

OCR

Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 Find the equation of the tangent to the curve $y = \frac{5x+4}{3x-8}$ at the point $(2, -7)$. [5]

2 It is given that θ is the acute angle such that $\cot \theta = 4$. Without using a calculator, find the exact value of

(i) $\tan(\theta + 45^\circ)$, [3]

(ii) $\operatorname{cosec} \theta$. [2]

3 The volume, V cubic metres, of water in a reservoir is given by

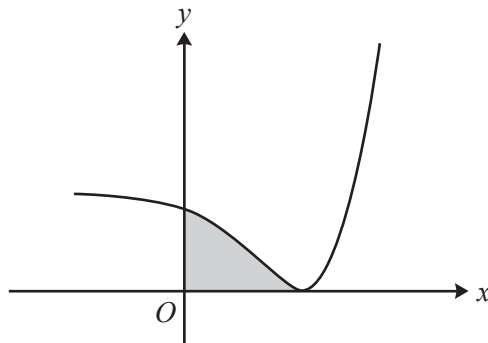
$$V = 3(2 + \sqrt{h})^6 - 192,$$

where h metres is the depth of the water. Water is flowing into the reservoir at a constant rate of 150 cubic metres per hour. Find the rate at which the depth of water is increasing at the instant when the depth is 1.4 metres. [5]

4 It is given that $|x + 3a| = 5a$, where a is a positive constant. Find, in terms of a , the possible values of

$$|x + 7a| - |x - 7a|. \quad [6]$$

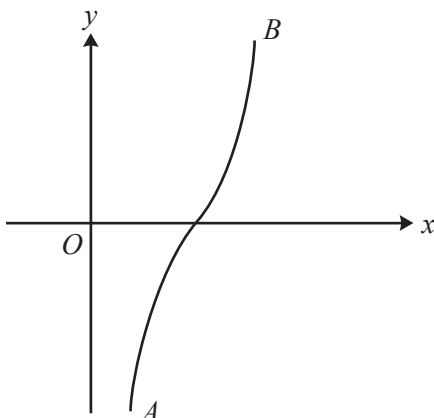
5



The diagram shows the curve $y = e^{3x} - 6e^{2x} + 32$.

(i) Find the exact x -coordinate of the minimum point and verify that the y -coordinate of the minimum point is 0. [4]

(ii) Find the exact area of the region (shaded in the diagram) enclosed by the curve and the axes. [4]



The diagram shows the curve $y = 8 \sin^{-1}\left(x - \frac{3}{2}\right)$. The end-points A and B of the curve have coordinates $(a, -4\pi)$ and $(b, 4\pi)$ respectively.

(i) State the values of a and b . [2]

(ii) It is required to find the root of the equation $8 \sin^{-1}\left(x - \frac{3}{2}\right) = x$.

(a) Show by calculation that the root lies between 1.7 and 1.8. [3]

(b) In order to find the root, the iterative formula

$$x_{n+1} = p + \sin(qx_n),$$

with a suitable starting value, is to be used. Determine the values of the constants p and q and hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. [5]

7 (i) Find the exact value of $\int_1^9 (7x+1)^{\frac{1}{3}} dx$. [4]

(ii) Use Simpson's rule with two strips to show that an approximate value of $\int_1^9 (7x+1)^{\frac{1}{3}} dx$ can be expressed in the form $m + n \sqrt[3]{36}$, where the values of the constants m and n are to be stated. [3]

(iii) Use the results from parts (i) and (ii) to find an approximate value of $\sqrt[3]{36}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. [2]

Question 8 begins on page 4.

8 The functions f and g are defined as follows:

$$f(x) = 2 + \ln(x+3) \text{ for } x \geq 0,$$

$$g(x) = ax^2 \text{ for all real values of } x, \text{ where } a \text{ is a positive constant.}$$

(i) Given that $gf(e^4 - 3) = 9$, find the value of a . [3]

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

(iii) Given that $ff(e^N - 3) = \ln(53e^2)$, find the value of N . [5]

9 It is given that $f(\theta) = \sin(\theta + 30^\circ) + \cos(\theta + 60^\circ)$.

(i) Show that $f(\theta) = \cos \theta$. Hence show that

$$f(4\theta) + 4f(2\theta) \equiv 8 \cos^4 \theta - 3. \quad [6]$$

(ii) Hence

(a) determine the greatest and least values of $\frac{1}{f(4\theta) + 4f(2\theta) + 7}$ as θ varies, [3]

(b) solve the equation

$$\sin(12\alpha + 30^\circ) + \cos(12\alpha + 60^\circ) + 4 \sin(6\alpha + 30^\circ) + 4 \cos(6\alpha + 60^\circ) = 1$$

for $0^\circ < \alpha < 60^\circ$. [4]

END OF QUESTION PAPER

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