Wednesday 3 June 2015 – Morning

FSMQ ADVANCED LEVEL

6993/01 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993/01

Other materials required:
- Scientific or graphical calculator

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 100.
- The Printed Answer Book consists of 20 pages. The Question Paper consists of 8 pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.
In any triangle $ABC$

Cosine rule \[ a^2 = b^2 + c^2 - 2bc \cos A \]

Binomial expansion

When $n$ is a positive integer

\[(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n\]

where \[ \binom{n}{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!} \]
Section A

1 Find the equation of the line which is perpendicular to the line $2x + 3y = 5$ and which passes through the point $(3, 4)$. [3]

2 (i) Find $\alpha$ in the range $0^\circ \leq \alpha \leq 180^\circ$ such that $\tan \alpha = -1.5$. [2]
(ii) Find $\beta$ in the range $0^\circ \leq \beta \leq 180^\circ$ such that $\sin \beta = 0.2$. [2]

3 Find the equation of the tangent to the curve $y = x^3 + 3x - 5$ at the point $(2, 9)$. [5]

4 (i) Find $\int_{1}^{2} (x^2 + 2x + 3) \, dx$. [4]
(ii) Interpret your answer geometrically. [1]

5 A train accelerates from rest from a point O such that at $t$ seconds the displacement, $s$ metres from O, is given by the formula $s = \frac{3}{2}t^2 - 2t + 3$.

(i) Show by calculus that the acceleration is constant. [3]
(ii) Find the velocity after 5 seconds. [2]

6 You are given that $n$ is a positive integer and $(n - 1), n, (n + 1)$ are three consecutive integers.

In each of the following cases form an equation in $n$ and solve it.

(i) The three integers add up to 99. [2]
(ii) When the product of the first integer and third integer is added to 5 times the second integer the sum is 203. [4]
7  (i) Solve algebraically the simultaneous equations \( y = 3 + 5x - x^2 \) and \( y = x + 7 \). \[4\]

(ii) Interpret your answer geometrically. \[1\]

8  The cubic polynomial \( f(x) = x^3 + ax + 6 \), where \( a \) is a constant, has a factor of \( (x + 3) \).

(i) Find the value of \( a \). \[2\]

(ii) Hence or otherwise, solve the equation \( f(x) = 0 \) for this value of \( a \). \[4\]

9  The equation of the circle \( C \) is \( x^2 + y^2 - 8x + 2y - 19 = 0 \).

(i) Express the equation of \( C \) in the form \( (x-a)^2 + (y-b)^2 = r^2 \). \[4\]

(ii) Hence or otherwise, use an algebraic method to decide whether the point \((8, 3)\) lies inside, outside or on the circumference of the circle.

Show all your working. \[2\]

10  Fig. 10 shows a partly open window OA, viewed from above. The window is hinged at O. When the window is closed, the end A is at point B. The window is kept open by a rod CD, where C is a fixed point on the line OB.

The point D slides along a fixed bar EF. When the window is closed, D is at F. When the window is fully open, D is at E.

\( OA = OB = 20 \text{ cm}, \ OC = 8 \text{ cm}, \ CD = 7 \text{ cm}, \ EF = 5 \text{ cm}, \ OE = 10 \text{ cm} \)

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig10.png}
\caption{Fig. 10}
\end{figure}

Find

(i) angle EOC when the window is fully open, \[3\]

(ii) the distance OD when angle EOC is 30°. \[4\]
Section B

11 Two curves, \(S_1\) and \(S_2\) have equations \(y = x^2 - 4x + 7\) and \(y = 6x - x^2 - 1\) respectively. The curves meet at A and at B.

![Diagram of two curves](image)

**(i)** Show that the coordinates of A and B are (1, 4) and (4, 7) respectively. 

Points P and Q lie on \(S_2\) and \(S_1\) between A and B. P and Q have the same x coordinate so that PQ is parallel to the y-axis, as shown in Fig. 11.

**(ii)** Find an expression, in its simplest form, for the length PQ as a function of \(x\).

**(iii)** Use calculus to find the greatest length of PQ.

**(iv)** Find the area between the two curves.

12 A distributor of flower bulbs has a large number of tulip bulbs and daffodil bulbs, mixed in the ratio 1 : 3 respectively. He packs the bulbs in boxes. He puts 10 bulbs, chosen at random, into each box.

**(a)** Find the probability that a box, chosen at random, contains

**(i)** exactly 4 daffodil bulbs,

**(ii)** at least 1 tulip bulb.

**(b)** Two boxes of bulbs are chosen at random.

Find the probability that there is a total of 3 tulip bulbs in the two boxes.
A gardener marks out a regular hexagon ABCDEF on his horizontal garden. Each side of the hexagon is 0.5 m. The gardener sticks a cane in the ground at each point of the hexagon. He joins the six canes at V where V is vertically above the centre, O, of the hexagon, as shown in Fig. 13. Each cane has a length of 2.4 m from the ground to V.

![Fig. 13](image)

Calculate, giving your answers to 3 significant figures,

(i) the vertical height of V above the ground, [3]

(ii) the angle between each cane and the ground, [2]

(iii) the angle between the plane VAB and the ground. [4]

The gardener stretches a horizontal wire around the structure to strengthen it. He fixes the wire to each cane at a point 1 m vertically above the ground.

(iv) Find the length of the wire. [3]
A company produces bottles of two liquids, X and Y. There are two ingredients, A and B, in each liquid.

The table shows the quantities, in centilitres (cl), of A and B needed for each bottle of liquid.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Y</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Each day the company can use 84 cl of A and 90 cl of B.

From this information an analyst writes down the inequality $4x + 3y \leq 84$.

(i) Explain what $x$ and $y$ stand for in this inequality and explain what the inequality models.

(ii) Use the information given to write down another inequality, other than $x \geq 0$ and $y \geq 0$.

(iii) On the grid given in the answer booklet, illustrate your two inequalities. Shade the region that is not required.

(iv) The company needs to produce the same number of bottles of X and of Y each day.

Find the maximum number of bottles of X and of Y that the company can produce.

(v) On one day the company does not have to produce the same numbers of bottles of X and of Y.

Write down the maximum number of bottles that can be produced and all the combinations that will give this maximum.