

AS Level Further Mathematics A**Y532 Statistics****Sample Question Paper**

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:

- a scientific or graphical calculator



* o o o o o o *

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by gms^{-2} . Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of pages.

Answer **all** the questions.

- 1 Two music critics, P and Q , give scores to seven concerts as follows.

Concert	1	2	3	4	5	6	7
Score by critic P	12	11	6	13	17	16	14
Score by critic Q	9	13	8	14	18	16	20

- (i) Calculate Spearman's rank correlation coefficient, r_s , for these scores. [4]

- (ii) Without carrying out a hypothesis test, state what your answer tells you about the views of the two critics. [1]

- 2 The probability distribution of a discrete random variable W is given in the table.

w	0	1	2	3
$P(W = w)$	0.19	0.18	x	y

Given that $E(W) = 1.61$, find the value of $\text{Var}(3W + 2)$. [7]

- 3 Carl believes that the proportions of men and women who own black cars are different. He obtained a random sample of people who each owned exactly one car. The results are summarised in the table below.

	Black	Non-black
Men	69	71
Women	30	55

Test at the 5% significance level whether Carl's belief is justified. [8]

- 4 (i) Four men and four women stand in a random order in a straight line. Determine the probability that no one is standing next to a person of the same gender. [3]

- (ii) x men, including Mr Adam, and x women, including Mrs Adam, are arranged at random in a straight line. Show that the probability that Mr Adam is standing next to Mrs Adam is $\frac{1}{x}$. [3]

- 5** (i) The random variable X has the distribution $\text{Geo}(0.6)$.
- (a) Find $P(X \geq 8)$. [2]
- (b) Find the value of $E(X)$. [1]
- (c) Find the value of $\text{Var}(X)$. [1]
- (ii) The random variable Y has the distribution $\text{Geo}(p)$. It is given that $P(Y < 4) = 0.986$ correct to 3 significant figures. Use an algebraic method to find the value of p . [3]
- 6** Sabrina counts the number of cars passing her house during randomly chosen one minute intervals. Two assumptions are needed for the number of cars passing her house in a fixed time interval to be well modelled by a Poisson distribution.
- (i) State these two assumptions. [2]
- (ii) For each assumption in part (i) give a reason why it might not be a reasonable assumption for this context. [2]
- Assume now that the number of cars that pass Sabrina's house in one minute can be well modelled by the distribution $\text{Po}(0.8)$.
- (iii) (a) Write down an expression for the probability that, in a given one minute period, exactly r cars pass Sabrina's house. [1]
- (b) Hence find the probability that, in a given one minute period, exactly 2 cars pass Sabrina's house. [1]
- (iv) Find the probability that, in a given 30 minute period, at least 28 cars pass Sabrina's house. [3]
- (v) The number of bicycles that pass Sabrina's house in a 5 minute period is a random variable with the distribution $\text{Po}(1.5)$. Find the probability that, in a given 10 minute period, the total number of cars and bicycles that pass Sabrina's house is between 12 and 15 inclusive. State a necessary condition. [4]

- 7 The discrete random variable X is equally likely to take values 0, 1 and 2. $3N$ observations of X are obtained, and the observed frequencies corresponding to $X = 0$, $X = 1$ and $X = 2$ are given in the following table.

x	0	1	2
Observed frequency	$N - 1$	$N - 1$	$N + 2$

The test statistic for a chi-squared goodness of fit test for the data is 0.3. Find the value of N .

[4]

- 8 The following table gives the mean per capita consumption of mozzarella cheese per annum, x pounds, and the number of civil engineering doctorates awarded, y , in the United States in each of 10 years.

x	9.3	9.7	9.7	9.7	9.9	10.2	10.5	11.0	10.6	10.6
y	480	501	540	552	547	622	655	701	712	708

source: www.tylervigen.com

- (i) Find the equation of the regression line of y on x .

[2]

You are given that the product moment correlation coefficient is 0.959.

- (ii) Explain whether this value would be different if x is measured in kilograms instead of pounds.

[1]

It is desired to carry out a hypothesis test to investigate whether there is correlation between these two variables.

- (iii) Assume that the data is a random sample of all years.

- (a) Carry out the test at the 10% significance level.

[6]

- (b) Explain whether your conclusion suggests that manufacturers of mozzarella cheese could increase consumption by sponsoring doctoral candidates in civil engineering.

[1]

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...day June 20XX – Morning/Afternoon

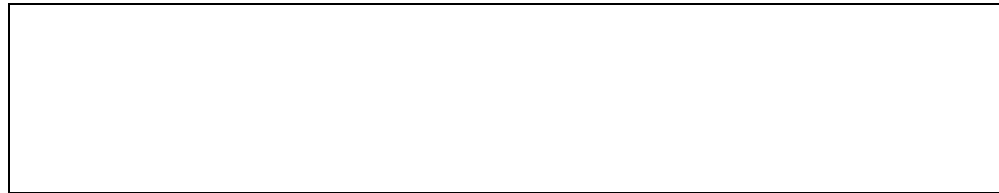
AS Level Further Mathematics A

Unit Y532 Statistics

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60



This document consists of 12 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for AS Level Further Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as *cao* may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance																
1	(i)		Rankings	*M1	1.1a	Rank both, can be reverse order																
			<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>1</td><td>3</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td></tr></table>	1	2	3		4	5	6	7	1	3	2	4	7	5	6				
			1	2	3	4		5	6	7												
			1	3	2	4		7	5	6												
$\sum d^2 = 8$	A1	1.1	$\sum d^2 = 8$ BC																			
$r_s = 1 - \frac{6 \times 8}{7 \times 48} = \frac{6}{7}$ or 0.857	dep*M1 A1 [4]	1.1 1.1	Use of correct formula awrt 0.857																			
1	(ii)		Strong agreement/association in rankings/views	B1 [1]	2.2b	Not strong correlation/relationship																
2			$x + y = 0.63$	M1	1.1	Use $\sum p = 1$ and $\sum xp = 1.61$ to																
			$0.18 + 2x + 3y = 1.61$			set up two equations in x and y																
				A1	1.1	and attempt to solve																
			$x = 0.46, y = 0.17$	A1	1.1	For both equations																
						Correct values of x, y , these or																
						exact equivalent BC																
			So $\text{Var}(W) = 0.18 + 2^2x + 3^2y$	M1	3.1a	Attempt $\sum x^2p$																
			-1.61^2	M1	1.1	Subtract their μ^2																
$= 0.9579$	A1	1.1	Answer, exact or 0.958 only (no FT)																			
$\text{Var}(3W + 2) = 3^2 \text{Var}(W) = 8.6211$	A1FT [7]	2.2a	FT $9 \times$ their $\text{Var}(W)$																			

Question			Answer	Marks	AO	Guidance										
3			H ₀ : gender and car colour not associated H ₁ : gender and car colour associated	B1	2.5	Hypotheses correctly stated (allow “independent”)	Yates’ Correction omitted (4.202), reject: can get B1M1A1M1A0A1M1A1 = 7 / 8									
			<table border="1"><tr><td>61.6</td><td>78.4</td><td>140</td></tr><tr><td>37.4</td><td>47.6</td><td>85</td></tr><tr><td>99</td><td>126</td><td></td></tr></table>	61.6	78.4	140		37.4	47.6	85	99	126		M1 A1	1.1 1.1	Correct method for finding E_f All E_f correct
			61.6	78.4	140											
			37.4	47.6	85											
			99	126												
			$\chi^2 = 6.9^2 \left(\frac{1}{61.6} + \frac{1}{78.4} + \frac{1}{37.4} + \frac{1}{47.6} \right)$	M1	3.3	Attempt at calculation of χ^2 , allow even if no Yates’ Correction										
			$[= 0.773 + 0.607 + 1.273 + 1.000]$ = 3.653	A1	3.4	Correct value of χ^2 , in range [3.65, 3.66]										
			< 3.841	A1FT	1.1	FT Compare their χ^2 explicitly with 3.841										
			Do not reject H ₀	M1	1.1	Correct first conclusion										
			Insufficient evidence of a relationship between gender and car colour	A1FT	2.2b	Correctly interpreted, need context, acknowledge uncertainty										
	[8]															

Question			Answer	Marks	AO	Guidance	
4	(i)		$2 \times 4! \times 4! [=1152]$	M1	1.1	Method (allow 2 omitted)	OR M1 Attempt at alternating male female
			$\div 8!$	M1	1.1		M1 denominator decreasing $1 \times \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1$
			$= \frac{1}{35}$ or 0.0286	A1	2.1	awrt 0.0286 Clear working or justification must be given SC2 for one number extra/omitted, e.g. $\frac{1}{70}$	A1 $= \frac{1}{35}$ or 0.0286
			[3]				
4	(ii)		$\frac{(2x-1)! \times 2}{(2x)!}$	M1	3.1a	Taking Mr and Mrs Adam as one item, ie 2x seen	
				M1	3.1a	Forming the probability and cancelling the factorial expressions	
			$= \frac{2}{2x} = \frac{1}{x}$	E1	2.1	AG An intermediate step must be shown	
			[2]				

Question			Answer	Marks	AO	Guidance	
5	(i)	(a)	0.4^7 $= 0.00164 \quad [1.6384 \times 10^{-3}]$	M1 A1 [2]	1.1a 1.1 Or $\frac{128}{78125}$	Allow M1 for 0.000655 or 0.4^8 OR $1 - (0.6 + 0.6 \times 0.4 + 0.6 \times 0.4^2 + \dots + 0.6 \times 0.4^6)$	
5	(i)	(b)	$\frac{5}{3}$	B1 [1]	1.1	Any equivalent exact fraction or awrt 1.67	
5	(i)	(c)	$\frac{10}{9}$	B1 [1]	1.1	Any equivalent exact fraction or awrt 1.11	
5	(ii)		$P(Y < 4) = 1 - q^3 = 0.986$ $q = \sqrt[3]{0.014} = 0.241\dots$ So $p = 1 - q = 0.759$ to 3 sf	M1 M1 A1 [3]	3.1a 2.1 1.1	$1 - q^4$ [but <i>not</i> $q^3 p$] gets M1M1A0 0.75898577...	
6	(i)		Cars must pass independently of one another... ...at a uniform rate	E1 E1 [1]	1.2 1.2	<i>Not</i> answers equivalent to “random” oe	Allow “constant average rate” but not “constant rate” E0 for any answer that implies fixed numbers in given time E0 for “events must occur randomly”, “independently”, “singly” or “at constant rate” oe

Question			Answer	Marks	AO	Guidance	
6	(ii)		E.g. Traffic lights or roundabouts might make cars go past in groups, not independently	E1	2.4	One reason why cars might not act independently, in context	
			E.g. The rate at which cars pass will be higher at commuting times, than in the middle of the day.	E1 [1]	2.4	One reason why the rate may not be uniform, in context	
6	(iii)	(a)	$e^{-0.8} \frac{0.8^r}{r!}$	B1 [1]	3.4		
6	(iii)	(b)	$e^{-0.8} \frac{0.8^2}{2!} = 0.144$	B1 [1]	1.1		[0.143785]
6	(iv)		$X \sim \text{Po}(24)$ $P(X \geq 28) = 1 - P(X \leq 27)$ $= 0.232$ to 3sf	M1 M1 A1 [3]	1.1a 1.1 1.1	BC awrt 0.232	
6	(v)		Distributions must be independent $Y \sim \text{Po}(11)$ $P(Y \leq 15) - P(Y \leq 11) = 0.9074 - 0.5792$ $= 0.328(2)$	B1 M1 M1 A1 [4]	1.2 1.1 3.4 1.1	Attempt at $\text{Po}(10 \times 0.8 + 2 \times 1.5)$ soi BC	Allow M1M1A0 for 0.2187

Question			Answer	Marks	AO	Guidance	
7			$\sum \frac{(O-E)^2}{E} = \frac{1}{N} + \frac{1}{N} + \frac{2^2}{N}$ $\frac{6}{N} = 0.3 \text{ so } N = 20$	M1 A1 M1 A1 [4]	3.1a 1.1 1.1 1.1	Correct formula used Correct $(O - E)^2$ values Obtain 20 only	
8	(i)		$y = 157x - 988$	B1 B1 [2]	1.1 3.3	Either coefficient correct to 3 s.f. All correct including letters	BC $[y = 157.11x - 988.15]$
8	(ii)		No; r is unaffected by linear coding	E1 [1]	2.3	Needs both “no” and “linear” seen, and no wrong or irrelevant statements	
8	(iii)	(a)	$H_0 : \rho = 0$ $H_1 : \rho \neq 0$ where ρ is the population correlation coefficient CV 0.5494 $0.9586 > 0.5494$ Reject H_0 Significant evidence of correlation between consumption of mozzarella cheese and number of civil engineering doctorates awarded.	B2 B1 M1 M1 A1 [6]	1.1 2.5 3.4 1.1 1.1 2.2b	Hypotheses correctly stated B1 if no symbol used Correct critical value seen Explicit comparison Correct first conclusion Contextualised, acknowledge uncertainty	
8	(iii)	(b)	No, correlation does not imply causality	E1 [1]	3.5b	Or: “there may be another factor that influences both”	

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1(i)	4				4
1(ii)		1			1
2	5	1	1		7
3	4	2		2	8
4(i)	2	1			3
4(ii)		1	2		3
5(i)(a)	2				2
5(i)(b)	1				1
5(i)(c)	1				1
5(ii)	1	1	1		3
6(i)	2				2
6(ii)		2			2
6(iii)(a)				1	1
6(iii)(b)	1				1
6(iv)	3				3
6(v)	3			1	4
7	3		1		4
8(i)	1			1	2
8(ii)		1			1
8(iii)(a)	3	2		1	6
8(iii)(b)				1	1
Totals	36	12	5	7	60

PS = Problem Solving

M = Modelling

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

AS Level Further Mathematics A**Y532 Statistics**

Printed Answer Booklet

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:

- Question Paper Y532 (inserted)
- Formulae AS Level Further Mathematics A

You may use:

- a scientific or graphical calculator



First name

Last name

Centre
numberCandidate
number**INSTRUCTIONS**

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INFORMATION

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1(i)	
	1(ii)

2

3

4(i)**4(ii)**

5(i)(a)	
5(i)(b)	
5(i)(c)	
5(ii)	

6(i)	
6(ii)	
6 (iii)(a)	
6 (iii)(b)	

6 (iv)	
6 (v)	

7

8(i)	
	8(ii)

8 (iii)(a)	
8(iii)(b)	

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