Tuesday 9 June 2015 – Morning

FSMQ INTERMEDIATE LEVEL

6989/01 Foundations of Advanced Mathematics (MEI)

Candidates answer on the Answer Sheet.

OCR supplied materials:
• Answer Sheet (MS4)

Other materials required:
• Eraser
• Scientific calculator
• Soft pencil
• Ruler

INSTRUCTIONS TO CANDIDATES

• Write your name clearly in capital letters, your centre number and candidate number on the Answer Sheet in the spaces provided unless this has already been done for you.
• Read each question carefully. Make sure you know what you have to do before starting your answer.
• There are forty questions in this paper. Attempt as many questions as possible. For each question there are four possible answers, A, B, C and D. Choose the one you consider correct and record your choice in soft pencil on the separate Answer Sheet.
• Read very carefully the instructions on the Answer Sheet.

INFORMATION FOR CANDIDATES

• Each correct answer will score one mark. A mark will not be deducted for a wrong answer.
• This document consists of 24 pages. Any blank pages are indicated.
Area of trapezium = \( \frac{1}{2} (a + b)h \)

Volume of prism = (area of cross-section) \( \times \) length

In any triangle \( ABC \)

Sine rule \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

Cosine rule \( a^2 = b^2 + c^2 - 2bc \cos A \)

Area of triangle = \( \frac{1}{2}ab \sin C \)

Volume of sphere = \( \frac{4}{3} \pi r^3 \)
Surface area of sphere = \( 4\pi r^2 \)

Volume of cone = \( \frac{1}{3} \pi r^2h \)
Curved surface area of cone = \( \pi rl \)

The Quadratic Equation

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[
x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}
\]
1 Three of the following statements are true and **one** is false. Which one is **false**?

A $7 - (3 - 2) = 6$

B $\frac{18}{-3} = -6$

C $15 - 2 \times 4 = 7$

D $(-5) \times (-3) = -15$

2 Three of the following statements are true and **one** is false. Which one is **false**?

A The lowest common multiple (LCM) of 15 and 18 is 90.

B The highest common factor (HCF) of 12 and 16 is 4.

C The square root of 97 lies between 9 and 10.

D The square of 2.5 is 6.5.

3 Which **one** of the following is the **correct** solution to the inequality $3x - 4 > 6 - 2x$?

A $x > -2$

B $x > 0.4$

C $x > 2$

D $x > 10$
4 The area of Spain is $1.95 \times 10^5$ square miles, correct to 3 significant figures.
The area of Portugal is $3.57 \times 10^4$ square miles, correct to 3 significant figures.

Which one of the following is the correct total area of Spain and Portugal in square miles, correct to 2 significant figures?

A $5.5 \times 10^9$
B $2.3 \times 10^5$
C $3.8 \times 10^4$
D $7.0 \times 10^9$

5 In still air at sea level, sound travels at $3.40 \times 10^2$ metres per second, correct to 3 significant figures.

Which one of the following, correct to 2 significant figures, is the correct time taken for sound to travel 2000 metres in still air at sea level?

A 5.9 seconds
B 1.7 seconds
C 17 seconds
D 0.59 seconds

6 A circular aluminium disc has radius 3.5 cm and is 0.2 cm thick.
The mass of 1 cm$^3$ of aluminium is 2.7 g.

Three of the following statements are true and one is false. Which one is false?

A The circumference of the disc is approximately 22 cm.
B The cross-sectional area of the disc is approximately 38.5 cm$^2$.
C The volume of the disc is approximately 81.4 cm$^3$.
D The mass of the disc is approximately 20.8 g.
7 Which one of the following is the correct solution of these simultaneous equations?

\[
\begin{align*}
2x + 3y &= 7 \\
5x - y &= 9
\end{align*}
\]

A \( x = 2, \ y = 1 \)  
B \( x = 5, \ y = -1 \)  
C \( x = \frac{20}{13}, \ y = -\frac{17}{13} \)  
D \( x = \frac{20}{13}, \ y = \frac{17}{13} \)

8 Three of the following statements are true and one is false. Which one is false?

A \( x(y + 1) - y(2 + x) = x - 2y \)
B \( 3(x - 1) - 2(2 - x) = x - 7 \)
C \( (x - 2)(2x + 3) = 2x^2 - x - 6 \)
D \( x(x - 2) - 2(1 - x^2) = 3x^2 - 2x - 2 \)

9 Three of the following equations have two real roots and one does not. Which one does not?

A \( x^2 - 7x + 13 = 0 \)
B \( x^2 - 8x + 14 = 0 \)
C \( 2x^2 - 9x + 10 = 0 \)
D \( 3x^2 - 10x + 8 = 0 \)
10. Which one of the following is a correct simplification of $1 + 2 \frac{1}{7} + 3 \frac{1}{9}$?

A $6 \frac{1}{6}$  
B $6 \frac{2}{3}$  
C $8 \frac{1}{3}$  
D $6 \frac{5}{6}$

11. A group of students conduct a survey on cars travelling past the school gates one morning. They want to investigate the number of people in each car.

The vertical line graph below shows their results.

Three of the following statements are true and one is false. Which one is false?

A The survey involved 36 cars.
B The mean number of people in each car was 1.8, correct to 1 decimal place.
C On another day 150 cars passed the school gate at this time. Based on the survey, an estimate for the number of people in these cars is about 270.
D A car is to be chosen at random at the same time the next day. Based on the survey, an estimate of the probability that there will be more than 2 people in the car is 0.6.
12 Which one of the following is a correct simplification of \( \frac{5x + 2}{5} - \frac{1 - 2x}{3} \)?

A \( \frac{3x + 1}{2} \)  
B \( \frac{5x + 1}{15} \)  
C \( \frac{25x + 1}{15} \)  
D \( \frac{7x + 1}{2} \)

13 The diagram shows two sections of an office desk. ABCD is a rectangle and DEF is a triangle. Units are metres.

Three of the following statements are true and one is false. Which one is false?

A The two sections have equal areas.
B The angle \( \alpha \) is 39°, correct to the nearest degree.
C The distance FE is 1.28 m, correct to 3 significant figures.
D The perimeter of the desk is 5.9 m, correct to 2 significant figures.
14. Four students are asked to write the four numbers $\sqrt{149}$, $12 \frac{1}{4}$, 12.2, $\frac{62}{5}$ in order of size, starting with the smallest number.

Aiden writes: $12 \frac{1}{4}$, $\sqrt{149}$, $\frac{62}{5}$, 12.2

Bobbie writes: $\sqrt{149}$, $12 \frac{1}{4}$, 12.2, $\frac{62}{5}$

Cathy writes: 12.2, $\sqrt{149}$, $12 \frac{1}{4}$, $\frac{62}{5}$

Debbie writes: 12.2, $\sqrt{149}$, $\frac{62}{5}$, $12 \frac{1}{4}$

Which one of the students has written the numbers in the correct order?

A. Aiden
B. Bobbie
C. Cathy
D. Debbie

15. Three of the following statements are true and one is false. Which one is false?

A. 50 miles is approximately 80 kilometres.
B. 10 kilograms is approximately 22 pounds.
C. 1 foot is approximately 25.4 centimetres.
D. 1 gallon is approximately 4.5 litres.
16 A group of students is asked to write down an equation from the following information.

A banana costs $b$ pence.  
An apple costs 5 pence more than a banana.  
6 bananas cost the same as 4 apples.

Which one of the following is the correct equation?

A $6b = 4(b + 5)$  
B $6b = 4b + 5$  
C $4b = 6b - 5$  
D $4b = 6(b + 5)$

17 Three of the following statements are true and one is false. Which one is false?

A The solution of the equation $3x - 2 = 1 - x$ is $x = \frac{3}{4}$.
B The solution of the equation $\frac{3}{x - 1} = 6$ is $x = 3$.
C The solution of the equation $3(x + 3) - 2(x - 1) = 5$ is $x = -6$.
D The solution of the equation $2(x - 1) = 4 - 3(2x + 1)$ is $x = \frac{3}{8}$. 
The village summer fete committee agreed to split the profits from the fete between the village Football, Cricket and Tennis clubs in the ratio 2:3:4.

The fete made a profit of £1260.

Three of the following statements are true and one is false. Which one is false?

A  None of the clubs received more than 50% of the profit.
B  The Tennis club received twice as much as the Football club.
C  The Cricket club received £630.
D  The Tennis club received £560.

You are given the vectors \( \mathbf{a} = \mathbf{i} + 3\mathbf{j}, \mathbf{b} = -3\mathbf{i} + \mathbf{j} \) and \( \mathbf{c} = -\mathbf{i} + 2\mathbf{j} \).

Three of the following statements are true and one is false. Which one is false?

A  \( \mathbf{a} + \mathbf{c} = 5\mathbf{j} \)
B  \( \mathbf{b} + \mathbf{c} - \mathbf{a} = -5\mathbf{i} \)
C  \( \mathbf{a} + \mathbf{b} = 2\mathbf{c} \)
D  \( \mathbf{a} - \mathbf{c} = \mathbf{b} \)
The triangle ABC has angle B = 90°, AB = 7 cm and BC = 3 cm. The square ACDE is drawn on the side AC of the triangle.

Three of the following statements are true and one is false. Which one is false?

A  The area of the triangle ABC is 10.5 cm².
B  The area of the square ACDE is 58 cm².
C  The length of the diagonal of the square, EC, is approximately 10.8 cm.
D  The angle ACB is 64.6°, correct to 3 significant figures.

Three of the following quadratic expressions have a factor \((x - 2)\), and one does not. Which one does not?

A  \(x^2 + x - 6\)
B  \(x^2 - x - 6\)
C  \(2x^2 - 3x - 2\)
D  \(2x^2 - 5x + 2\)
22. In the triangle shown, PQ = 6 cm, PR = 7 cm and angle P = 42°.

Which one of the following is the correct length for QR, correct to 3 decimal places?

A. 0.862 cm  
B. 4.751 cm  
C. 4.684 cm  
D. 5.402 cm

23. Three of the following statements are true and one is false. Which one is false?

A. 12 459 is 12 500, correct to the nearest thousand.  
B. 32.3478 is 32.3, correct to 3 significant figures.  
C. 435.678 is 435.7, correct to 1 decimal place.  
D. 543.6 is 544, correct to the nearest whole number.
24. A company buys 6 filing cabinets to fit along a wall. The width of each cabinet is 50 cm, correct to the nearest cm. The length of the wall is 300 cm, correct to the nearest cm.

Three of the following statements are true and one is false. Which one is false?

A. The width of one cabinet could be 503 mm.
B. The length of the wall is not greater than 300.5 cm.
C. The total width of all 6 cabinets is at least 297.5 cm.
D. It is certain that 5 cabinets will fit.

25. A farmer uses 80 metres of portable fencing to make a rectangular enclosed space of length $l$ metres and width $w$ metres. The fencing forms three sides of the rectangle and a wall forms the fourth side, as shown in the diagram.

Which one of the following is the correct formula for the area, $A$ m$^2$, of the rectangle?

A. $A = w^2$
B. $A = w(80 - w)$
C. $A = w(80 - 2w)$
D. $A = 80w$
26 You are given that \( a = 3, \ b = 5, \ c = -2. \)

Three of the following statements are true and one is false. Which one is false?

A \( c^2 = 4 \)

B \( ab + bc + ca = -1 \)

C \( b^2 - a^2 = c^4 \)

D \( 2a - 3c = 0 \)

27 Rosie and Sam are rearranging equations.

Rosie rearranges \( F = \frac{9}{5} C + 32 \) to give \( C = \frac{5}{9}(F - 32). \)

Sam rearranges \( \frac{1}{f} = \frac{1}{x} + \frac{1}{y} \) to give \( x = \frac{yf}{y - f}. \)

Which one of the following statements is true?

A Both Rosie and Sam are correct.

B Rosie is incorrect and Sam is correct.

C Rosie is correct and Sam is incorrect.

D Both Rosie and Sam are incorrect.
28 Three of the following statements are true and **one** is false. Which one is **false**?

A \[4x^2y + 6xy^2 = 2xy(2x + 3y)\]

B \[\frac{3}{5}x + \frac{5}{6} = \frac{1}{30}(18x + 25)\]

C \[x^2 - 25 = (x - 5)(x + 5)\]

D \[x(x + 2y) - y(2x + y) = x^2 + y^2\]

29 The diagram shows the graphs of three lines, P, Q and R.

The equation of Q is \(2x - 3y + 12 = 0\).
The equation of R is \(y = 3 - 2x\).

Three of the following statements are true and **one** is false. Which one is **false**?

A The line P has negative gradient.

B The solution of the simultaneous equations \(2x - 3y + 12 = 0\) and \(y = 3 - 2x\) is given by the coordinates of the point where the lines Q and R intersect.

C The x-coordinate of the point where R cuts the x-axis is the solution of the equation \(3 - 2x = 0\).

D The line Q has gradient \(-\frac{2}{3}\).
Some students are asked to explain how to work out the value of the expression \( \frac{n^2 - 1}{n} \) for a given value of \( n \).

Abbie says:  square \( n \), then subtract one divided by \( n \).
Bob says:  square \( n \), subtract 1 and then divide the result by \( n \).
Chloe says:  square \( (n - 1) \) and then divide by \( n \).
Dannie says:  square \( n \), divide by \( n \) and subtract 1.

Which one of the students has given the correct way to work out the value of the expression?

A  Abbie
B  Bob
C  Chloe
D  Dannie

Emily and James each take a sample of students from their school in order to carry out an investigation.

Emily chooses her tutor group and uses all students in the group. She claims that she has produced a random sample.

James chooses the first 10 names from the alphabetical list of all students in the school. He claims that he has produced a random sample.

Which one of the following is a correct statement?

A  Emily and James are both correct.
B  Emily is correct, but James is incorrect.
C  Emily is incorrect, but James is correct.
D  Emily and James are both incorrect.
A shop reduces the prices of all items by 20% in a sale.

Three of the following statements are true and **one** is false. Which one is **false**?

A  An item priced at £200 before the sale is priced at £160 in the sale.
B  An item that is priced at £300 in the sale was priced at £360 before the sale.
C  20% off means a reduction of one-fifth of the price.
D  A reduction of £10 is made on an item originally priced at £50.

Three of the following statements are true and **one** is false. Which one is **false**?

A  For $0^\circ \leq \theta \leq 360^\circ$ the solution of the equation $\cos \theta = 0.7$ is $45.6^\circ$ or $314.4^\circ$, correct to 1 decimal place.
B  $\sin 50^\circ = \cos 40^\circ$
C  For $0^\circ \leq \theta \leq 180^\circ$, $0 \leq \sin \theta \leq 1$.
D  The graph shows the function $y = 5 \sin x - 1$. 

![Graph of $y = 5 \sin x - 1$](image)
The diagram shows a prism.

PQRS is a horizontal rectangle. PLMS is a vertical rectangle.
PQ = SR = 10 cm, PL = SM = 5 cm and PS = QR = LM = 8 cm.

Three of the following statements are true and one is false. Which one is false?

A  \( LQ = \sqrt{125} \)
B  Angle \( \angle LQP = 23^\circ \), correct to the nearest degree.
C  \( \tan LQP = \frac{1}{2} \)
D  \( LR^2 = 10^2 + 5^2 + 8^2 \)

35  A and B are two points on a river bank, with B 500 m downstream from A.

\[
\begin{array}{c}
\text{A} & \quad 500 \text{ m} & \quad \text{B} \\
\end{array}
\]

Mike rows his small boat from A to B then back to A. In still water he rows at 1 m s\(^{-1}\). The river flows at 0.6 m s\(^{-1}\).

Three of the following statements are true and one is false. Which one is false?

A  Mike rows in still water at 3.6 km h\(^{-1}\).
B  The total time Mike takes to row from A to B then back to A is just over 26 minutes.
C  Mike takes twice as long to row from B back to A as to row from A to B.
D  Mike’s speed from B back to A is one quarter of his speed from A to B.
36 The table below gives some values for the function \( y = x^3 + 4x^2 + x - 2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^3 )</td>
<td>-64</td>
<td>-27</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( 4x^2 )</td>
<td>64</td>
<td>16</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>-4</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( -2 )</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>-6</td>
<td>0</td>
<td>-2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In order to answer this question you are advised to complete the table and draw the graph on the grid below.

Three of the following statements are true and one is false. Which one is false?

A The equation \( x^3 + 4x^2 + x - 2 = 0 \) has three roots, one of which is an integer.

B There are two points on the curve \( y = x^3 + 4x^2 + x - 2 \) where the gradient is zero.

C When \( x = -3 \) the gradient is negative.

D The equation \( x^3 + 4x^2 + x - 2 = -5 \) has only one root.
Gavin is driving along a road that has two traffic lights. The probability that he stops at the first lights is 0.5. If he stops at the first lights then the probability that he stops at the second lights is 0.6. If Gavin does not stop at the first lights then the probability that he stops at the second lights is 0.3.

In order to complete this question you are advised to work out and fill in the missing probabilities of the probability tree below.

Three of the following statements are true and **one** is false. Which one is **false**?

A  The probability that Gavin stops at both lights is 0.3.

B  The probability that Gavin does **not** stop at either light is 0.35.

C  The probability that Gavin stops at exactly one of the lights is 0.35.

D  The probability that Gavin stops at at least one of the lights is 0.35.
Here is a distance-time graph for a commuter train and an express train.

The commuter train goes from station P to R, stopping at Q on the way. The express train travels from R to P without stopping at Q.

Three of the following statements are true and one is false. Which one is false?

A The commuter train travels faster from P to Q than it does from Q to R.
B The express train passes the commuter train at station Q.
C The commuter train waits in station Q for 2 minutes.
D The express train travels at a constant speed of 135 km h\(^{-1}\).
Chicken eggs are labelled in one of four categories depending on their mass as shown in the table.

<table>
<thead>
<tr>
<th>Size</th>
<th>Mass, m gram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very large</td>
<td>$m \geq 73$</td>
</tr>
<tr>
<td>Large</td>
<td>$63 \leq m &lt; 73$</td>
</tr>
<tr>
<td>Medium</td>
<td>$53 \leq m &lt; 63$</td>
</tr>
<tr>
<td>Small</td>
<td>$m &lt; 53$</td>
</tr>
</tbody>
</table>

One evening a chicken farmer collects all the eggs laid by his chickens that day as follows.

<table>
<thead>
<tr>
<th>Size</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very large</td>
<td>124</td>
</tr>
<tr>
<td>Large</td>
<td>237</td>
</tr>
<tr>
<td>Medium</td>
<td>311</td>
</tr>
<tr>
<td>Small</td>
<td>181</td>
</tr>
</tbody>
</table>

This pie chart displays these data.

Three of the following statements are true and one is false. Which one is false?

A  The modal category is ‘Medium’.
B  The percentage of eggs labelled ‘Very large’ is 14.5%, correct to 1 decimal place.
C  In the pie chart, the angle of the sector representing ‘Small’ is 76°, correct to the nearest degree.
D  In the sectors of pie chart, the difference between the biggest and smallest angle is 24°, correct to the nearest degree.
A market gardener wants to estimate the size of his potato crop. He chooses one line of plants, digs them all up and weighs each potato.

The masses are summarised in the table.

<table>
<thead>
<tr>
<th>Mass of potato, $x$ gram</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; x \leq 50$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$50 &lt; x \leq 100$</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>$100 &lt; x \leq 150$</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>$150 &lt; x \leq 200$</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>$200 &lt; x \leq 250$</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>$250 &lt; x \leq 300$</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

In order to answer this question you are advised to complete the cumulative frequency table above and draw the cumulative frequency graph on the grid below.

In order to answer this question you are advised to complete the cumulative frequency table above and draw the cumulative frequency graph on the grid below.

Three of the following statements are true and one is false. Which one is false?

A  The total number of potatoes weighed is 250.

B  The median mass is approximately 165 g.

C  The interquartile range for the mass of these potatoes is approximately 100 g.

D  44% of potatoes weigh 150 g or more.