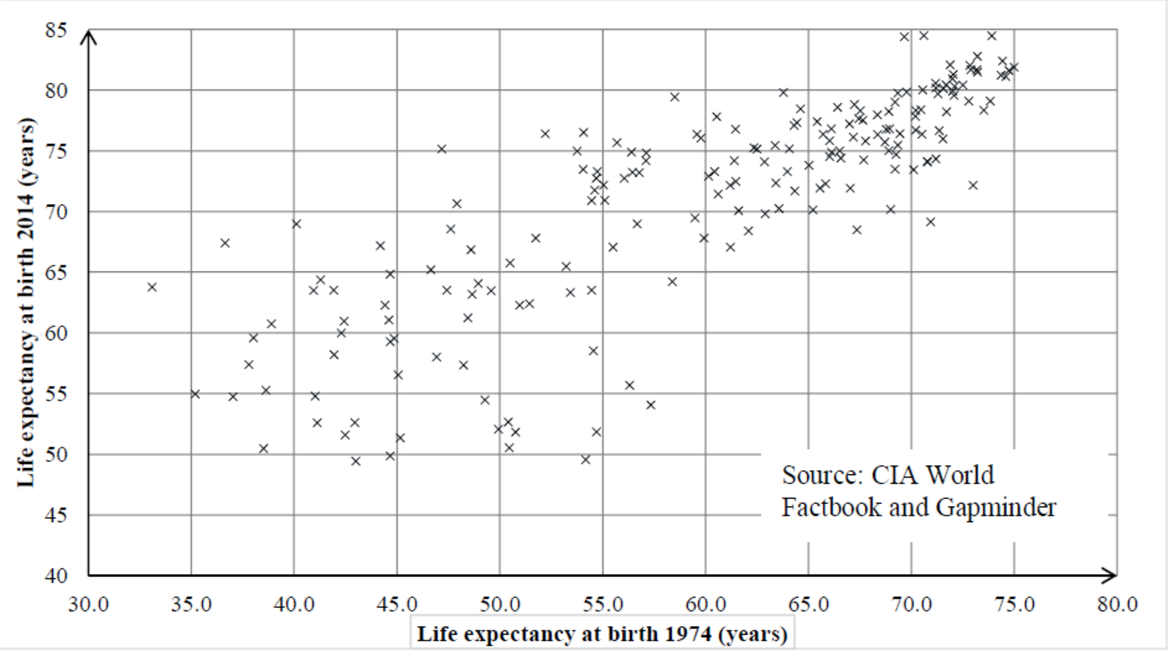
16 (f)


Life expectancy at birth 2014 (years)
85
80
75
70
65
60
55
50
45
40
30.0
35.0
40.0
45.0
50.0
55.0
60.0
65.0
70.0
75.0
80.0
Life expectancy at birth 1974 (years)
Source: CIA World Factbook and Gapminder

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