

OCR

Oxford Cambridge and RSA

Monday 22 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (54 marks)

- 1 (a) (i) A curve has polar equation $r = 2a \cos \theta + 2b \sin \theta$, where $a > 0$ and $b > 0$.

Show, by considering its cartesian equation, that the curve is a circle which passes through the origin. Find the centre and radius of the circle in terms of a and b . [5]

- (ii) For the case $a = b = 1$, use integration to show that the region bounded by a minor arc of the circle and the lines $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$ has area $1 + \frac{\pi}{3}$. [5]

- (b) Given that $f(t) = \ln(1+t)$, obtain expressions for $f'(t)$, $f''(t)$ and $f'''(t)$. Hence show that the Maclaurin series for $\ln(1+t)$ begins

$$t - \frac{t^2}{2} + \frac{t^3}{3} \dots$$

Deduce the first two non-zero terms of the Maclaurin series for $\ln\left(\frac{1+t}{1-t}\right)$. [8]

- 2 (a) (i) By considering $\left(z + \frac{1}{z}\right)^5$, where $z = \cos \theta + j \sin \theta$, show that

$$\cos^5 \theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos \theta). \quad [5]$$

- (ii) Use de Moivre's theorem to find an expression for $\cos 5\theta$ in terms of powers of $\cos \theta$. [5]

- (b) (i) Obtain the roots of the equation $w^5 = 4\sqrt{2}$ in the form $re^{j\theta}$. Show the points corresponding to these roots in an Argand diagram. [4]

- (ii) For each root w , let $v = w\sqrt{2}e^{j\pi/10}$.

Show the points corresponding to the values of v on your Argand diagram.

Find, in simplified form, an equation for which the values of v are the roots. [4]

- 3 This question concerns the matrix \mathbf{M} where $\mathbf{M} = \begin{pmatrix} 5 & -1 & 3 \\ 4 & -3 & -2 \\ 2 & 1 & 4 \end{pmatrix}$.

(i) Obtain the characteristic equation of \mathbf{M} .

Find the eigenvalues of \mathbf{M} .

[7]

These eigenvalues are denoted by $\lambda_1, \lambda_2, \lambda_3$, where $\lambda_1 < \lambda_2 < \lambda_3$.

(ii) Verify that an eigenvector corresponding to λ_1 is $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ and that an eigenvector corresponding to λ_2 is

$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. Find an eigenvector of the form $\begin{pmatrix} a \\ 1 \\ c \end{pmatrix}$ corresponding to λ_3 . [5]

(iii) Write down a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{M} = \mathbf{PDP}^{-1}$. (You are not required to calculate \mathbf{P}^{-1} .)

Hence write down an expression for \mathbf{M}^4 in terms of \mathbf{P} and a diagonal matrix. You should give the elements of the diagonal matrix explicitly. [3]

(iv) Use the Cayley-Hamilton theorem to obtain an expression for \mathbf{M}^4 as a linear combination of \mathbf{M} and \mathbf{M}^2 . [3]

Section B (18 marks)

- 4 (i) Starting with the relationship $\cosh^2 t - \sinh^2 t = 1$, deduce a relationship between $\tanh^2 t$ and $\operatorname{sech}^2 t$. [1]

You are given that $y = \operatorname{artanh} x$.

(ii) Show that $\frac{dy}{dx} = \frac{1}{1-x^2}$. [4]

(iii) Show, by integrating the result in part (ii), that $y = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$. [4]

(iv) Show that $\int_0^{\frac{\sqrt{3}}{6}} \frac{1}{1-3x^2} dx = \frac{1}{\sqrt{3}} \operatorname{artanh} \frac{1}{2}$. Express this answer in logarithmic form. [4]

(v) Use integration by parts to find $\int \operatorname{artanh} x dx$, giving your answer in terms of logarithms. [5]

END OF QUESTION PAPER

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