

**GCSE (9-1)**

**Mathematical Skills Handbook**

**SCIENCE**

**J247, J248, J249, J250, J257, J258, J259, J260**

For first teaching in 2016

**Mathematical Skills  
Handbook**

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# Introduction

In order to be able to develop their skills, knowledge and understanding in GCSE Science, learners need to have been taught, and to have acquired competence in, the appropriate areas of mathematics relevant to the subject as indicated in each specification. These can also be found in the DfE criteria for the sciences.

There will be a minimum weighting of assessment of quantitative skills in each qualification. These weightings are:

- 10% Biology
- 20% Chemistry
- 30% Physics
- 20% Combined Science GCSE (with a 1:2:3 ratio of each of the sciences).

All mathematical content will be assessed within the lifetime of the specification.

The content of this handbook follows the structure of the table outlined in the specifications, with mathematical skill discussed in turn. The discussion of each skill begins with a description and explanation of the mathematical concepts. This is followed by specific scientific contexts where these skills are used in the context of the curriculum which are then demonstrated in a step by step manner with particular emphasis on the mathematical skills being practised and developed. Notes on common difficulties and misconceptions, as well as suggestions for teaching, may be included in either section as appropriate.

As this handbook shows, all required mathematical skills can be covered alongside the subject content in an integrated fashion. It is recommended that teachers aim to specifically assess learners' understanding and application of the mathematical concepts as a matter of course, in order to discover and address any difficulties that they may have. This is particularly relevant for learners who are not taking the Higher Tier in GCSE Mathematics so the skills they need to develop may only be covered in Science lessons.

## Definition of GCSE (9–1) Mathematics

The GCSE (9–1) Mathematics specification aims to enable learners to develop:

1. a sound understanding of concepts
2. fluency in procedural skill
3. competency to apply mathematical skills in a range of contexts
4. confidence in mathematical problem solving.

This guide aims to inform and guide teachers as to what to expect when demonstrating these ideas from the expected mathematical knowledge at Key Stage 3 and 4, and how they may complement the work that is being done in Maths lessons. Learners often find it difficult to apply their mathematical knowledge in unfamiliar settings and by teaching these skills with significant background knowledge of key areas of difficulty it can empower and inspire learners to become more independent and better problem-solvers.

# Chapter 1 : Arithmetic and numerical computation

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The Science specifications state in appendix 5f that learners are expected to:

- a. Recognise and use expressions in decimal form
- b. Recognise expressions in standard form
- c. Use ratios, fractions and percentages
- d. Make estimates of the results of simple calculations

## Prior Key Stage 3 learning

Learners should be able to:

- order positive and negative integers, decimals and fractions
- understand and use place value for decimals, measures and integers of any size
- interpret and compare numbers in standard form
- work interchangeably with terminating decimals and their corresponding fractions
- interpret fractions and percentages as operators
- use ratio notation, including reduction to simplest form.

## Mathematical skills

### a) Recognise and use expressions in decimal form

This area is extensively covered in Key Stage 3 Mathematics but it does not mean that knowledge should be assumed as there are a number of misconceptions. When calculating expressions using decimal expressions, learners' instinct is to reach for their calculators and type the numbers in to get an answer. It should be stressed that this is not necessarily bad practice but that they should be able to perform basic arithmetic without using a calculator if applicable. A key misconception is the idea of multiplying and dividing a decimal by powers of 10. For example when attempting to calculate:

$$0.00123 \times 1000$$

the often quoted rule of "add a zero for every zero you multiply" doesn't work here. If this simple rule was applied blindly the answer of 0.001 230 00 would have the effect of multiplying by 1. The key point to make to learners is that zeros do not get added or 'taken away' but the numbers 'move' to the left or right of a stationary decimal point.

For example:

$$0.001\ 23 \times 1000$$

means we have to move the numbers three times to the left as we are multiplying. We have therefore

$$0.0123$$

by moving once and hence multiplying by 10. We do this another two more times to complete the calculation to get

$$0.001\ 23 \times 1000 = 1.23$$

If we divide then we move to the right. A key idea for the learners to embrace is the idea of testing whether their answer is sensible or not. If they make a mistake and move the digits to the right to get:

$$0.000\ 001\ 23$$

It must be incorrect because the number has got a lot smaller. Encourage learners to check their answers for sensibility.

### b) Recognise expressions in standard form

Calculations involving standard form are covered by all learners in GCSE Mathematics. They are required to recognise and interpret numbers in standard form. The issues learners have with dealing with standard form are largely due to the issues they have with dealing with indices. A key misconception is to ‘over-use’ the addition rule for the multiplication of indices. Another key mistake is for the learners not to leave the first number in the form  $a \times 10^b$ ,

where  $0 < a < 10$ , and  $b$  is an +/- integer. For example they may write:

$$(2 \times 10^4) \times (8 \times 10^3) = 16 \times 10^7$$

Here the learner has correctly dealt with the indices and has got the correct numerical answer but the number isn’t in standard form. The 16 is incorrect and has to be ‘scaled’ down by dividing by 10. However if we divide by 10 we have to multiply the  $10^7$  by 10 to keep the numerical value the same. Hence:

$$16 \times 10^7 = 1.6 \times 10^8$$

Learners should be encouraged to convert to standard form as it is the best way to make comparisons on quantities. The following examples illustrate when a number is in standard form:

- $16 \times 10^3$  N is not in standard form
- $0.16 \times 10^5$  N is not in standard form
- $1.6 \times 10^4$  N is in standard form

These two examples illustrates the effective use of S.I. measurements using a prefix to enable easy comparison on the magnitude of quantities:

- 1600 N
- 1.6 kN is preferred notation with a prefix

### Calculator use

Learners with access to a scientific calculator should be able to use it to convert between different decimal and standard form calculations. For Sharp calculators the ‘Change’ button is used whilst for Casio calculators it is the ‘S→D’ button that performs this operation. Texas Instruments have a specific scientific notation function, EE. Learners should be encouraged to use this calculator function to help with the accuracy of their answers.

To write numbers in standard form on a calculator either use a ‘ $\times 10x$ ’ button (Casio) or the ‘EXP’ button (Sharp).

For other models please encourage learners to investigate for themselves the appropriate functions.

## Teaching activity: Standard form

At the beginning of a lesson when learners will be using standard form, display the examples (or something similar) from earlier and ask the learners to classify them as correct standard form or not. This simple five minute starter task if repeated whenever a lesson contains use of standard form will help consolidate the ideas without impeding on the time spent on the actual science.

### c) Use ratios, fractions and percentages

The Science specifications specifically state that learners should be able to understand and apply the concept of proportions in a scientific context. Direct and inverse proportions is covered briefly in Key Stage 3 Maths and then again in the Higher Tier of the GCSE Mathematics course.

The simplest type of proportion is direct. An easy way of understanding this is if two variables  $a$  and  $b$  are directly proportional to one another then if we double  $a$  we have to double  $b$ , if we halve  $a$  we have to halve  $b$ , etc. A key point to mention to the learners is this only applies if we multiply or divide the quantities; it doesn't work with addition and subtraction.

For example:

If  $a$  is directly proportional to  $b^2$  and we know that when  $a = 6$ ,  $b = 10$ , how can we find the relationship between them?

We know that when  $a = 12$  (doubling 6) then  $b^2 = 200$  (doubling  $10^2$ ) and so on.

The *ratio* of  $a$  and  $b^2$  is a *constant*. Therefore we know that:

$$\frac{a}{b^2} = \frac{12}{200} = \frac{6}{100} = \frac{3}{50} = 0.06$$

We can write this more succinctly as:

$$a = 0.06b^2$$

and we have found the relationship.

For inverse proportion the situation is reversed. If  $a$  and  $b$  are inversely proportional then if we double  $a$  we have to halve  $b$ , and if we triple  $b$  we have to divide  $a$  by three and so on. Therefore this time their *product* remains constant.

For example:

If  $a$  is inversely proportional to  $\sqrt{b}$  and we know that when  $a = 2$ ,  $b = 9$ .

We then have:

$$a \times \sqrt{b} = 2 \times \sqrt{9} = 2 \times 3 = 6$$

this can be written more succinctly as:

$$a = \frac{6}{\sqrt{b}}$$

A specific example in Physics is that the pressure,  $P$ , and the volume,  $V$ , of an ideal gas are inversely proportional to each other at a constant temperature. This is summarised as:

$$PV = \text{constant}$$

Where at Key Stage 5,  $n$ ,  $R$  and  $T$  are constant.

## Teaching activity: proportionality

Display on the board:

" $x$  is directly proportional to  $y^2$  when  $x = 6, y = 10$ "

Challenge learners to find as many different pairs of  $x$  and  $y$  that satisfy this relationship, for example  $x = 24, y = 300$  (tripling). Give them five minutes. As an extension ask learners to find the relationship between  $x$  and  $y$ . You can change the type of proportion and numbers appropriately. This is a simple activity at the end of a lesson (or beginning to consolidate) using proportion that helps learners grasp the mathematical skills.

Another key area of difficulty is percentage increases and decreases which occurs in the context of percentage yields in Chemistry and Biology experiments. This is often because learners are 'over-taught' the method on how to find percentage increases and decreases, actually the calculations are relatively easy.

The key is to understand that the multiplier 1 represents a change of 0%. A multiplier of 1.43 therefore represents an increase of 43% whilst a multiplier of 0.83 represents a decrease of 17% (note that it is the difference between the multiplier and 1 which is the change – it isn't a percentage decrease of 83%). Amounts and percentages can then be found using a very simple formulae:

$$\text{Original Amount} \times \text{Multiplier} = \text{New Amount}$$

This formula can be stated in a formula triangle and then applied to situations where the percentage change is required. For example if initially the mass of a radioactive isotope is 5.6g and after a week it is 4.7g then to work out the percentage decrease we have:

$$5.6 \times \text{Multiplier} = 4.7$$

$$\text{Multiplier} = \frac{4.7}{5.6} = 0.84$$

This represents a percentage decrease of  $(1 - 0.84) = 16\%$  decrease.

Say we wanted to know the population of rabbits which were initially 82 and experienced an increase of 16% then:

$$82 \times \text{Multiplier} = \text{New Population}$$

$$82 \times 1.16 = \text{New} = 95.12$$

### d) Make estimates of the results of simple calculations

Being able to make estimation for the size of a given measure is a notoriously difficult concept, even for teachers. The best advice for making 'plausible' estimates is to start at a 'known' quantity and then extrapolate from that fact to the object to be estimated.

For example, let's say the mass of a tiger is to be estimated. If the mass of an average human is 'known' as 65kg then perhaps a sensible estimation is to say that the tiger is 1 to 2 times the mass of the human, say 1.5 and hence the mass of the tiger could be estimated as  $65 \times 1.5 = 97.5\text{kg}$ .

In making estimations in calculations care should be taken with how the numbers are rounded as it could lead to confusion as to whether there is an under or overestimation. For example the calculation:

$$\begin{array}{r} 4.9 \\ \hline 1.10 \end{array}$$

This could be estimated as  $5/1.1 = 4.54$ . The trouble is that 4.9 has been overestimated. As it is a division calculation, if the numerator is made larger it will give us larger answer thus an overestimate. The denominator being made smaller e.g.  $4.9/1 = 4.9$  has the effect of making the answer larger too because dividing by a small number yields a large result. The actual answer is 4.45.

## Contexts in Science

### Biology

*Question:* The diameter of a human egg cell is  $120\ \mu\text{m}$ . What is the diameter in mm? ( $1\ \mu\text{m} = 1 \times 10^{-3}\ \text{mm}$ )

*Answer:* Using the information given we can use proportional reasoning.

If  $1\ \mu\text{m} = 1 \times 10^{-3}\ \text{mm}$  then  $120\ \mu\text{m} = 120 \times 1 \times 10^{-3}\ \text{mm}$ .

Now we need to convert this number to standard form. We start with:

$$120 \times 10^{-3}\ \text{mm}$$

Now we divide the first number by 10 but multiply the power of ten by 10 to keep the value the same. Hence

$$12.0 \times 10^{-2}\ \text{mm}$$

We repeat these so that the first number lies between 1 and 10:

$$1.2 \times 10^{-1}\ \text{mm}$$

### Chemistry

*Question:* Katie investigates a chemical reaction between carbon dioxide and hydrogen to make methane and water. She predicts that 11.0 g of carbon dioxide should make 4.0 g of methane. In an experiment she finds that 11.0 g of carbon dioxide makes 2.2 g of methane. Calculate the percentage yield of methane.

*Answer:* The formula for percentage yield is given by

$$\frac{\text{actual yield}}{\text{predicted yield}} \times 100$$

We are calculating the amount of actual yield to predicted yield as a percentage (much like if you scored  $\frac{56}{84}$  on a test; the percentage would be  $\frac{56}{84} \times 100$ ). The answer therefore is

$$2.2 \div 4.0 \times 100 = 55\%$$

## Physics

*Question:* The voltage through a transformer is increased before transmission through the National Grid. It is increased from 2.5 kV to 40 kV. How much would this increase in voltage change the current?

*Answer:* This is a question about proportionality. Using the formula:

$$\text{Power} = \text{Current} \times \text{Voltage}$$

means that if the power is constant then the product of current and voltage is also constant. This means voltage and current are inversely proportional to each other. So to find out how much the voltage has increased we calculate:

$$\frac{40}{2.5} = 16$$

so the voltage has increased by 16 times. As the current is inversely proportional this means that the current has been decreased by a factor of 16 or the original current has been divided by 16.

# Topic Check In 1

## Questions

**Do not use a calculator.**

Calculate the following, showing all your working.

1.  $55 + 36$
2.  $251 - 72$
3.  $25 \times 18$
4.  $216 \div 12$
5.  $5 - 7 - 32 + 24$
  
6. Lucy writes:  
$$\begin{array}{r} -6 \\ \times -8 \\ \hline -48 \end{array}$$
The answer is incorrect. Explain how you know she has made a mistake.
  
7. Explain why  $1.4 \times 10^{-3}$  is the same as 0.0014.
  
8. Joe is attempting the following calculation.  
$$\begin{array}{r} 1148 \\ \div 14 \\ \hline \end{array}$$
His incorrect solution is shown below.

Line 1	532
Line 2	$\begin{array}{r} 14 \\ ) 1148 \end{array}$
Line 3	$14 \times 50 = 700$
Line 4	$14 \times 30 = 420$
Line 5	$14 \times 2 = 28$

What is the correct answer? Explain the mistake Joe has made.

9. Amanda is using these numbers to make a new number.  
1                  -2                  4                  -8
  - She can only use the + and – operations.
  - She cannot use any number more than once.
  - She cannot use multiplication, division or powers.
  - She cannot put numbers together, e.g. she can't use 48.

What is the largest number that she can make?

10. A box has a mass of 60 g and contains test tubes. Each test tube has a mass of 15 g. A full box has a mass of 1.5 kg. How many test tubes are in the box?  
**REMEMBER: 1 kg = 1000 g**

## Answers

1. 91
2. 179
3. 450
4. 18
5. - 10
6. Negative multiplied by negative gives positive
7.  $1.4 \times 10^{-3} = 1.4 \div 1000 = 0.0014$

8. 82. He should have added 50 and 30

This method of long division is also known as ‘repeated subtraction’ or ‘chunking’ and will be what learners are used to from Key Stage 2 mathematics.

To have correctly worked this out learners will have picked ‘easy’ multiples of 14 to subtract from the number they are dividing from (1148 in this case).

So:

$$\begin{aligned}14 \times \mathbf{50} &= 700 \\1148 - 700 &= 448\end{aligned}$$

This process is then repeated with a smaller multiple such as 30:

$$\begin{aligned}14 \times \mathbf{30} &= 420 \\448 - 420 &= 28\\\text{Then finally } 14 \times \mathbf{2} &= 28.\end{aligned}$$

The multiples are added to give the correct answer:  $50 + 30 + 2 = 82$

9.  $15 = 1 - -2 + 4 - -8$

10. Students more likely to have changed everything to grams, not kg

$$\frac{1.5 \times 1000 - 60}{1.5 \times 1000} = 96 \text{ test tubes}$$

# Chapter 1 RAG

Question	Topic	R	A	G
1	Able to use manual methods for addition			
2	Able to use manual methods for subtraction			
3	Able to use manual methods for multiplication			
4	Able to use manual methods for division			
5	Able to add and subtract a list of numbers			
6	Understand how to multiply directed numbers			
7	Able to recognise equivalent expressions			
8	Understand the importance of place value in calculations			
9	Understand the effect of subtracting a negative value			
10	Able to apply arithmetic skills to solve problems			

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7	Able to recognise equivalent expressions			
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9	Understand the effect of subtracting a negative value			
10	Able to apply arithmetic skills to solve problems			

# Chapter 2 : Handling Data

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The Science specifications state in appendix 5f that learners are expected to:

- a. Use an appropriate number of significant figures
- b. Find arithmetic means
- c. Construct and interpret frequency tables and diagrams, bar charts and histograms
- d. Understand the principles of sampling as applied to scientific data
- e. Understand simple probability
- f. Understand the terms mean, mode and median
- g. Use a scatter diagram to identify a correlation between two variables
- h. Make order of magnitude calculations

## Prior Key Stage 3 learning

Learner's should be able to:

- describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs.

## Mathematical skills

### a) Use an appropriate number of significant figures

Rounding is quite straightforward for learners but there are a couple of misconceptions that can still arise. A common misconception is to 'lose' a significant figure mistaking it to be a 0 place holder. For example take  $4.99 \times 10^5$  which is currently correct to 3 significant figures. Rounding this number to 2 significant figures would yield  $5.0 \times 10^5$ . The 0 in this context represents a significant figure, not a place holder and hence should be retained. Some learners will find it irresistible to quote the answer as  $5 \times 10^5$  but this is the answer if it is rounded to 1 significant figure.

Learners must understand that the lowest level of accuracy in the inputs of a calculation will determine the level of accuracy in the answer. If there are 3 inputs to a particular calculation and they are quoted as being correct to 2, 3 and 4 significant figures respectively then the answer can only be quoted reliably correct to 2 significant figures. Additionally if the inputs are given to a set number of decimal places then it's the lowest level of decimal places that the answer must be given to.

Learners can dislike using significant figures and generally prefer to use decimal places. It is useful to highlight the advantages of significant figures over decimal places by showing that the accuracy does not change when changing units. i.e 3.2 mm is 0.32 cm is 0.0032 m all to 2 sf, but 0.32 cm is 0.00 m to 2 dp.

### b) Find arithmetic means

The mean is calculated using a simple formula:

$$\bar{x} = \frac{\sum x}{n}$$

where  $x$  is the data values and  $n$  is the number of data values. This is potentially a rather off-putting formula to present to the learners but most of them will be familiar with the rather ad-hoc message; ‘Add them all up, divide by how many’ from Key Stage 3 and actually this is an excellent way of teaching the topic. There are few misconceptions with this when dealing with raw, listed data as the calculations involved are quite simple.

Outliers (items of data significantly different from the mean) can also pose problems. It should be emphasised that there is no hard and fast rule on how to deal with outliers; they should be treated case by case. First of all the classification of an outlier is itself an ambiguous task. For experiments a simple checklist is this:

Was the suspected outlier recorded in error?

Was the suspected outlier recorded in different conditions to the rest?

If the answer to either of these questions is YES then the outlier should be omitted from the data set and the average should be calculated without this member. If a potential outlier is spotted at the time of the experiment then learners should question whether the experiment should be repeated.

### Teaching activity

At the beginning of a lesson where this skill will be used write down a list of random numbers (these can be generated online or from Excel for example) on the whiteboard; make sure that there is one number significantly larger than the rest and one number significantly smaller than the rest. Ask them to calculate the mean in the cases where i) all the numbers are included in the calculation, ii) all the numbers except the ‘outliers’ are included. A brief discussion can then be had on which calculation is more valid and the key points as listed above can be emphasised.

### Calculator use

Many different scientific calculators have a Statistics mode where the mean can be calculated automatically. Whilst these functions are incredibly useful to gain full summary statistics (standard deviation, sum of squares, correlation coefficients etc.) the computational advantage of doing this for just the mean is nil. Having said that, because the data is entered into a table rather than as part of a long addition sum it can make checking for data entry errors easier.

### c) Construct and interpret frequency tables and diagrams, bar charts and histograms

Learners will be familiar with frequency tables, bar charts and histograms from Key Stage 3. There are a few misconceptions however for the construction of histograms and bar charts as there is a subtle often ignored difference.

Bar charts are used when the data is *discrete*. The data can only take specific values. Variables such as eye colour, number of offspring etc. can only take certain values, there are no ‘in-between’ values; number of offspring could be 0,1,2, etc. but couldn’t be 1.5. Eye colour can only be blue, green etc. (unless you specify a wave frequency with each colour and then it becomes continuous). With bar charts there are gaps between the bars when it is plotted.

Histograms are used when the data is *continuous*. Height, weight, and age are all examples of continuous data where they are measured to a specified accuracy (age is considered continuous because actually it is a measure of time, even though we only state integer years when we say our age). With histograms there are no gaps between the bars.

In Maths, learners will investigate data that has been recorded in unequal group widths and so will be familiar with the heights of the histogram bars being frequency density rather than frequency. If the data has been recorded in equal width groups then frequency density or frequency can be used.

#### d) Understand the principles of sampling as applied to scientific data

There are many types of sampling. A key aspect of them all is that there is a degree of *randomness* involved. One cannot avoid all traces of bias but from random sampling these biases can be eliminated as much as possible.

A useful way to obtain this ‘randomness’ is to use a random number generator or a list of random numbers either provided to the learner or generated from a spreadsheet or calculator. Imagine that a 1 m<sup>2</sup> of grass is to be sampled to count the number of worms. To sample without bias, the area needs to be split up into smaller squares of *equal* size (say for example 0.0625m<sup>2</sup> squares labelled from 1 to 16:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

If we want to take 4 samples then we require 4 random numbers to eliminate the bias caused if we were to choose the squares to be sampled. Using the RANDOM button on a calculator or using the ‘=randbetween(1,16)’ function on a spread sheet program these can be obtained. Below is four random numbers generated by Excel:

16

7

7

6

Here 7 has been repeated so another random number has to be generated:

4

Therefore samples should be taken from squares 16, 7, 6 and 4. It should be noted that true random numbers are very difficult if not impossible to obtain. The random numbers generated in a list or from a spreadsheet are computed from an algorithm that means that for the purist they do not represent true randomness but are *pseudo-random*. In fact some mathematicians argue that there is no such thing as true randomness, even rolling a die can be predicted as we know the laws of physics can predict the outcome if we know all of the initial conditions.

### e) Understand simple probability

The fundamental misconception when dealing with probability is the idea of ‘randomness’. Unfortunately in the last couple of years the word ‘random’ has entered the vernacular as a term being something bizarre happening. Learners could confuse a random event as being a ‘rare’ event rather than the actual meaning. There are a number of different definitions for the word ‘random’ but to keep it simple emphasise to the learners that a random process doesn’t produce rare results but it produces results that are impossible to *predict*. Randomness is based on our inability to predict what will happen at a *given* moment of time. In genetics there is a probability of a given gene being passed on to the offspring, imagine that this particular characteristic has a probability of  $\frac{1}{4}$ . This means that if there are 4 offspring then *on average* 1 of them will inherit this particular characteristic. If there were 1200 offspring then *on average* 300 of them would have the characteristic. The global long-time behaviour of a random process can be predicted but for any given offspring it cannot be predicted that they inherit the characteristic. This can be modelled with a roll of a four-sided die with a 4 representing an inheritance and 1, 2, 3 representing no inheritance. In fact any random process can be represented by rolling a die.

### f) Understand the terms mean, mode and median

The mean, median and mode are all measures of *central tendency* of a data set. They act as a representative value for the whole data set. Some of them are better than others. Given a list of data these quantities are easy to calculate and well known to the learners but there are some subtleties. The list below represents some data:

25, 24, 27, 28, 19, 31, 25, 31

The mean has been mentioned in M1.2 and is the sum of the data values divided by how many and hence is:

$$\bar{x} = \frac{210}{8} = 26.25$$

To find the median, the data has to be first ordered in size:

19, 24, 25, 25, 27, 28, 31, 31

The median is the *middle* value or in a formal manner the  $\frac{n+1}{2}^{\text{th}}$  piece of data where  $n$  is the number of pieces of data. In this example there are 8 pieces of data and hence the median lies on the  $(8+1)/2^{\text{th}}$  piece of data which is the  $4.5^{\text{th}}$  piece of data. This doesn’t really make sense until you realise that the  $4.5^{\text{th}}$  data is halfway between the  $4^{\text{th}}$  and  $5^{\text{th}}$  items; 25 and 27 and hence the median (usually denoted by  $Q_2$ ) is:

$$Q_2=26$$

The mode is the easiest to spot and is the most ‘popular’ item of data, the most frequent item. In the above example there is no single most frequent item of data with 25 and 31 both occurring twice. The data set is therefore *bi-modal* with modes 25 and 31.

As a general rule of thumb the mean is the most useful statistical measure and it uses *all* the items of data. If there are outliers however then the median is more representative because it is less sensitive to outliers. If for example a data value of 100 was added above the mean would change to 34.4 but the median would move to 27, a far more representative measure.

The choice in which to use is dependent on the context. Mode is not often used with numerical data, but useful with categorical data (i.e. most common species in a sample). Mean is often quoted because it uses all the data, but it is very sensitive to outliers so median may be more useful.

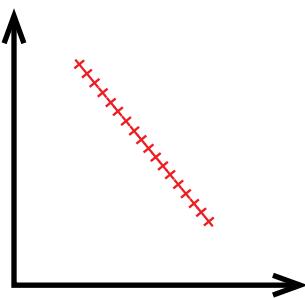
## Teaching activity

Resources – Activity 1 PowerPoint

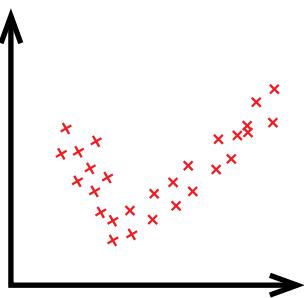
Display the set of numbers in the PowerPoint slide on the board at the beginning of the lesson. Learner's have to decide which the appropriate measure of average to use is. They do not have to calculate the mean/median/mode etc. although this could be incorporated into this activity. This could be used at the start of a lesson when calculating averages forms a significant part of the lesson.

### g) Use a scatter diagram to identify a correlation between two variables

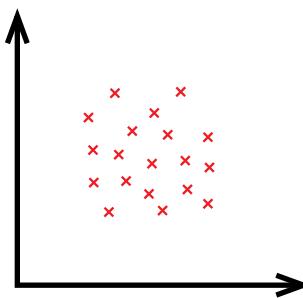
There are many types of correlation and in the absence of a mathematical procedure to calculate the correlation coefficient a lot of the interpretation comes down to judgement. The following graphs illustrate different types of correlation for two variables plotted against one another:



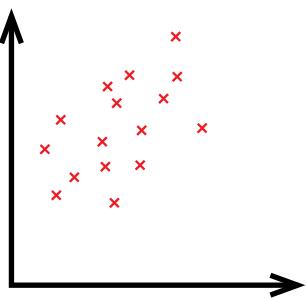
Perfect negative linear correlation



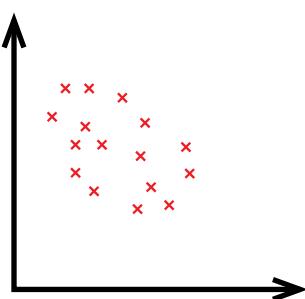
Quadratic correlation



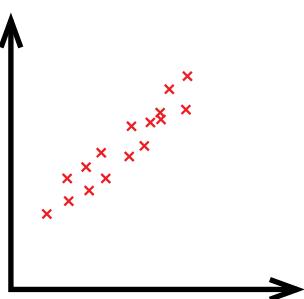
No correlation



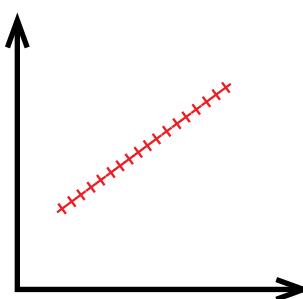
Weak positive linear correlation



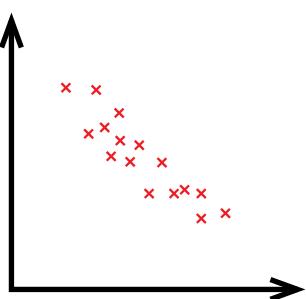
Weak negative linear correlation



Strong positive linear correlation



Perfect positive linear correlation



Strong negative linear correlation

A number of points need to be made. Just because the data is not in a straight-line does not mean that there is no correlation. From the second diagram above it is obvious that there is a strong quadratic correlation but extremely weak *linear* correlation. For the most part, linear correlation is what we are looking for most of the time. If both sets of data are *normally distributed* then the shape of scatter will be approximately elliptical in shape and for most biological variables this will be the case. Actually this is a condition for the correlation coefficient (see later section) to be used; that the data be normally distributed and in the absence of a formal mathematical test one can look at the data and see it is approximately an elliptical shape (like the 4<sup>th</sup> and 5<sup>th</sup> diagrams above).

An incredibly important point however is that *correlation does NOT necessarily imply causation*. Even if two variables display a high level of linear correlation it does not mean that there is a link between them. For example if the average heights and average weights of different mammals were plotted you would expect there to be a positive correlation; the taller the mammal, the heavier it is. This could reasonably be expected to be a causation; a tall animal will be heavier (in general) than a smaller one. However if for a month we measured the temperature at midday and also the number of cars passing by a particular street there could feasibly be a strong correlation between the two measurements despite there being no link to each other whatsoever. There may be a link of course but this will require more thought, the point to make is that a strong correlation does not necessarily mean the variables are linked or dependent on one another. Another good example to use is ice cream sales and burglaries. Data shows a strong positive correlation between these, the more ice creams sold the more burglaries occur, in fact there is a third variable, temperature, that may come into play.

### Teaching activity

Resources – Activity Sheet 2, mini-whiteboards

The purpose of this activity is to demonstrate the idea of correlation versus causation. This activity is to be used as a starter activity and as stimuli for a debate about what inferences can be made about correlation. There are a number of ways you can proceed with the activity. Hand out Activity Sheet 3 which has 54 different variables on it. In a spreadsheet create a random number generator by writing the formula =randbetween(1,54).

Generate two sets of random numbers. The first set of numbers corresponds to variable A and the second to variable B. Learners match the numbers to a variable on the sheet. On the mini-whiteboards learners then decide to write what kind of correlation there is (if any). The choices are:

Variable A implies Variable B

Variable B implies Variable A

Variable A implies Variable C implies Variable B

Variable B implies Variable D implies Variable A

No correlation

For the cases of indirect causation learners should be able to come up with Variables C and D which may cause the correlation. For example:

Variable A = Test score at English

Variable B = Amount of alcohol drunk

Here a learner could argue that B implies C implies A by saying that the more alcohol drunk means less sleep and hence lower concentration and hence a lower test score at English. Variable C in this case is amount of sleep. Learners show their mini-whiteboard with their answer on. Ask individual learners why they have made their choice. There is no correct answer to this and different learners may have different ideas. The aim of this activity is to give learners a grasp of the issues correlation may have and for them to think beyond the ‘numbers’ and think about the actual variables that are being measured. An interesting and entertaining debate will ensue. Awards could be made for the most imaginative causation between two incredibly obscurely linked variables.

## **h) Make order of magnitude calculations**

In scientific processes the scales can vary significantly. It is important to understand how many orders of magnitude one measurement is compared to another. For example, the width of a particular type of phytoplankton is 5  $\mu\text{m}$ , the length of a bee is approximately 1 cm and the height of a human is approximately 1.6 m. How can these different measures be compared? A useful way of quantifying this difference is by using orders of magnitude. The plankton is measured in micrometres and hence is  $10^{-6}\text{m}$ . The bee is measured in cm and hence is  $10^{-2}\text{m}$ . Humans are measured in metres which is  $10^0\text{m}$ . Therefore bees are 10 000 times larger than plankton and as  $10\ 000 = 10^4$  then bees are *four orders of magnitude larger* than plankton. Additionally humans are 100 times larger than bees, which mean that as  $100 = 10^2$  then bees are *two orders of magnitude* larger than the ant. One could say that the size of a bee to a plankton is larger than the size of a human to a bee by using this method of comparison.

## **Contexts in Science**

### **Sampling**

*Question:* A group of learners decide to use lichens to try and work out how polluted their school grounds are. They read about a scale called the Lichen Diversity Value (LDV). It is worked out in this way:

- choose four trees at random in the area
- hold a quadrat on the north side of the trunk of one tree
- count the total number of all the lichens in the quadrat
- then do this on the east, south and west side of the tree
- repeat this for each tree

Suggest how the learner could choose four trees at random.

*Answer:* To obtain true randomness, random numbers need to be generated. Assign a number to each tree and then use the random number button on a calculator to get four random numbers that represent the trees that are to be sampled.

## Mean

**Question:** Martha investigates how well different balls bounce. She drops different balls from the same height and measures the height the balls bounce. She repeats the experiment 3 times. Her results are shown in the table:

Ball	Drop Height (cm)	1st Reading bounce (cm)	2nd Reading bounce (cm)	3rd Reading bounce (cm)	Mean bounce (cm)
Blue	100	61	62	60	61
Green	100	60	31	59	50
White	100	84	86	85	85
Yellow	100	26	24		26

Marta missed one result for the yellow ball. Calculate the missing result.

**Answer:** To calculate the mean we add up all the data and divide by how many. Let's call the missing piece of data  $x$ . Therefore we have

$$\frac{26 + 24 + x}{3} = 26$$

To find  $x$  we have to rearrange this formula. First we have to multiply the mean by 3:

$$26 + x = 3 \times 26 = 78$$

and finally subtract (26+24)

$$x = 78 - 50 = 28$$

and the missing number is 28.

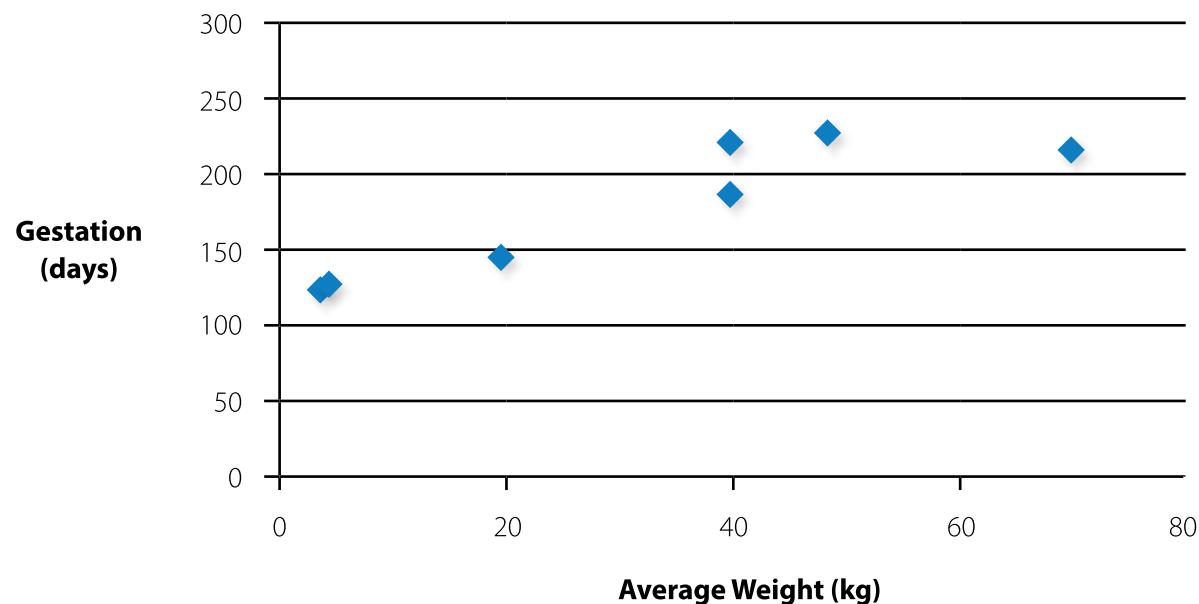
## Correlation

*Question:* The following table shows average gestation periods and average female weight of seven different primates:

	Gestation (days)	Average female size (kg)
human	266	50
chimpanzee	227	40
gorilla	257	70
orangutan	260	40
baboon	187	20
monkey, Rhesus	164	5
monkey, Patas	167	5.5

Source: <http://www.sjsu.edu/faculty/watkins/gestation.htm>

These are plotted on a scatter graph by Priya:



Describe the relationship between Gestation period and Average Weight and suggest a possible reason why this is so.

*Answer:* There appears to be a positive correlation as the data appear to follow a positive line. Therefore one could make a conjecture that the heavier the species then the longer the gestation period because the larger the animal the longer it has to be developed in the womb.

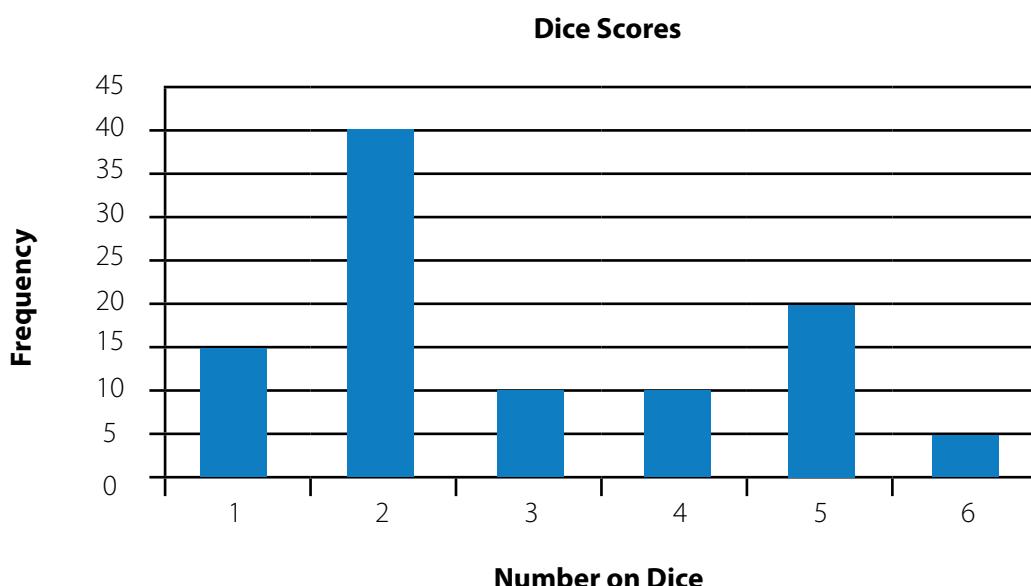
# Topic Check In 2

## Questions

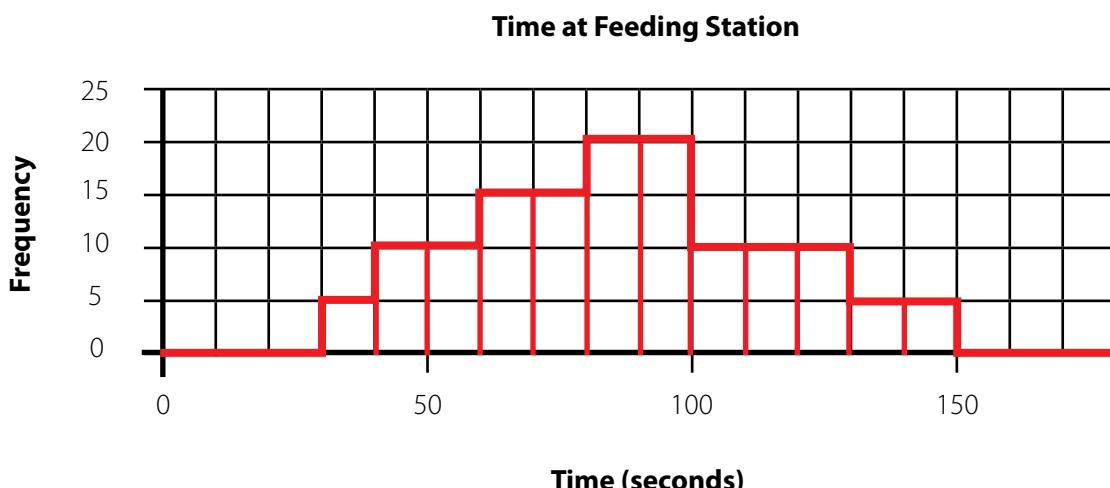
- Round 1 532.2364 to 3 significant figures.
- Find the mode, median and mean of this data set?

2      6      3      11      7      3      8      4      1

- The bar chart shows the results of a rolled dice. How many times was it rolled?



- The histogram shows the time in seconds that individual birds spend at a feeding station. How many birds spend less than 1 minute at the feeding station.



5. Rashid's school has 1500 learners. He wants to give a questionnaire to a sample of 5% of the school learners. How many learners should be in the sample?
6. Research has indicated that there is a probability of  $\frac{3}{4}$  that parents will have a baby born with brown eyes. How many brown eyed babies would you expect in a sample of 300 babies?
7. An experiment finds that using thicker insulation material on a box increases the amount of time for the temperature of the box to cool from 50°C to room temperature. Sketch the points you might see on a results graph of an experiment comparing insulation thickness to time taken for the box to cool.
8. A sunflower measuring 4 mm grows to 40 cm over 4 weeks. Explain in words how many times bigger the sunflower is after the 4 weeks.
9. The mass of one molecule of water is  $2.99 \times 10^{-23}$  g. Estimate how many molecules of water there are in 1 g of water.
10. Tom measures the expansion of a brass rod when heated. His 5 results are recorded below.

0.738	0.751	0.726	0.733	0.729
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Calculate the average expansion of the brass rod for Tom's experiment.

## Answers

1. 5320
2. Mode 3, Median 4, Mean 5
3. 100
4. 25 birds
5. 75
6. 225 babies
7. Points plotted showing positive correlation
8. 100 times bigger
9.  $3.34 \times 10^{22}$  molecules
10. 0.735

# Chapter 2 RAG

Question	Topic	R	A	G
1	Able to round values to stated significant figures			
2	Able to calculate mode, median and mean averages from a data list			
3	Able to interpret bar charts			
4	Able to interpret histograms			
5	Able to calculate the number of candidates for a stratified random sample			
6	Able to use probability to determine expected outcomes			
7	Able to recognise types of correlation			
8	Able to determine orders of magnitude			
9	Able to use standard form			
10	Able to calculate average for experimental data			

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# Chapter 3 : Algebra

The Science specifications state in appendix 5f that learners are expected to:

- Understand and use the symbols:  $=$ ,  $<$ ,  $<<$ ,  $>>$ ,  $>$ ,  $\propto$ ,  $\sim$
- Change the subject of an equation
- Substitute numerical values into algebraic equations using appropriate units for physical quantities
- Solve simple algebraic equations

## Prior Key Stage 3 learning

Learners should be able to:

- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use standard mathematical formulae; rearrange formulae to change the subject

## Mathematical skills

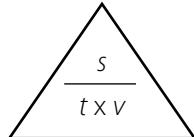
### a) Understand and use the symbols

$=$ ,  $<$ ,  $<<$ ,  $>>$ ,  $>$ ,  $\propto$ ,  $\sim$

Learners should be aware and familiar with the expressions  $=$ ,  $<$ ,  $>$  and know when to use them from Key Stage 3 mathematics. The symbols  $<<$ ,  $>>$  and  $\sim$  will be less well understood however. Take  $a = 5$ ,  $b = 6$ ,  $c = 0.1$  and  $d = 110$ . It is obvious that  $c < a < b < d$  but if these represent distances in mm then what is more important is their relative size to each other. Both  $a$  and  $b$  are quite close to each other (technically they are of the same order) so as an approximation we can say  $a \sim b$ . However  $c$  is a lot smaller than the rest so we can write  $c << a, b, d$ . Also as  $d$  is a lot larger then we can write  $d >> a, b, c$ .

### b) Change the subject of an equation

There are a number of techniques for rearranging formulae that are taught at Key Stage 3. Each learner will have their preferred method and if their method works then this should be encouraged. For simple formulae like speed = distance  $\div$  time ( $v = s/t$ ) a formula triangle can be used. In fact this is a good starting point for learners to start practising rearranging equations as there a lot of examples in Science where a simple relationship like this exists. We write the formula in a triangle



If they want to find the time taken they physically cover up  $t$  in the triangle and going from left to right can read the formula as

$$t = \frac{s}{v}$$

If they want to find the distance  $s$ , they can cover it up and what is left is  $t \times v$  and hence

$$s = t \times v$$

There are many examples of relationships like this, for example  $\text{density} = \text{mass}/\text{volume}$  or  $\text{current} \times \text{resistance} = \text{voltage}$  which can all be treated in the same way.

## Teaching activity

At the beginning of the term or at an appropriate time get learners to draw all of the formula triangles in the back of their books so they can quickly reference them when they are needed. Examples include the three above but also more obscure formulae like

$$\text{Percentage Change} = \text{Actual Change} \div \text{Original}$$

A more complicated example in Physics is one of the SUVAT equation for velocity

$$v = u + at$$

To change the subject to  $t$ , it should be noted that first  $u$  on the LHS has to be ‘unlocked’ before  $t$  has any chance of ‘being on its own’. The opposite and adding  $u$  is subtracting it (remembering as it is a formula it has to be done on both sides):

$$v - u = at$$

The final part is to see that  $t$  is still ‘locked’ by  $a$ . Therefore the opposite is division by  $a$  and hence:

$$t = \frac{v - u}{a}$$

and the subject has been changed.

An example of a non-linear equation to rearrange is Einstein’s famous  $E = mc^2$  equation relating energy and mass. Rearranging this for  $c$  means first of all that we have to divide by  $m$  to get the  $c$  part by itself. Notice the square isn’t applied to the  $m$ , otherwise it would have been written as  $E = (mc)^2$  or  $m^2c^2$ . Therefore:

$$\frac{E}{m} = c^2$$

finally the  $c$  is ‘locked’ into a square so to ‘undo’ the square we have to square root both sides:

$$c = \sqrt{\frac{E}{m}}$$

and the subject has been changed.

### c) Substitute numerical values into algebraic equations using appropriate units for physical activities

Learners should be aware of the principles from Key Stage 3 Maths but a few misconceptions may remain. The most common problem is their dealing with powers and negative quantities in formulae. The expression  $x^2$  for example, whilst innocuous enough can cause issues when a negative number is substituted. Substituting  $x = -2$  for example should be calculated as  $(-2)^2 = 4$  not  $-2^2 = -4$ . These problems can come to the fore in formulae where powers occur such as the inverse square law, kinetic energy and SUVAT equations.

Additionally in the context of a formula such as  $p = mv$  learners should be aware that the mass is being *multiplied* to the velocity despite the absence of a multiplication sign. In general the laws of BIDMAS should be adhered to where the operations should be completed in the order of Brackets, Indices, Division, Multiplication, Addition and Subtraction.

The formula for linear momentum is given by  $p = mv$  where  $m$  is the mass and  $v$  is the velocity. If the mass of an object is 2kg and the velocity is 4m/s then the linear momentum is calculated as:

$$p = mv$$

$$p = 2 \times 4 = 8 \text{ kg m/s}$$

Substituting values into expressions involving powers and fractions:

A SUVAT equation after rearrangement can be written as;

$$a = \frac{v^2 - u^2}{2s}$$

and substituting the following to find  $a$ ;  $v = 5$ ,  $u = -4$  and  $s = 2$ . The numerator and denominator have to be calculated separately first for although there are no brackets present, it is implied that there are brackets for each part of the fraction hence:

$$a = \frac{(5^2 - (-4)^2)}{2 \times 2} = \frac{25 - 16}{4} = \frac{9}{4} = 2.25 \text{ m/s}^2$$

NB: Natural Display calculators allow these substitutions to be entered in fraction mode, avoiding any issues.

#### d) Solve simple algebraic equations

Learner's will certainly be familiar with solving simple linear equations from Key Stage 3. A strategy is treat them like formulae and rearrange the formula to find the unknown. The difference here is that there is only one 'letter' so a lot of the work is numerical.

Solving an equation usually involves substituting values into a formulae and realising that there is one unknown unaccounted for. Finding the value of this unknown is the same as solving the equation. Take for example the formula:

$$E = U + pV$$

If we were to substitute some values in for  $E$ ,  $U$  and  $p$  ( $E = 7$ ,  $U = 2$ ,  $p = 3$ ) the formula becomes an equation for  $V$ ; the only variable which remains unknown:

$$7 = 2 + 3V$$

To find  $V$  we have to 'unlock' what is happening to  $V$ . By this we mean we have to 'undo' the operations that are linking  $V$  to the other numbers. First we subtract the 2 from both sides to get the  $3V$  by 'itself':

$$5 = 3V$$

To 'undo' the multiplication by 3 we divide by 3 and solve the equation:

$$V = \frac{5}{3}$$

Using the formula for magnification, if for example the magnification factor is known as 3.4 and the image size is known as 10cm we can solve the equation to find the object size:

$$\text{Magnification} = \frac{\text{Image size}}{\text{Object size}}$$

Substituting these values in:

$$3.4 = \frac{10}{\text{Object size}}$$

Multiplying both sides by the object size:

$$3.4 \times \text{Object size} = 10$$

Then dividing by 3.4:

$$\text{Object size} = \frac{10}{3.4} = 2.9 \text{ cm}$$

Hence the object size is 2.9cm

For the higher tier quadratic equations may need to be solved; although there are methods for solving these easily (by factorising/completing the square/graphical) a useful technique in the context of science is to use the formula. Take the following problem. A particle is projected from ground-level vertically upwards with an initial velocity of m/s. How long does it take to first reach 10 m?

Initially recognising the correct equation as  $s = ut + \frac{1}{2} at^2$  and then substituting the values in ( $a = -g = -9.8 \text{ m/s}^2$ ):

$$10 = 5t - 4.9t^2$$

This is a quadratic equation as the highest power of the unknown is 2. This is not in a suitable form to perform the formula. First we 'set to zero' collecting all terms on one side usually with the  $t^2$  as positive (it makes the calculations a little simpler). So putting all the terms on the LHS:

$$4.9t^2 - 5t + 10 = 0$$

The quadratic formula is:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where  $a$  is the number in front of the square term,  $b$  is the number in front of the linear term and  $c$  is the constant, hence in this example  $a = 4.9$ ,  $b = -5$ ,  $c = 10$ . Hence substituting these in:

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4.9)(10)}}{2(4.9)}$$

the brackets with the negative numbers are necessary here to avoid any errors involving negative numbers. A first stage of calculation is:

$$t = \frac{5 \pm \sqrt{(25 - 196)}}{9.8}$$

next

$$t = \frac{5 \pm \sqrt{(-171)}}{9.8}$$

Learners might be tempted to ignore the negative inside the square root but this is a BIG mistake. The equation does not have any real solutions and a learner may be tempted to conclude that their calculations are incorrect. This is not the case! There are no solutions because in the context of the original problem the particle doesn't reach the height of 10 m. The initial velocity of 5 m/s wasn't enough to project it to higher than 10 m. Instead try an initial velocity of 20 m/s. The equation is then:

$$4.9t^2 - 20t + 10 = 0$$

and applying the formula:

$$t = \frac{20 \pm \sqrt{(400 - 196)}}{9.8}$$

$$t = \frac{20 \pm \sqrt{204}}{9.8}$$

$$t = \frac{20 \pm 14.3}{9.8}$$

There are two answers here one is

$$t = \frac{20 + 14.3}{9.8} = 3.5\text{s}$$

and

$$t = \frac{20 - 14.3}{9.8} = 0.58\text{s}$$

Which is correct? The answer is both of course but the problem stated when the particle *first* reached 10 m and hence only 0.58 is correct to the problem! Notice all values have been taken to 2 sig fig as this was the level of accuracy g was quoted to. Many commonly used GCSE Maths calculators have equation solving functions, both for linear equations and quadratic equations.

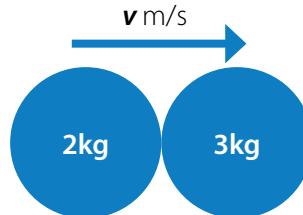
## Contexts in Science

### Momentum

*Question:* A ball of mass 2 kg travels at a velocity of 7 m/s. Another ball of mass 3 kg travels at a velocity of 2 m/s in the same direction as shown in the diagram



The balls collide and stick together.  $v$  is the velocity of both balls together.



What is the value of  $v$ ?

*Answer:* Applying the law of conservation of momentum in elastic collisions we know that the momentum before the collision must equal the momentum afterwards. The formula for momentum is  $p = mv$  where  $m$  is the mass and  $v$  the velocity. Therefore before the collision we have

$$P_{\text{before}} = (2 \times 7) + (3 \times 2) = 20 \text{ Nm}$$

and afterwards

$$P_{\text{after}} = (2 + 3)v$$

We can form an equation to solve for  $v$  by equating the momentum before and after:

$$5v = 20$$

This simple linear equation can be solved by ‘undoing’ the multiplication by 5 by dividing both sides by 5 and hence

$$v = 20 \div 5 = 4 \text{ m/s}$$

## More calculations

This is the area where learners will most frequently be required to use equations in different arrangements. Key equations that learners need to be able to manipulate include:

$$\text{amount of substance} = \frac{\text{mass}}{\text{molar mass}} \quad (n = \frac{m}{M})$$

$$\text{amount of substance} = \frac{\text{mass}}{\text{molar gas mass}}$$

$$\text{amount of substance} = \text{concentration} \times \text{volume} \quad (n = cV)$$

Longer, unstructured calculations may require using one or more of these equations multiple times, in different arrangements.

Learners who are able to grasp the mathematical principle of rearranging equations in this early stage of the course, as opposed to learning the different arrangements of each equation individually, will be able to apply this skill more confidently in other areas.

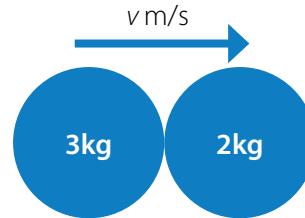
# Topic Check In 3

## Questions

1. Use the symbols  $<$ ,  $>$ ,  $<<$ ,  $>>$ ,  $=$  and  $\approx$  between the following sets of numbers

A	5		12		20
B	4		6		70
C	$\frac{1}{2}$		0.5		1
D	78.9		6.3		5.9

2. Given  $v^2 = u^2 + 2as$  find  $v$  when  $u = 10$ ,  $a = 2$  and  $s = 75$
3. Rearrange  $s = ut + \frac{1}{2}at^2$  to make  $u$  the subject
4. Use  $E = mc^2$  to find an approximation of  $c$  if the total mass before the fission reaction is  $392 \times 10^{-27}$  kg and the total mass after the fission reaction is  $391 \times 10^{-27}$  kg and the energy released is  $8.7 \times 10^{-11}$  J.
5. Calculate the magnification factor if the object size is 0.002 mm and the image size is 2 mm.
6. Calculate the mass of 2 mol of Carbon Dioxide [given  $A_r$  of C = 12,  $A_r$  of O = 16].
7. The flight distance from London to New York is 5576 km and it takes 6.4 hours. Show that an aeroplane flying from London to New York has  $v \sim 900$  km h $^{-1}$ .
8. A ball of mass 3 kg travels at a velocity of 7 m/s towards another ball of mass 2 kg which is stationary. After the collision they travel together at the same velocity  $v$ . Calculate the new velocity  $v$ .



9. A particle is projected from ground level vertically upwards with an initial velocity of 15 m/s. Using  $a = -g = -9.8$  m/s estimate how long the particle remains 5m above the ground.
10. A car is stopped at a traffic light. As the light goes green, a cyclist passes the car at 6 m/s. The car immediately accelerates at 4 m/s. When does the car pass the cyclist?

**Answers**

1.

A	5	<	12	<	20
B	4	<	6	<<	70
C	$\frac{1}{2}$	=	0.5	<	1
D	78.9	>>	6.3	>	5.9

2.  $v = \sqrt{10^2 + 2 \times 2 \times 75} = 20$

3.  $u = \frac{s - \frac{1}{2}at^2}{t}$  or  $u = \frac{2s - at^2}{2t}$

4.  $c = \sqrt{\frac{8.7 \times 10^{-11}}{392 \times 10^{-23} - 391 \times 10^{-23}}} \approx 2.9 \times 10^8$

5.  $Magnification = \frac{2}{0.002} = 1000$

6.  $Mass = (12 + 16 + 16) \times 2 = 88\text{g}$

7.  $v = \frac{5576}{6.4} = 871.25\dots \sim 900\text{kmh}^{-1}$

8.  $7 \times 3 = 5v$

$v = \frac{21}{5} = 4.2\text{ ms}^{-1}$

9.  $5 = 15t + \frac{1}{2} \times (-9.8) \times t^2$

$4.9t^2 - 15t + 5 = 0$

$t = 0.38, t = 2.68$

Time above 5 m is 2.3 s

10. car  $s = 0 + \frac{1}{2} \times 4 \times t^2$

cyclist  $s = 6t + 0$

when they are at the same place  $\frac{1}{2} \times 4 \times t^2 = 6t$

 $2t^2 - 6t = 0, t = 0$  or  $t = 3$ . Car passes the cyclist after 3 s, 18 m from traffic lights.

# Chapter 3 RAG

Question	Topic	R	A	G
1	Able to use inequality symbols			
2	Able to substitute values into formulae			
3	Able to change the subject of formulae			
4	Able to determine the value of an unknown by rearrangement and substitution			
5	Able to use magnification formula			
6	Able to use formula for molecular mass			
7	Able to give the solution from a formula to an appropriate degree of accuracy			
8	Able to set up and solve an equation for the conservation of momentum			
9	Able to set up and solve a quadratic equation in context			
10	Able to use equations of motion to solve problems			

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# Chapter 4 : Graphs

The Science specifications state in appendix 5f that learners are expected to:

- a. Translate information between graphical and numeric forms
- b. Understand that  $y = mx + c$  represents a linear relationship
- c. Plot two variables from experimental or other data
- d. Determine the slope and intercept of a linear graph
- e. Draw and use the slope of a tangent to a curve as a measure of rate of change
- f. Understand the physical significance of area between a curve and the  $x$ -axis and measure it by counting squares as appropriate

## Prior Key Stage 3 learning

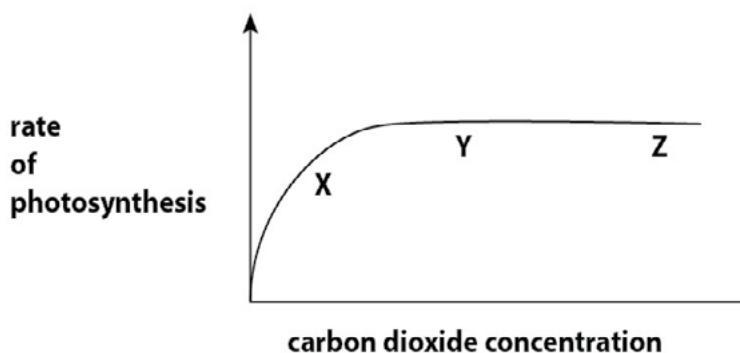
Learners should be able to:

- work with coordinates in all four quadrants recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in  $x$  and  $y$  and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form  $y = mx + c$ ; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of  $y$  for given values of  $x$  and vice versa and to find approximate solutions of simultaneous linear equations
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piecewise linear, exponential and reciprocal graphs

## Mathematical skills

### a) Translate information between graphical and numeric forms

A key feature of any line graph displaying a relationship between 2 variables is that the gradient of the curve represents the rate at which a quantity is changing; the steeper the curve the more quickly ‘something’ is changing and vice versa; the shallower the curve the more slowly the ‘something’ is changing. For example in a Biology context the following graph shows the effect of carbon dioxide concentration on the rate of photosynthesis:



From the beginning to point **X** the gradient is steep but getting shallower meaning that the rate of photosynthesis is ‘slowing down’. From point **Y** onwards the graph is horizontal and hence the rate of photosynthesis is staying the same. These principles can be applied to any line graph in most contexts.

### b) Understand that $y = mx + c$ represents a linear relationship

Learners should be expected to know:

- A linear relationship can be written as  $y = mx + c$
- $m$  is the gradient of the line
- $c$  is the  $y$  intercept of the line

This concept is a two-way process. Learners should be able to work out a relationship given a graph and learners should be able to sketch a graph given a linear relationship. The former is dealt with in the next sections but the latter is explained now.

Learners should understand that a positive  $m$  represents a line going ‘up’ from left to right and a negative  $m$  a line going ‘down’ from left to right. When sketching the graph they should always start from the  $y$ -intercept and then use the gradient to determine another point on the graph. Once this extra point has been determined the line can be drawn as it is only necessary for 2 points to be known to draw a straight line.

In the context of SUVATS the relationship between the velocity and time of an object travelling under uniform acceleration is given by:

$$v = u + at$$

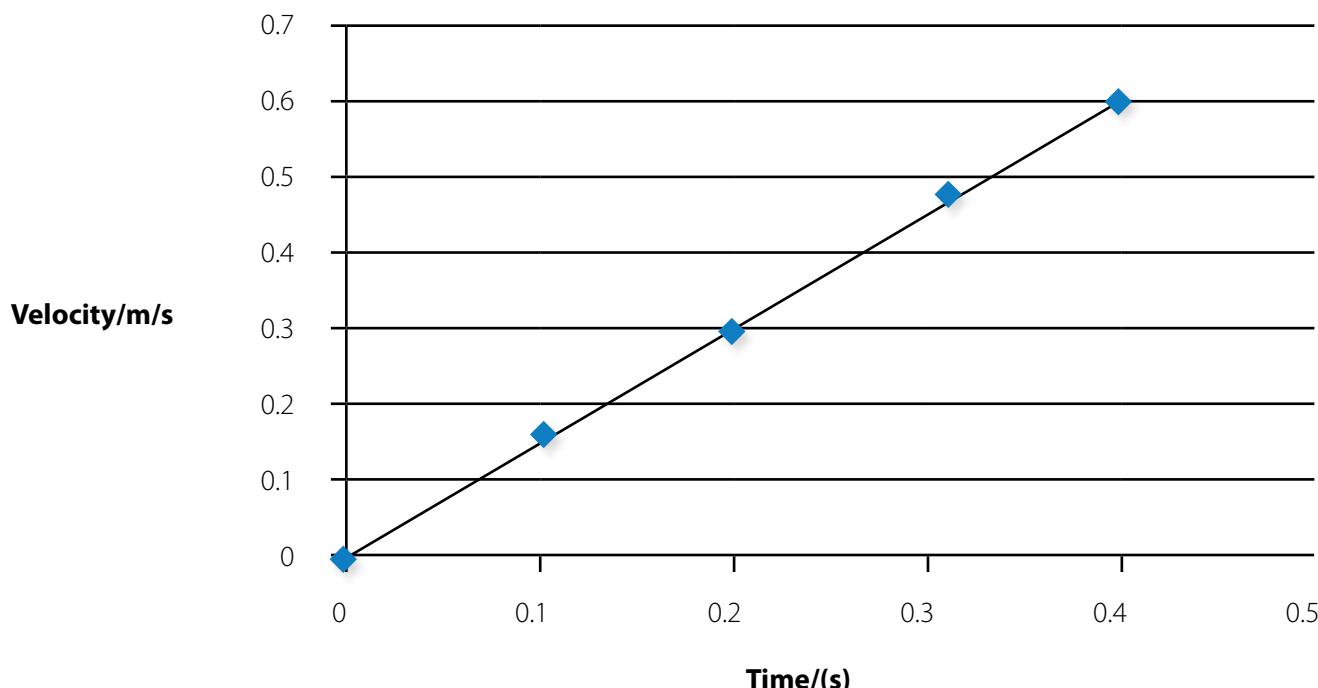
This may not at first be considered as a linear graph but by re-ordering the terms on the left hand side to give:

$$v = at + u$$

the learner can recognise this as in the form  $y = mx + c$  but with  $v$  and  $t$  representing  $y$  and  $x$  respectively and  $a$  representing the gradient and  $u$  representing the initial velocity. Imagine that the particular relationship is

$$v = 0.2t + 3$$

To sketch the graph (accurately in fact) start from the  $y$  intercept in this case (0,3). The gradient of 0.2 means that for every horizontal step of length 1 the graph moves up 0.2. Hence another point on the graph is (1,3.2). This point can then be plotted and the line can be drawn:



For the relationship  $v = -1.1t + 3$  the negative gradient means that for every positive horizontal step of size 1 the vertical step is -1.1 hence another point on the graph is (1, 1.9) as well as the y intercept (0, 3).

### c) Plot two variables from experimental or other data

Plotting a graph on paper requires little mathematical knowledge but is more of an exercise in steady-hand movement and knowledge of the conventions. Rulers should be used and consideration of the scale of the graph should be taken into account *before* a ruler is used to draw the axes. It is important that graphs are drawn to take up as much of the graph paper as possible. Axes need labels in the form Quantity/Units and a title for the graph is necessary to ensure the reader understands what is being graphed.

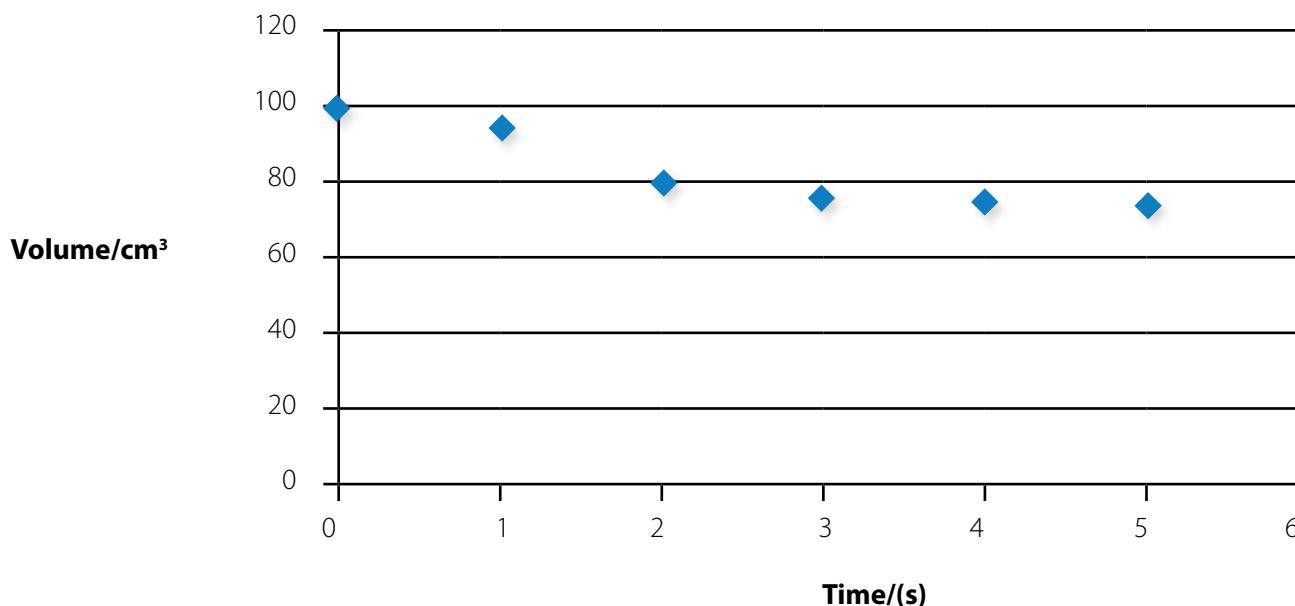
When connecting individual data points it is important that a *smooth* line is drawn between the points so consideration of the shape of the overall points is necessary. Students should make an estimate of the shape before connecting the data points.

If any of the variables do not start from 0 a zig-zig line should *not* be drawn, rather the scale starts at a sensible starting value.

Take for example the following experimental data taken from a reaction experiment:

	A	B	C
1	Time	Volume	
2	0	100	
3	1	96	
4	2	80	
5	3	75	
6	4	73	
7	5	72	
8			
9			

The graph can then be made using Excel or on paper:



#### d) Determine the slope and intercept of a linear graph

Learners should be able to:

- Find the y-intercept of a linear graph
- Find the gradient of a linear graph

Finding the y-intercept should be self-explanatory. Learners examine where the line crosses the y-axis and this value is the y-intercept. To find the gradient the following formula is useful:

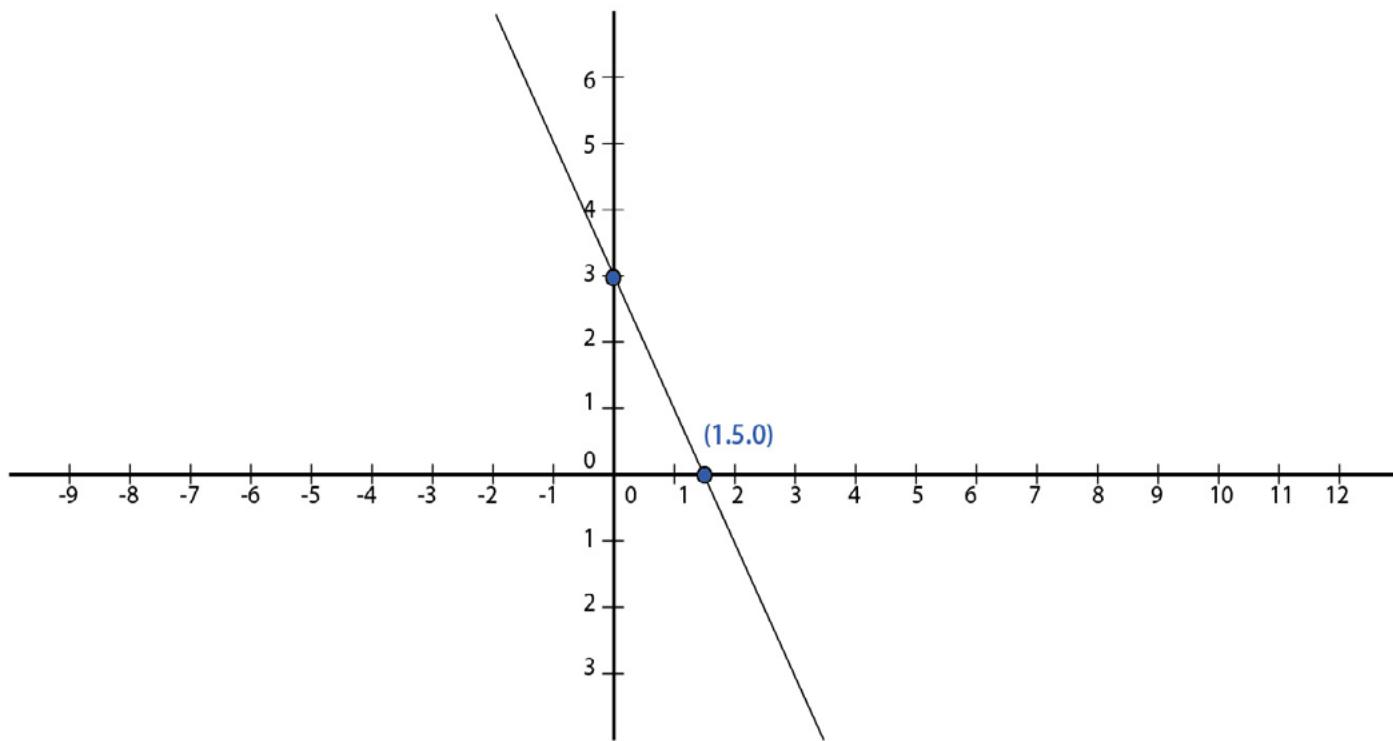
$$\text{Gradient} = \frac{\text{'Rise'}}{\text{'Run'}}$$

The ‘Rise’ represents the vertical step between 2 points and the ‘Run’ represents the horizontal step. Both of these quantities could be negative and care has to be taken in these cases. The principle is that we take two points on the line and choose a ‘starting point’. From this point, measure the horizontal distance to the other point (‘run’) and the vertical distance to the other point (‘rise’) and then perform the division to find the gradient.

The gradient is always a measure of the rate of change between the two variables. The gradient of a graph  $y=mx+c$  basically measures the rate of change of  $y$  with respect to  $x$ , in words how quickly  $y$  changes as  $x$  changes. A positive gradient means a quantity that increases as  $x$  increases whilst a negative gradient is a decreasing quantity as  $x$  increases.

In Physics we can use this to find the initial velocity and acceleration from a constant acceleration graph

Examine the following velocity –time graph representing an object moving under constant acceleration:



The relationship between velocity and time is of the form

$$v = u + at$$

and hence the initial velocity is the y-intercept and the acceleration is the gradient.

The y-intercept is clearly  $(0, 3)$  hence the initial velocity is  $3 \text{ ms}^{-1}$ . To find the gradient and hence the acceleration choose

another a point on the line. The obvious choice is the point on the x intercept ( $1.5, 0$ ).

The ‘Rise’ is  $-3$  as to get from  $(0, 3)$  to  $(1.5, 0)$  requires a ‘drop’ of  $3$  and hence a rise of  $-3$ . The ‘Run’ is  $1.5$  as to get from  $(0, 3)$  to  $(1.5, 0)$  requires a ‘run’ of  $1.5$ . The acceleration can be calculated as:

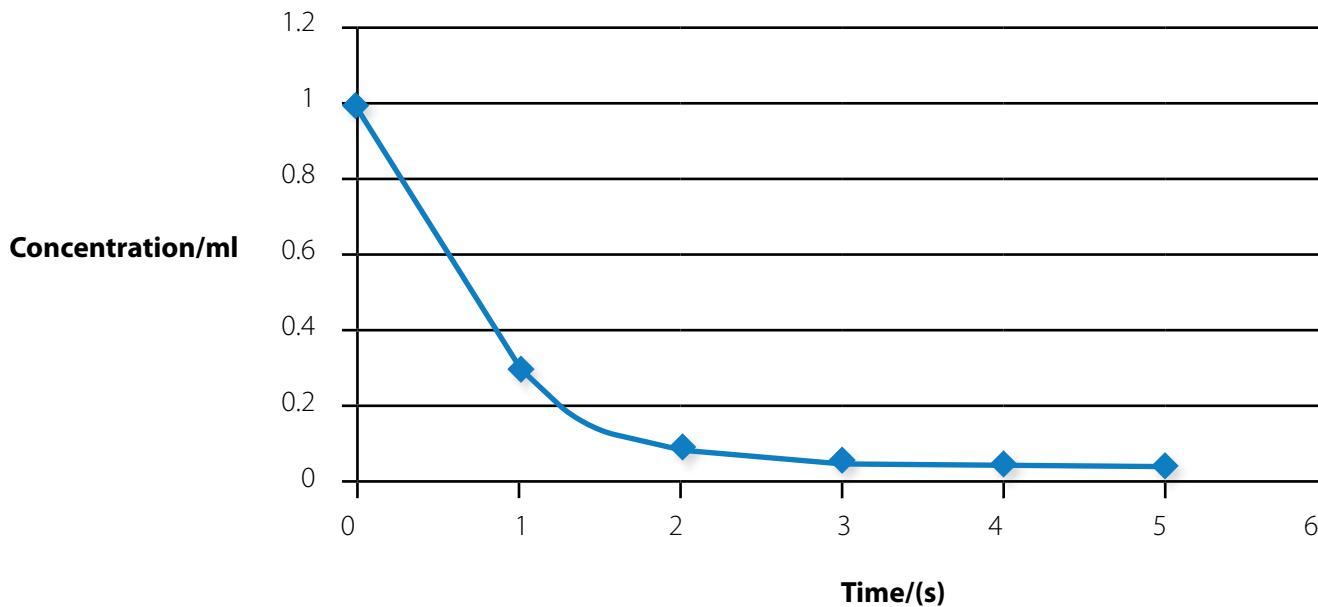
$$\text{Acceleration} = \frac{-3}{1.5} = -2 \text{ ms}^{-2}$$

For example Hooke’s law states that  $F = kx$ . In a graph of  $F$  against  $t$ ,  $k$  represents the gradient and is also the rate of change of  $F$  with respect to  $x$ . In words, how quickly  $F$  changes as  $x$  changes. This is  $y = mx + c$  where  $c$  is equal to zero, this represents a direct proportionality.

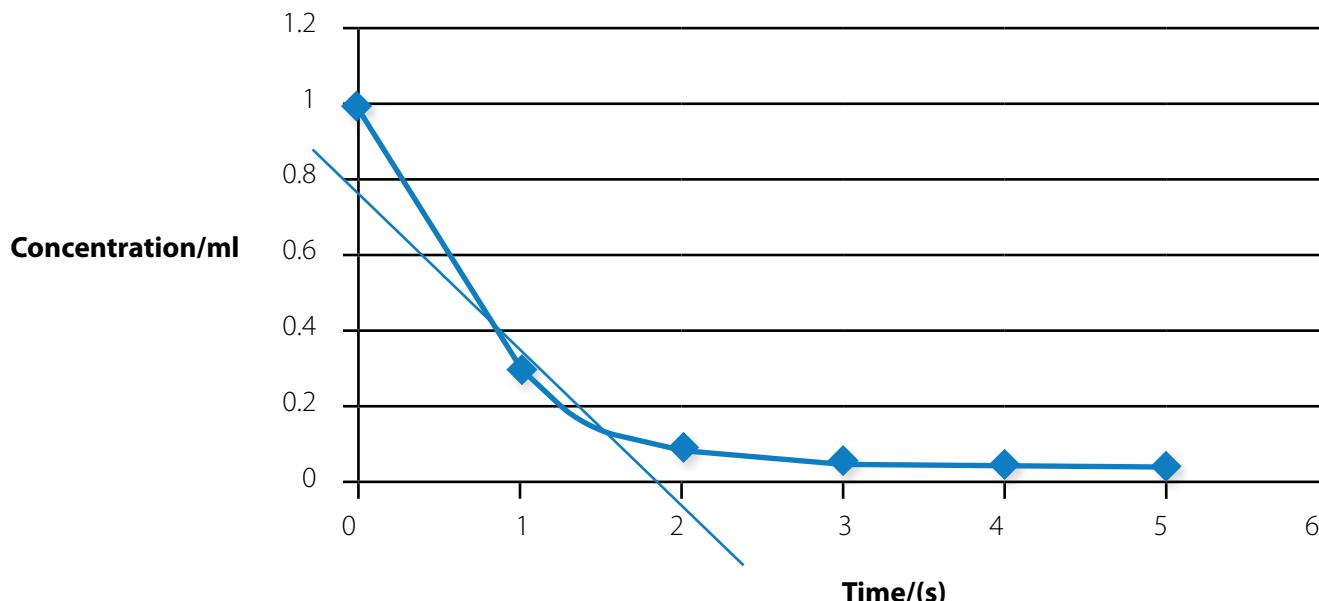
### e) Draw and use the slope of a tangent to a curve as a measure of rate of change

For linear graphs the gradient is the same throughout and hence the rate of change is easy to accurately obtain (see previous sections). For non-linear graphs in the absence of an application of Calculus a tangent has to be drawn by hand and eye to approximate the instantaneous rate of change. Non-linear graphs have an ever increasing gradient and hence the rate of change will change from point to point (unlike linear graphs where the rate of change remains constant). Therefore to find a rate of change at a given point requires some approximation using a tangent (or chord).

Below is a typical graph for a reaction displaying a concentration against time:



To find the rate of reaction at an *instantaneous* moment in time a tangent has to be drawn to approximate the gradient (which represents the rate) at the given time. To approximate the rate at  $t = 1$  a tangent has to be drawn at the point where  $t = 1$ :



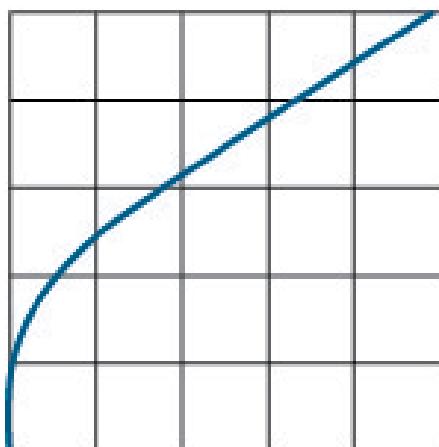
To find the gradient of the tangent two points have to be found on it. From the graph two such points that make the calculations easier are the x and y intercepts which can approximately be read as (1.8,0) and (0,0.7) respectively. The 'rise-run' calculation then becomes:

$$\text{Rate} = \frac{0 - 0.7}{1.8 - 0} = -\frac{0.7}{1.8} \sim 0.39$$

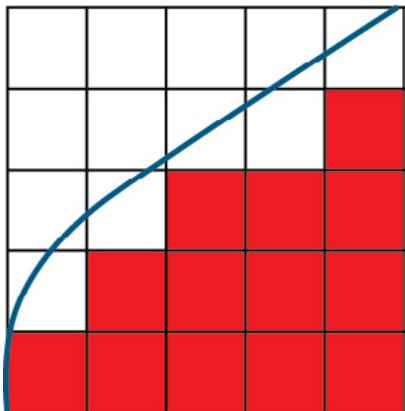
and hence the instantaneous rate of reaction at  $t = 1$  is 0.39.

**f) Understand the physical significance of area between a curve and the x-axis and measure it by counting squares as appropriate**

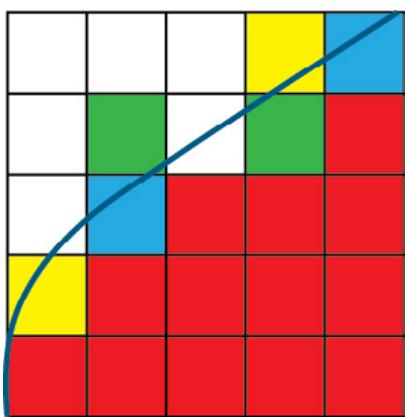
The most common example where this will be examined will be in interpreting the area under the graph of a velocity-time graph which is the distance travelled. If the graph consists of straight line segments (ie periods of constant acceleration) then the area can be calculated exactly through knowledge of areas of rectangles and triangles (see Chapter 5). However if the acceleration is not constant then the graph will be a curve and without resorting to Calculus techniques (not taught even in A Level Science) then the best way to estimate this area is by counting the squares (if the graph is printing on a grid!). This should be relatively straightforward for a learner. The key point to remember to count  $\frac{1}{2}$  squares and not 'pretend' they are one square. For example the area underneath the curve below is to be estimated.



First full squares are counted (in red):



There are 13 full squares. To make the best estimate possible we match the half squares as best we can to make a full one. This is done below with matching colours representing a full square pair:

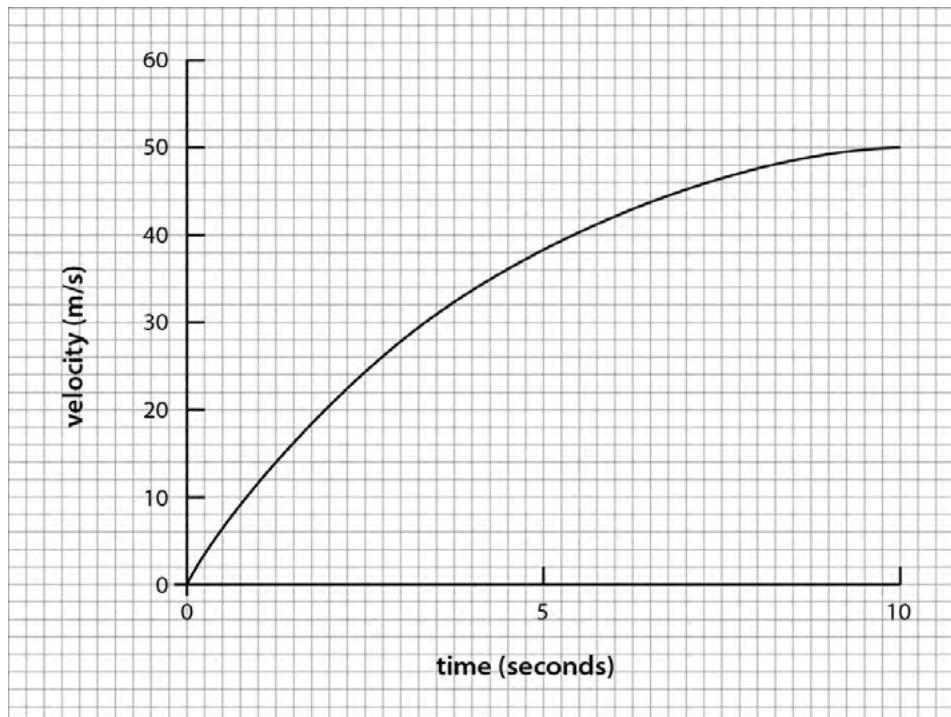


There are 3 pairs and one  $\frac{1}{2}$  square which cannot be matched to another. Hence a good estimate for the area is  $13+3+\frac{1}{2} = 16.5$  squares.

## Contexts in Science

### Velocity-Time Graphs

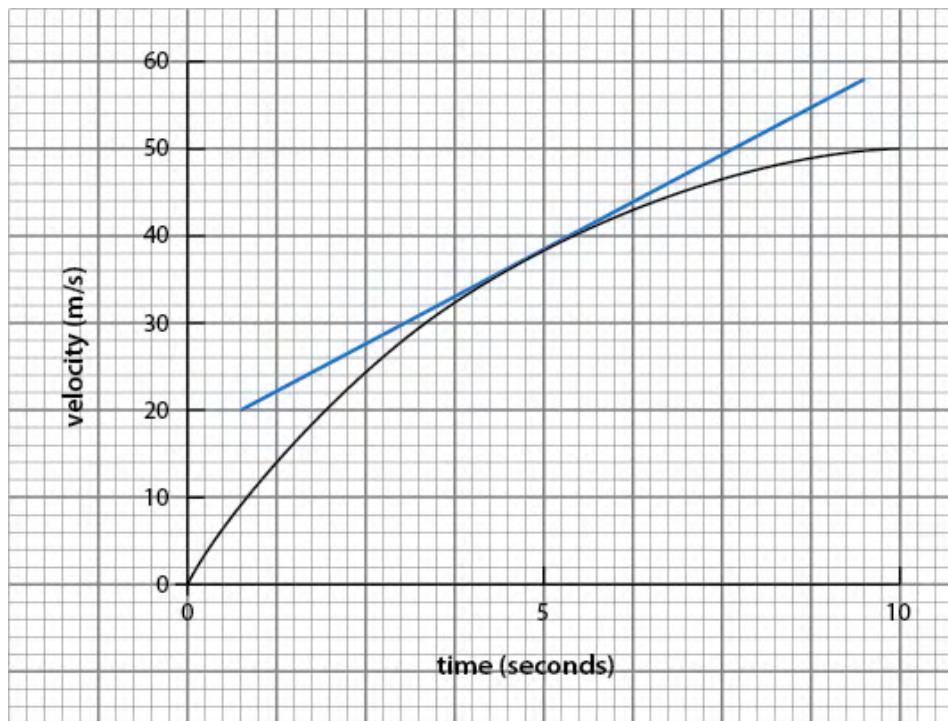
A free-fall skydiver falls from a plane and reaches terminal velocity after 15 seconds. The graph of motion is shown below:



- Use the graph to find the acceleration at 5 seconds
- Use the graph to find the distance travelled between 0 and 10 seconds

#### Answer

- To estimate the acceleration at 5 seconds a tangent has to be drawn at that point; see below

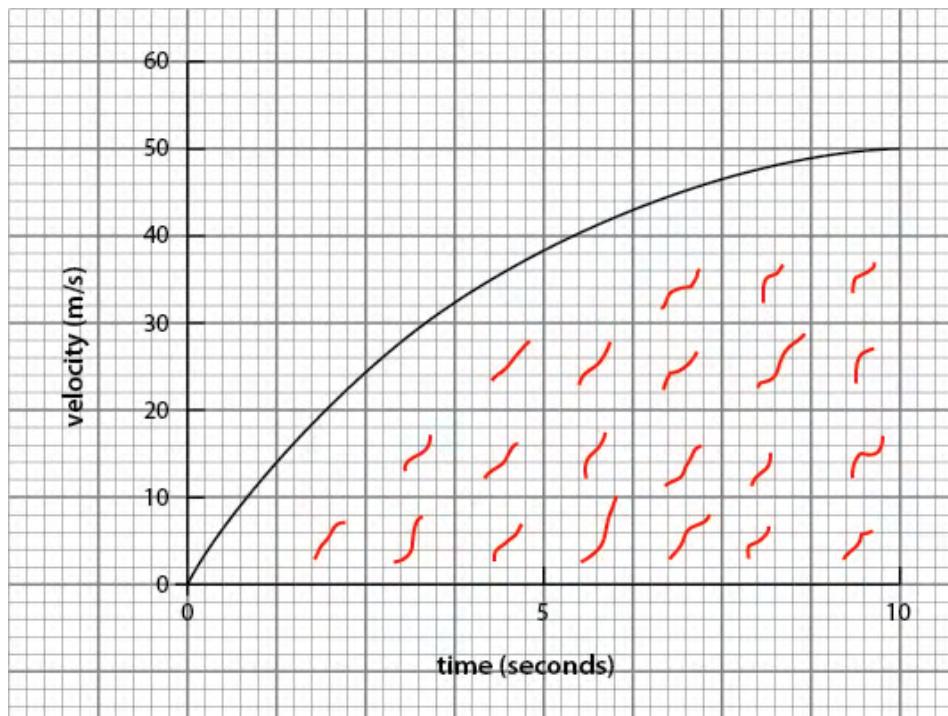


Each horizontal small square is  $5 \div 20 = 0.25$  seconds. Therefore the end points of the tangent drawn are  $(0.75, 20)$  and  $(9.5, 5.8)$ . The gradient can be calculated using the 'rise/run' formula:

$$\text{Acceleration} = \frac{58 - 20}{9.5 - 0.75} = \frac{38}{8.75} = 4.34 \text{ m/s}^2$$

- b) The best way to count the square here is to count the large square which are equal to

$$\frac{5}{4} \times 10 = 12.5 \text{ metres. Hence we have}$$

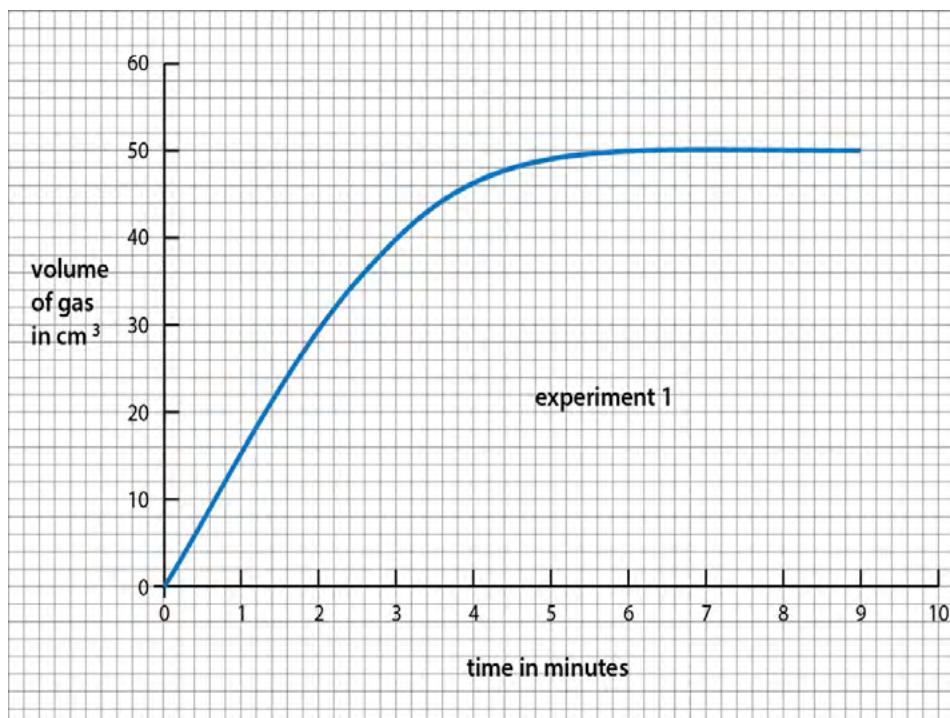


The ‘squiggles’ are shown to indicate that counting has taken place. There are 21 full squares and approximately 8 ‘half’ squares = 4 full squares. Therefore the distance travelled is

$$(21 + 4) \times 12.5 = 312.5\text{m}$$

### Reaction graphs in Chemistry

A student investigates the reaction between calcium carbonate and hydrochloric acid. He measures the volume of gas made every minute. Look at the graph. It shows his results for the experiment.



What is the rate of reaction between 0 and 2 minutes in  $\text{cm}^3/\text{minute}$ ?

#### Answer

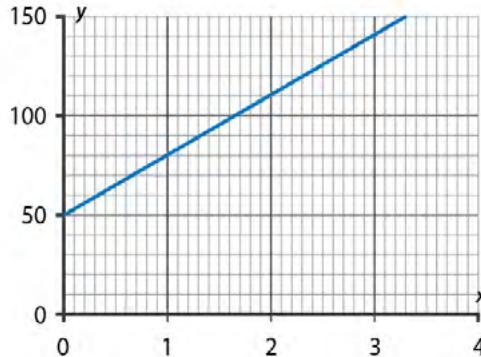
In this example between 0 and 2 mins the graph is approximately a straight line and the gradient and hence the rate of reaction can be estimated without using a tangent technique. At 2 mins the volume of gas is approximately  $29\text{ cm}^3$ . Hence using the ‘rise/run’ formula:

$$\text{Rate of reaction} = \frac{29 - 0}{2 - 0} = 14.5\text{ cm}^3/\text{min}$$

# Topic Check In 4

## Questions

- Find the value of  $y$  when  $x=2$ .
- Write down the  $y$ -intercept for the line.
- State the gradient for the line.



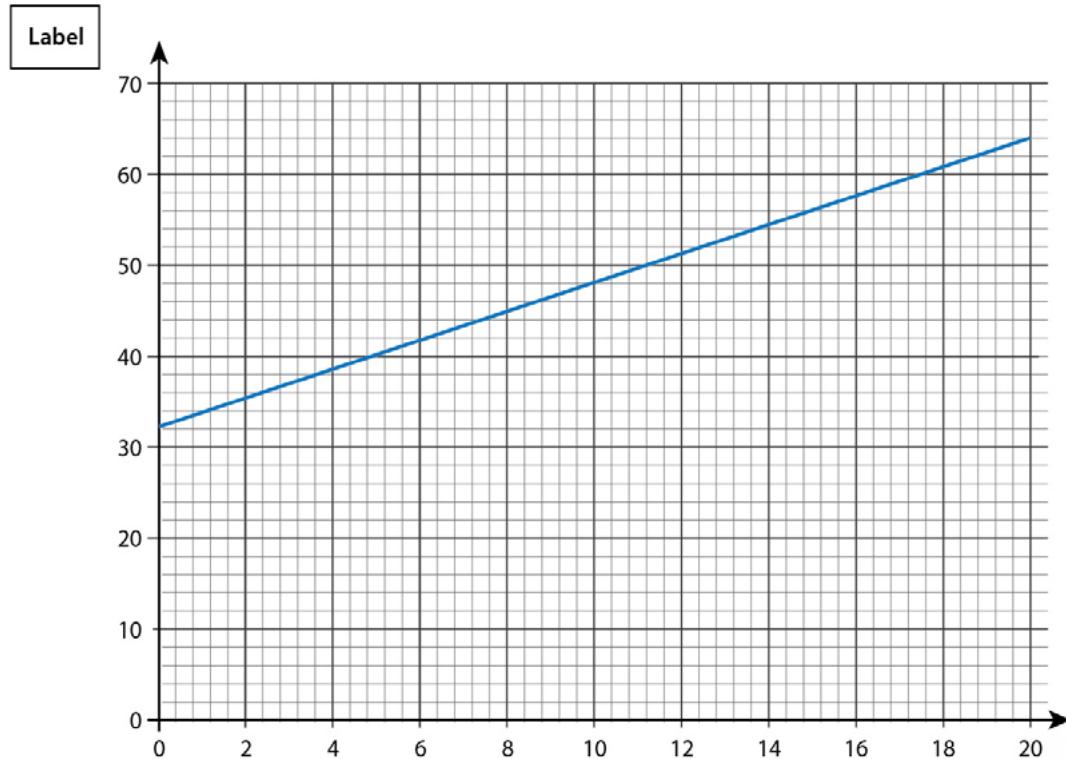
The table of values below is for the straight line graph  $v = 3+2t$ .

- Work out the missing values.

$t$	0	3		30
$v$			19	

- Write down the gradient of the graph of  $y = 7x - 2$ .

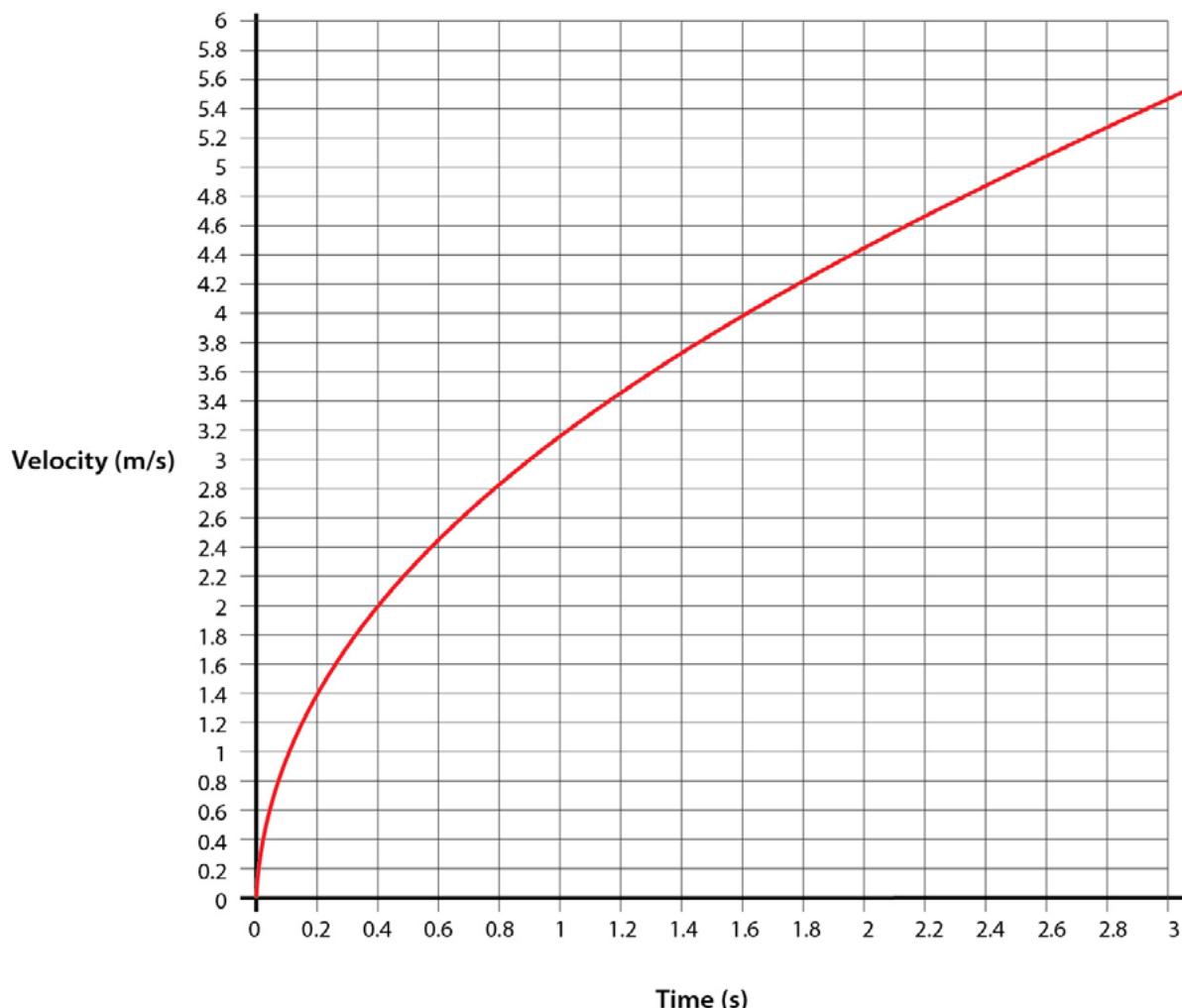
Use the graph of changing between degrees Celsius and degrees Fahrenheit for questions 6 and 7.



Water freezes at  $0^{\circ}\text{C}$  and also at  $32^{\circ}\text{F}$ .

6. Write down the label for the vertical axis.
7. Use the graph to change  $14^{\circ}\text{C}$  to degrees Fahrenheit.

The graph below shows the velocity – time graph of a particle accelerating from rest. Use this graph for questions 8–10.



8. How do you know that the particle is not undergoing constant acceleration?
9. Use the graph to estimate the acceleration of the particle after 1.6 second.
10. Estimate the distance travelled in the first two seconds.

## Answers

1. 110
2. 50
3. 30
4. 3, 9, 8, 63
5. 7
6. Fahrenheit
7. 54
8. The velocity – time graph is not a straight line. oe
9.  $\approx 1.25 \text{ ms}^{-2}$
10. 6 m

# Chapter 4 RAG

Question	Topic	R	A	G
1	Able to read values from graphs.			
2	Able to find y-intercept from graph			
3	Able to find the gradient from linear graph			
4	Able to substitute values into a linear formula			
5	Understand the general equation of a straight line			
6	Able to label axis of graph			
7	Able to use a conversion chart			
8	Understand that the gradient of a velocity time graph represents acceleration			
9	Able to estimate acceleration by calculation of the tangent of a velocity time curve			
10	Able to estimate distance travelled from area under a velocity time graph			

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9	Able to estimate acceleration by calculation of the tangent of a velocity time curve			
10	Able to estimate distance travelled from area under a velocity time graph			

# Chapter 5 : Geometry and trigonometry

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The Science specifications state in appendix 5f that learners are expected to:

- a. Use angular measures in degrees
- b. Visualise and represent 2D and 3D forms including two dimensional representations of 3D objects
- c. Calculate areas of triangles and rectangles, surface area and volume of cubes.

## Prior Key Stage 3 learning

Learners should be able to:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)
- use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D

## Mathematical skills

### a) Use angular measures in degrees

Learners should be able to understand that angular measure is measured in degrees and be able to use a protractor to measure angles. They should also know how to calculate angles in polygons which is helpful when analysing the structure of atoms. For an  $n$  sided regular polygon the formula for the size of each internal angle is given by

$$\text{Internal angle} = \frac{180(n - 2)}{n}$$

So for the example of a hexagon (the base structure of graphite) the internal angles between sides (and therefore bond angles) is given by

$$\frac{180(6 - 2)}{6} = \frac{180(4)}{6} = 120 \text{ degrees}$$

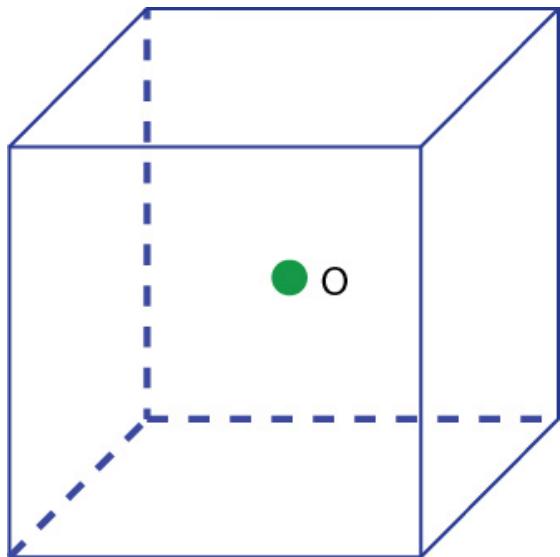
At GCSE learners normally learn the sum of the internal angles of a polygon  $180(n - 2)$ , and then determine the individual interior angles of a regular polygon by dividing the sum by the number of sides.

Often teachers will approach this from the other direction. The exterior angle of a regular polygon is  $360/n$ , and then the interior angle is  $180 - \text{exterior}$

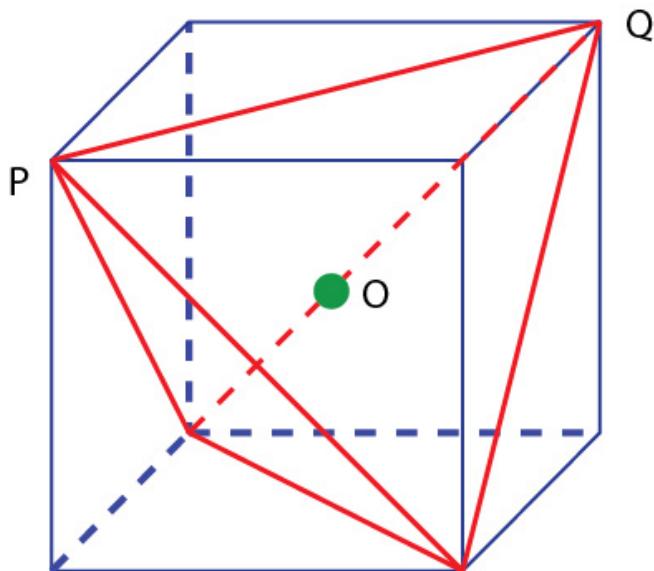
## Extension – Bond angles in a Tetrahedron

The structure of Diamond is tetrahedral. The bond angles are more difficult to calculate in a 3 dimensional shape but as an extension task the following mathematical explanation can be given to inquisitive, eager learners!

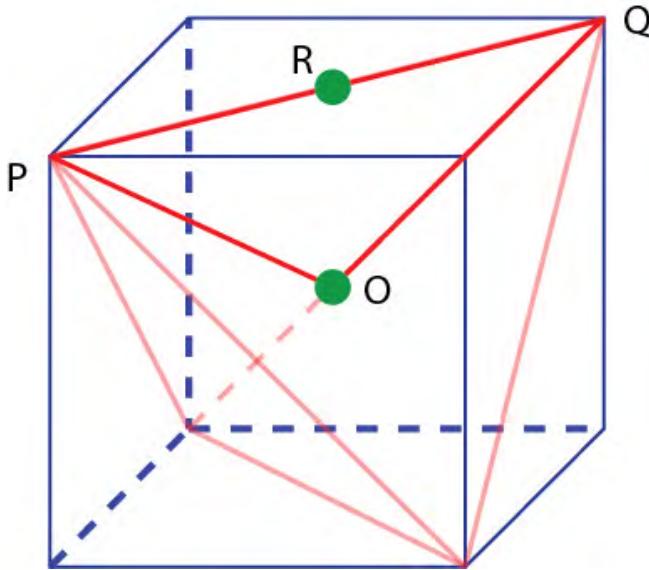
The bond angles in a tetrahedron require trigonometry. Take a cube of side length 2 with a centre O:



Now imagine a tetrahedron with vertices on the vertices of the cube:



Now imagine the triangle OPQ:



The angle POQ is the bond angle and R is the midpoint of PQ. Using Pythagoras the length PQ can be calculated as:

$$\sqrt{2^2 + 2^2} = 2\sqrt{2}$$

so the length of PR is  $\sqrt{2}$  and the length OR is 1 as it O is the centre of the cube. Hence the triangle OPR is right-angled and the angle POR can be found by trigonometry:

$$\angle POR = \tan^{-1} \frac{\sqrt{2}}{1} = \tan^{-1} \sqrt{2} \approx 54.75^\circ$$

Hence the bond angle POQ is double  $\approx 109.5^\circ$ .

### b) Visualise and represent 2D and 3D forms including two dimensional representations of 3D objects

When looking at bond structures of the allotropes of carbon molecules learners will be expected to understand some basic facts about polygons (2D shapes) and polyhedral (3D shapes).

Graphite consists of a layered lattice of carbon molecules that are bonded in hexagons. This is possible because the hexagon shape *tessellates* the plane. There are only a few shapes that are able to do this and due to the atomic structure of Carbon it means that hexagons can be formed.

Diamond molecules form bonds between in a tetrahedron shape (special triangular based pyramid). This is a regular (all sides the same) 3 dimensional shapes whose sides are equilateral triangles and there are four of them. It is an example of a *Platonic solid*. There are actually only 5 Platonic solids which have equal sides and faces; tetrahedron (4 faces), cube (6 faces), octahedron (8 faces), dodecahedron (12 faces) and a icosahedron (20 faces).

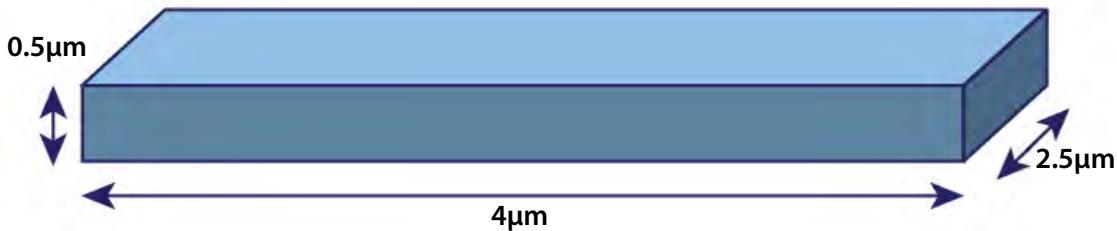
The ‘Bucky-Ball’ is an example of an *Archimedean solid*. These solids have two or more regular faces. The ‘Bucky-ball’ is a *truncated icosahedron* which has 12 faces that are pentagons and 8 faces that are hexagons. It can be made from the Platonic Icosahedron by ‘cutting off’ each vertex.

### c) Calculate areas of triangles and rectangles, surface area and volume of cubes

The following list of formulae will be required:

1.  $\text{Area of a Triangle} = \frac{1}{2} \text{base} \times \text{perpendicular height} = \frac{1}{2} bh$
2.  $\text{Area of triangle} = \frac{1}{2} ab \sin C, a, b \text{ are sides containing the interior angle } C$
3.  $\text{Surface area of cuboid} = 2(wh + wl + hl), w \text{ is width, } l \text{ is length and } h \text{ is height}$
4.  $\text{Volume of Cuboid} = hlw$

In a Biology context imagine a particular cell is approximated as a cuboid with length  $4\mu\text{m}$ , height  $0.5\mu\text{m}$  and width  $2.5\mu\text{m}$



The volume is calculated as:

$$V = hlw = 0.5 \times 4 \times 2.5 = 5 \mu\text{m}^3$$

The surface area is calculated as:

$$S.A = 2(wh + wl + hl) = 2(2.5 \times 0.5 + 2.5 \times 4 + 0.5 \times 4) = 26.5\mu\text{m}^2$$

Therefore the surface area to volume ratio is 26.5:5 or 53:10

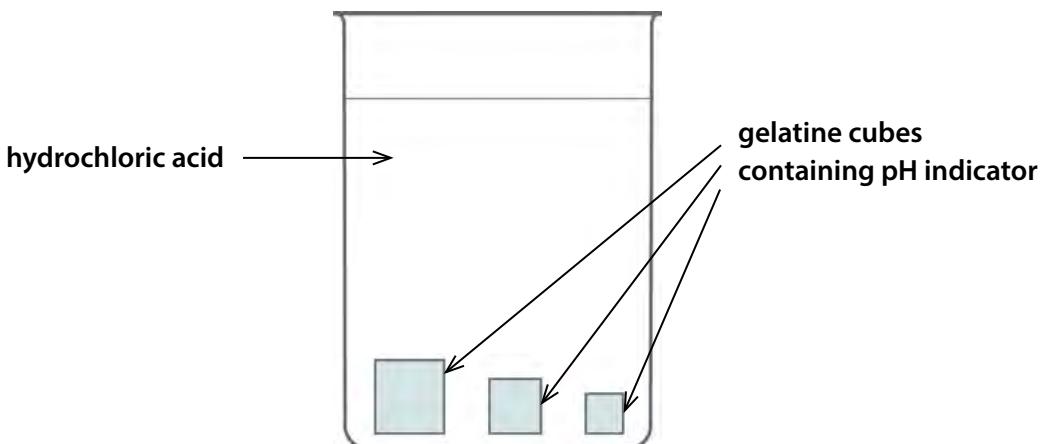
### Contexts in Science

#### Surface Area/Volume Ratio

Some students investigate the effect of the ratio of surface area:volume on the rate of diffusion in animal cells.

They use hydrochloric acid and gelatine cubes stained blue with pH indicator.

They put different size cubes into a beaker of hydrochloric acid and time how long it takes for the cubes to completely change colour.



This table shows their results.

length of 1 side of cube in cm	surface area: volume ratio in $\text{cm}^{-1}$	time to completely change colour in seconds
1	.....	132
2	3	328
3	2	673

Calculate the surface area: volume ratio for the cube with sides of 1cm.

### Answer

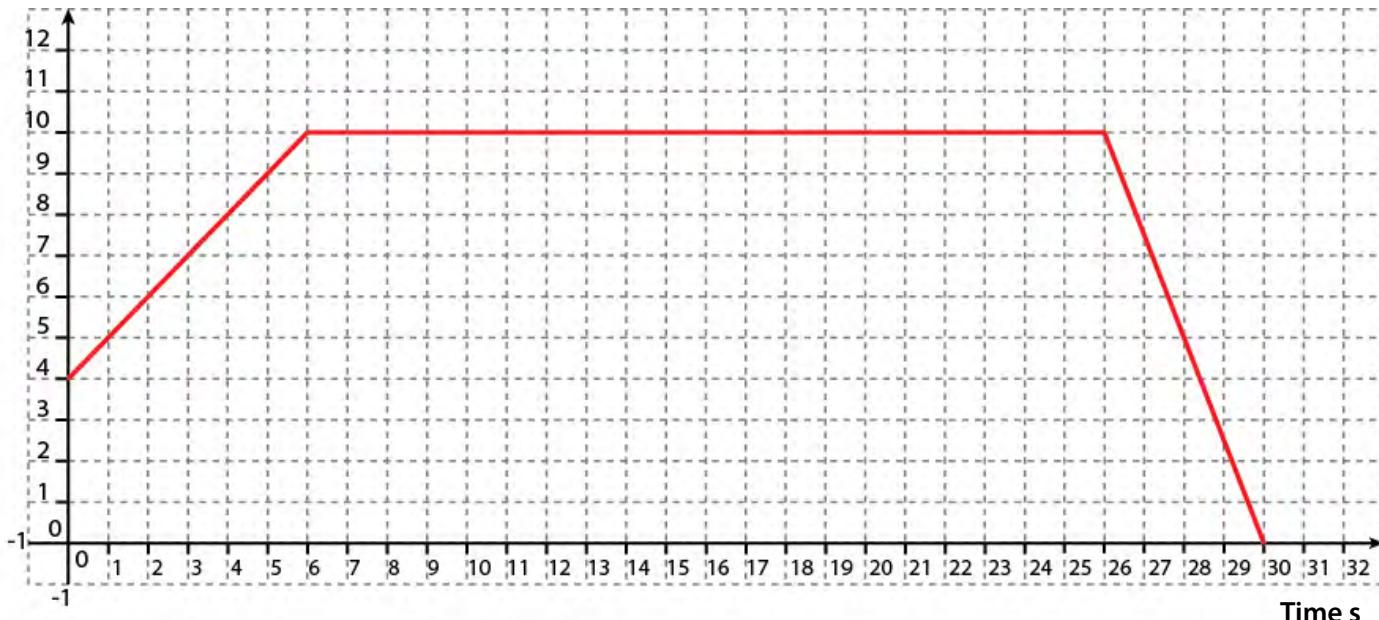
The volume of a cube is given by the length  $x$  cubed, ie  $x^3$ . In this example the volume is  $1^3 = \text{cm}^3$ . The surface area is given by  $6 \times 1 = 6\text{cm}^2$  as there are 6 faces to a cube each with  $1\text{cm}^2$  area.

Hence the surface area to volume ratio in this question is calculated by  $\frac{\text{surface area}}{\text{volume}} = \frac{6}{1} = 6\text{cm}^{-1}$ .

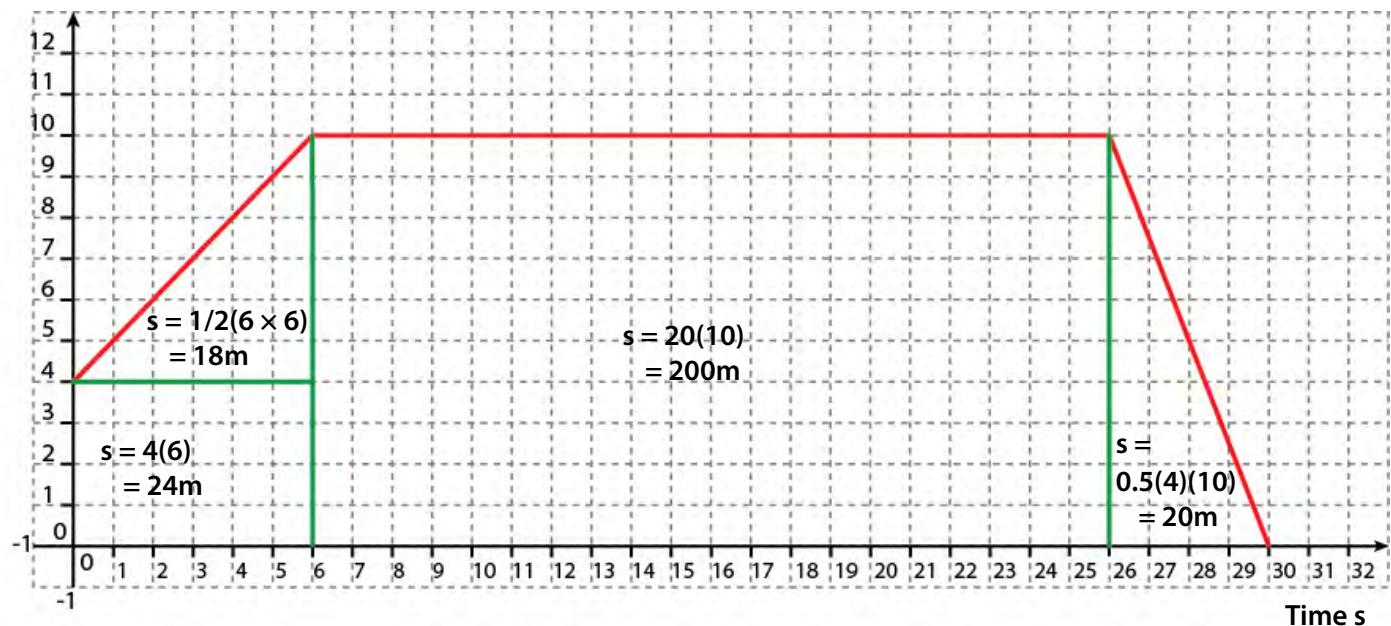
### Speed – time graphs

Calculate the distance travelled of a car given by the following speed-time graph

#### Speed m/s



The area underneath this graph can be calculated directly by dividing it into areas of rectangles and triangles like so:

**Speed m/s**

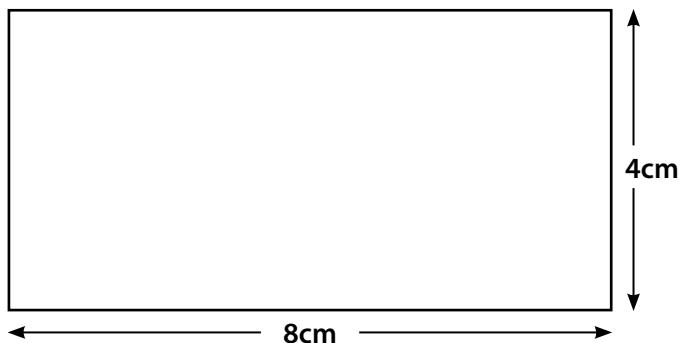
Hence the total distance travelled is  $24 + 18 + 200 + 20 = 262$  metres.

# Topic Check In 5

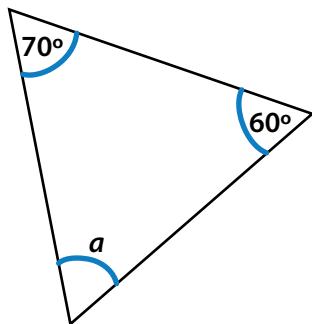
## Questions

### Chapter 5

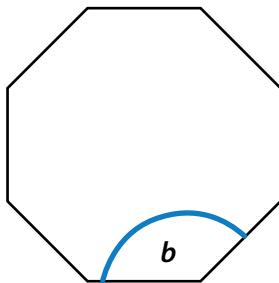
1. Calculate the area of the rectangle shown.



2. Work out angle  $a$ .

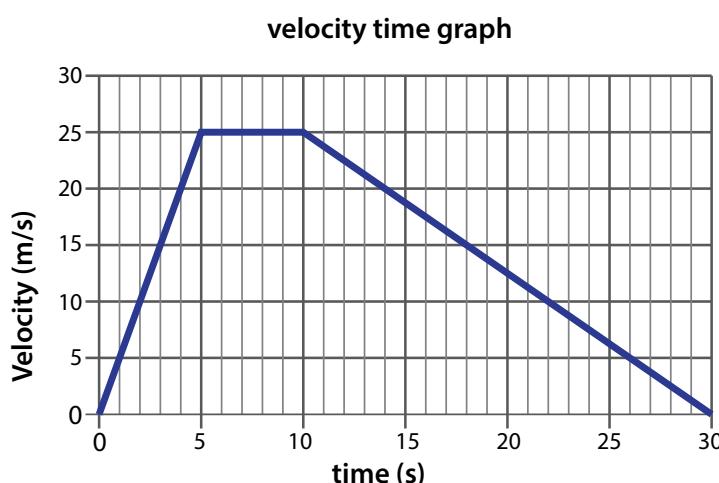


3. The shape opposite is a regular octagon.  
Calculate the size of interior angle  $b$ .



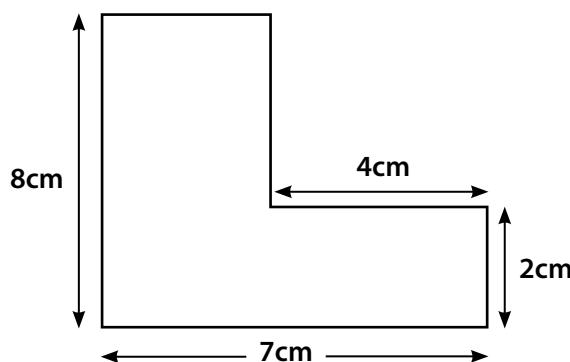
4. Calculate the volume of a cuboid which has sides 3 cm, 5 cm and 10 cm.

5. Calculate the surface area of a cube which has all its side lengths 4 cm.
6. The graph below shows a velocity time graph.

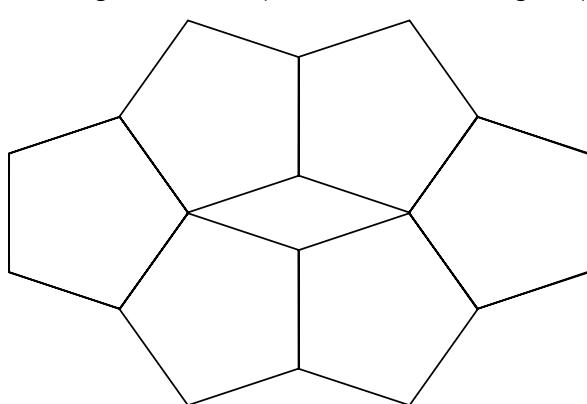


Calculate the total distance travelled in the 30 seconds.

7. Calculate the area of the shape.



8. Calculate the surface area to volume ratio of a cube with sides of 10 cm.
9. Explain why a decagon will not tessellate.
10. The diagram shows a pattern of identical regular pentagons and a rhombus.



Find the interior angles of the pentagons and the rhombus.

## Answers

1.  $32 \text{ cm}^2$
2.  $50^\circ$
3.  $135^\circ$
4.  $150 \text{ cm}^3$
5.  $96 \text{ cm}^2$
6.  $437.5 \text{ m}$
7.  $32 \text{ cm}^2$
8.  $0.6 \text{ cm}^{-1}$
9. Interior angles of a decagon are  $144^\circ$  so 3 decagons will overlap.
10. Pentagon angles  $108^\circ$ , rhombus angles  $144^\circ$  and  $36^\circ$

# Chapter 5 RAG

Question	Topic	R	A	G
1	Able to calculate area of a rectangle			
2	Know and use the fact that the sum of the interior angles of a triangle is $180^\circ$			
3	Able to calculate the interior angle of a regular polygon			
4	Able to calculate the volume of a cuboid			
5	Able to calculate the surface area of a cube			
6	Able to calculate the area under a velocity time graph to find distance travelled			
7	Able to calculate the area of a composite 2D shape			
8	Able to find the ratio of surface area to volume			
9	Able to apply angle rules to solve problems			
10	Able to apply angle rules to solve problems			

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