GCE

Mathematics (MEI)

Unit **4798**: Further Pure Mathematics with Technology

Advanced GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

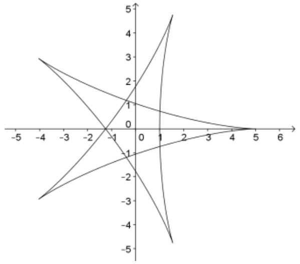
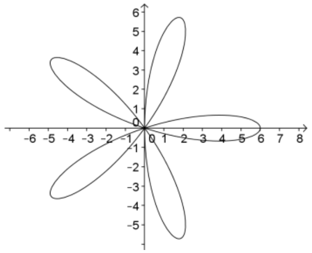
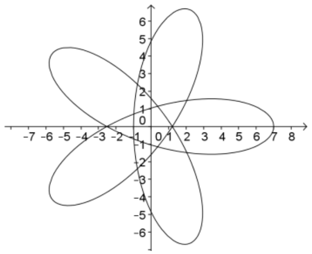
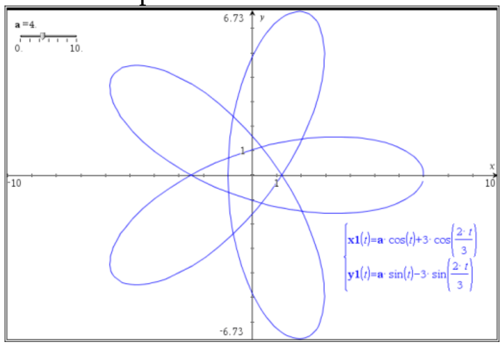
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

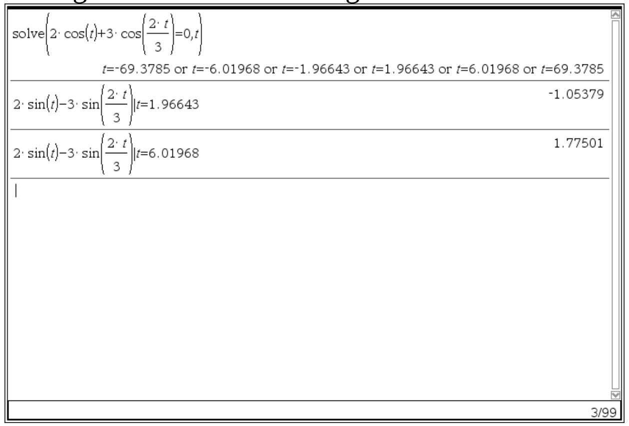
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

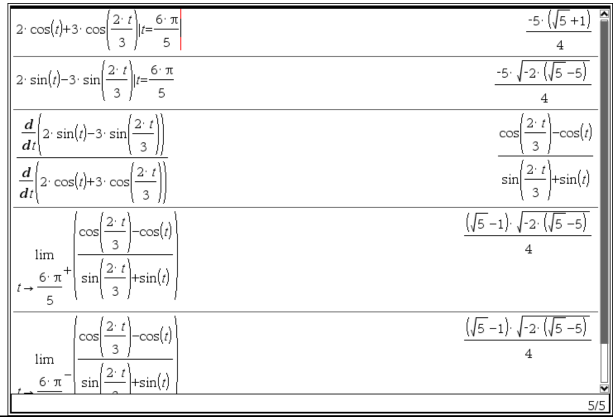
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		a=2 	G1	Correct shape
			a=3 	G1	Correct shape
			a=4 	G1	Correct shape 
			Rotational symmetry order 5 displayed in all 3.	G1	
			Intersection with +ve x-axis marked correctly.	G1	
			Any correct comment from: • curves being bounded / closed, • rotational/reflectional symmetry.	B1	
			a=2 has cusps.	B1 **[7]**	

Question	Answer	Marks	Guidance
(ii)	$x=0 \Rightarrow 2\cos(t)+\cos\left(\frac{2t}{3}\right)=0$ $\Rightarrow t=1.96643$ or $t=6.01968$ $\Rightarrow y=-1.05379$ or $y=1.77501$ By symmetry points are $(0,-1.77501),(0,-1.05379),(0,1.05379),(0,1.77501)$	M1 A1 A1 M1 A1 **[5]**	Must give evidence of solving x=0 for t. 
(iii)	$r^2=x^2+y^2$ $=6a\cos t\cos(2t/3)-6a\sin t\sin(2t/3)+a^2+9$ $=6a\cos(5t/3)+a^2+9$ Maximum distances are when $\frac{5t}{3}=0,2\pi,4\pi,\ldots$ i.e. $t=0,\frac{6\pi}{5},\frac{12\pi}{5},\ldots$ Minimum distances are when $\frac{5t}{3}=\pi,3\pi,5\pi\ldots$ i.e. $t=\frac{3\pi}{5},\frac{9\pi}{5},3\pi,\ldots$ Which are independent of a. For a=2 maximum distance is 5, minimum distance is 1.	M1 A1 A1 M1 A1 E1 B1 **[7]**	Accept method based on $\frac{dr}{dt}$ or $\frac{dr^2}{dt}$.
(iv)	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ $\frac{dy}{dx}=\frac{\cos(2t/3)-\cos(t)}{\sin(2t/3)+\sin(t)}$	M1 A1 M1	Limit from one direction only scores max M1A1 M0 A0 B1 {or any equivalent expression} Can be implied by subsequent working.

Question			Answer	Marks	Guidance
			There are two branches either side of $t=\frac{6\pi}{5}$. Considering the gradient on each branch:		
			$\lim\limits_{t\to\frac{6\pi}{5}^{+}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{(\sqrt{5}-1)\sqrt{-2(\sqrt{5}-5)}}{4}$ $\quad(=\sqrt{5-2\sqrt{5}})$	A1	Accept appropriate numerical values.
			$\lim\limits_{t\to\frac{6\pi}{5}^{-}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{(\sqrt{5}-1)\sqrt{-2(\sqrt{5}-5)}}{4}$		Accept statement that the curve is defined.
			The curve is defined at $t=\frac{6\pi}{5}$ so there is a cusp.	B1 **[5]**	For reference $\left(\frac{-5(\sqrt{5}+1)}{4},\frac{-5\sqrt{-2(\sqrt{5}-5)}}{4}\right)$ 

Question			Answer	Marks	Guidance
2	(i)		Solving $z-\frac{z^3}{6}=0.3+0.4\mathrm{i}$ $z=0.280679+0.404886\mathrm{i}$	M1 A1 **[2]**	soi Single solution given.
	(ii)		Solving $\sin z=0.3+0.4\mathrm{i}$ $z=0.280630+0.405112\mathrm{i}$ Error in real part: 0.000050 Error imaginary part: (–) 0.00023	B1 M1 A1 **[3]**	Accept + or – Accept 0.00049 and 0.00022 based on earlier rounding.
	(iii)		Column for *a*. Column for $z=a+0.4\mathrm{i}$ Columns for $z-\frac{z^3}{6}$ and for $\sin z$ Column for $\mathrm{Re}\left(z-\frac{z^3}{6}-\sin z\right)$ Error is < 0.001 for 0.5+0.4i. Error is > 0.001 for 0.6+0.4i. Smallest value is *a* = 0.6 correct to 1 dp.	M1 M1 M1 M1 M1A1 A1 **[7]**	More or fewer columns can be used. soi
	(iv)		$z_{n+1}=z-\frac{\sin(z)-0.3-0.4i}{\cos(z)}$ $z_1=0.3+0.4\mathrm{i}$ $z_2=0.28054+0.40507\mathrm{i}$ $z_3=0.28063+0.40511\mathrm{i}$ $z_4=0.28063+0.40511\mathrm{i}$ therefore 3 iterations	M1 M1 A1 A1 A1 **[5]**	Correct form for NR. Correct form for $\mathrm{f}(z)$ and $\mathrm{f}'(z)$. Evidence of at least 1 correct iteration. All correct.

Question			Answer	Marks	Guidance
	(v)		$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \ldots$ $\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \frac{z^7}{7!} + \ldots$	B1	Correct expansions for sin and sinh soi.
			$\sin(\mathrm{i}\,z) = (\mathrm{i}\,z) - \frac{(\mathrm{i}\,z)^3}{3!} + \frac{(\mathrm{i}\,z)^5}{5!} - \frac{(\mathrm{i}\,z)^7}{7!} + \ldots$	M1	Substituting (i*z*) into correct expansion for sin(*z*).
			$= \mathrm{i}\,z + \frac{\mathrm{i}\,z^3}{3!} + \frac{\mathrm{i}\,z^5}{5!} + \frac{\mathrm{i}\,z^7}{7!} + \ldots$	A1	Simplification.
			$= \mathrm{i}\sinh z$	A1	Identifying this is equivalent to $\mathrm{i}\sinh z$.
			The solution to $\sinh z = 0.4 - 0.3\mathrm{i}$ is $z = 0.40511 - 0.28063\mathrm{i}$.	B1	Or equivalent argument.
			For $z = 0.40511 - 0.28063\mathrm{i}$: $\sin(\mathrm{i}\,z) = \sin(0.28063 + 040511\mathrm{i})$ $= 0.3 + 0.4\mathrm{i}$ $\mathrm{i}\sinh z = 0.3 + 0.4\mathrm{i}$	E1 **[6]**	

3	(i)		Example program:
			
			Define program1(m)=
			Prgm
			 Local n
			 For n,3,m
			  If remain(2^(n),n)=2 Then
			   Disp n
			  EndIf
			 EndFor
			EndPrgm
				M4	If the answers are incorrect allocate method marks as follows:
					M1 Appropriate structure program
					M1 Loop for *n* or equivalent
					M1 values of 3-30
					M1 Check (If) statement
					
					More efficient programs might be possible.

			3, 5, 7, 11, 13, 17, 19, 23, 29	A1 **[5]**	

Question			Answer	Marks	Guidance
	(ii)		Changing the If statement: If remain(2^(n),n)=2 and isPrime(n)=false Then n = 341 Fermat's Little Theorem: If p is prime then $a^p \equiv a \pmod{p}$ 341is not prime therefore FLT doesn't apply.	M1 A1 A1 B1 E1 **[5]**	Changing the If statement Correct If statement given

	(iii)		Example program:
			
			Define program3(n)=
			Prgm
			
			Local a,b
			For a,3,n
			For b,3,n
			If remain(2^a,b)=2 and remain(2^b,a)=2 and a≠b and isPrime(a)=true and isPrime(b)=true Then
			Disp a,b
			
			EndIf
			EndFor
			EndFor
			EndPrgm
			
			11,31
			19,73
			23,89
			31,11
			37,73
			73,19
			73,37
			89,23
				M5
				
				A2
					If the answers are incorrect allocate method marks as follows:
					M1 Appropriate structure program
					M1 Loops for a and b or equivalent
					M1 values of 3-100
					M1 Check (If) statement: congruences
					M1 Check (If) statement: inequality and a,b prime
					
					More efficient programs might be possible.
					
					A1 at least 1 correct. A2 all correct.
					
					Programs that treat these as unordered pairs and give:
					11,31
					19,73
					23,89
					37,73
					will receive full marks.

Question			Answer	Marks	Guidance
			$2^{11} = 2048$ $= 66 \times 31 + 2$ $2^{31} = 2147483648$ $= 195225786 \times 11 + 2$	M1 A1 **[9]**	
	(iv)		$2^a \equiv 2 (\bmod b) \Rightarrow 2^{ab} \equiv 2^b (\bmod b)$ $\Rightarrow 2^{ab} \equiv 2 (\bmod b)$ by FLT. $2^{ab} \equiv 2 (\bmod b)$ and $2^{ab} \equiv 2 (\bmod a)$ from the first part of the question. This can be written as $2^{ab} = k_1 b + 2$ and $2^{ab} = k_2 a + 2$. $k_1 b = k_2 a$ and as a and b are distinct primes this value can be written as $k_3 ab$ for some integer k_3. $2^{ab} = k_3 ab + 2$ i.e. $2^{ab} \equiv 2 (\bmod ab)$ 1387 or 2701 or 2047 (by multiplying pairs from iii). (Other answers such as 561).	E1 E1 E1 E1 E1 B1 **[6]**	Other arguments are possible. B1 for any one of these values.

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