GCE

Mathematics (MEI)

Advanced GCE A2 7895-8

Advanced Subsidiary GCE AS 3895-8

OCR Report to Centres June 2016

This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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OCR REPORT TO CENTRES

4751 Introduction to Advanced Mathematics (C1)

General Comments:

As last June, having had a whole year to prepare for this examination, candidates were in general confidently applying the basic techniques required, with many candidates gaining most of the marks available in section A.

All questions were found to be accessible, with candidates rarely omitting to answer a question or part question.

Although the majority of candidates used the surds competently in question 5, there were a lot of errors when trying to simplify their roots in 9(ii) and 11(ii) and it was fortunate for them that subsequent work was ignored after acceptable answers in those parts. In general, candidates who opted to complete the square rather than use the formula did so badly. Quite a few candidates did not appreciate fully the difference between 'solving an equation in x' and giving coordinates, with many losing a mark in 9(i) because of this. Also there was often confusion between roots and factors, especially evident in the language candidates used in 9(ii).

As usual, some candidates made arithmetical errors on this non-calculator paper. This was evident in the fractions in question 2, as expected, but also in evaluating the coefficients in the binomial theorem, and the radius in question 10(ii), as well as the quadratic formula and the discriminant in the later parts of question 11.

Comments on Individual Questions:

Section A

Question No. 1
Nearly all candidates interpreted the zero power correctly in the first part. Most interpreted the fractional power correctly in the second part, although a number of candidates began by attempting to cube 9, which usually ran into difficulties as candidates did not have the assistance of their calculators; they had similar issues when attempting to find the square root 729. The most common error was candidates believing that 3 cubed was 9. Coping with the fraction and negative power in the last part was usually done correctly; notable errors were inverting the fraction whilst losing the power altogether or losing the power from either the numerator or denominator.

Question No. 2
This was a good source of marks for the majority of candidates, who found the demand of solving a pair of simultaneous equations relatively straightforward, although errors in coping with the fractional answer to x to find the y-value were quite common, as was occasionally forgetting to find the y-value. Very few candidates found the y-value first. Those who used substitution and wrote down $2x + 3(7 – 3x) = 12$ nearly always went on to get the correct answer for x – although it was particularly disheartening the number of times that $7x = 9$ became $x = 7/9$. Those who substituted for y and had $y = 7 – 3((12 – 3y)/2)$ were usually less successful, due to the fraction and the number of negative terms in the equation. Elimination methods were less frequently seen and not as successful – candidates often did not multiply all values by the required constant or they added or subtracted their pair of equations incorrectly.

Question No. 3
Nearly all candidates knew how to solve a linear inequality for the first part, and earned at least one of the two marks. When the rearrangement was done so that that the $2x$ term appeared on the right, already positive (so $-11 > 2x$) the vast majority of candidates went on to get the correct answer. However, when candidates arranged to $-2x > 11$, a considerable number neglected to reverse the inequality sign when dividing by the negative value of 2. While the majority of

candidates scored both marks in the second part, a number failed to expand $(5c^2d)^3$ correctly, with many of these failing to cube the 5. It was common for candidates to achieve at least two correct elements – with nearly all getting c^{10} and an equal split between those getting one of 250 or d^{-2}. Some candidates failed to deal with the two *d* terms correctly in both the numerator and denominator with many of these giving an answer of d^2.

Question No. 4
The majority of the candidates were very familiar with the topic of rearranging to make a different variable the subject of a formula, and coped well with this example. Nearly all candidates correctly multiplied by (2*c* – 5) to give *a*(2*c* – 5) = 3*c* + 2*a*. However it was surprising that a large number of candidates went on to make *c* rather than *a* the subject of the formula (albeit the majority did this correctly and scored 3 of the 4 marks available). Where errors occurred it was usually sign errors from moving terms from one side to the other and a small minority did not simplify their answers fully, giving say an answer of *a* = 3*c* / (2*c* – 5 – 2). It was pleasing to see that the majority of candidates correctly factorised their *a* (or *c*) terms as this has in the past caused issues.

Question No. 5
The first part was nearly always correct with the vast majority scoring at least one mark for correctly stating that $\sqrt{50} = 5\sqrt{2}$. Some candidates had difficulty with $3\sqrt{8}$ and a number incorrectly gave this as $5\sqrt{2}$ which typically came from the incorrect working of $3\sqrt{8} = 3(2\sqrt{2}) = (3+2)\sqrt{2} = 5\sqrt{2}$. In the second part, most candidates clearly knew how to rationalise the denominator with nearly all correctly indicating the need to multiply both numerator and denominator by $(4+\sqrt{3})$; only a small minority incorrectly multiplied by either $(4 - \sqrt{3})$ or $\sqrt{3}$. Nearly all correctly achieved a value of 13 for the denominator but some had issues with either expanding or simplifying the numerator. A significant minority who achieved $\frac{26+13\sqrt{3}}{13}$ did not simplify this correctly with $2 + 13\sqrt{3}$ being a common incorrect answer.

Question No. 6
Binomial expansion was done well in comparison with previous years. Most candidates remembered to use the correct coefficients and were comfortable multiplying them with powers of 5. There were not too many arithmetic errors.

Question No. 7
Most candidates managed to solve the equation. Quite a number of candidates multiplied out the brackets and rearranged to form a quadratic equation in the traditional form. This was then usually solved by factorisation, but occasionally using the formula. Those who used the given form and took the square root of both sides were more inclined to find just one root, by ignoring the possibility that the square root of 9 could be −3. The quality of the parabolas varied enormously, but most candidates determined the coordinates of the turning point and made a good attempt. Some candidates did not consider the turning point and often had skewed parabolas with a minimum on the *y*-axis. A few candidates sketched cubics and received no marks.

Question No. 8
This question was well done by many of the candidates, but a few had no idea how to use the information about the remainder. Nearly all used the fact that f(2) = 11 to find one equation. A few errors were made in applying the remainder theorem by putting f(1) = 8, rather than f(−1). Those who tried to determine this second equation by doing the long division were usually unsuccessful. Having found the two equations, candidates nearly always came up with the correct solutions.

Section B

Question No. 9
(i) About the same number of candidates gave the coordinates of intersection of the two graphs as gave the requested roots of the given equation in x. A few misread from the graph and/or struggled with the scale.

(ii) In the main this part was completed well, with almost all candidates gaining the first two marks for multiplying by (x+2) and expanding to prove the stated equality. A significant number of candidates were unable to progress further, unsure of how to solve a cubic equation. Stronger candidates produced a well-organised solution, leading directly to the fully factorised expression (sometimes in only a few lines of working, having used the root of $x = -1$ from the graph and/or part (i) to obtain the first factor). The majority were able to find the correct quadratic following division by (x + 1), with a few using synthetic division and a sizeable minority finding the solution by inspection. At this stage many found the correct final solution, but a significant number failed to include $x = -1$ in their final solution to this question, or stated incorrectly that (x + 1) was a root.

(iii)(*A*) Many candidates were able to translate correctly although there were issues with the intersections on the x-axis for some. Quite a few candidates pointed out the intersection but did not write down the coordinates as requested.

(iii)(*B*) Many candidates failed to recognise that they should substitute (x − 3) for x in the original equation, with a variety of different methods attempted. Substituting (x + 3) was quite a common error, as was adding 3 to the original equation, or changing the constant term to −5. Some used estimated roots. Many failed to gain full marks because they omitted 'y =' from their final answer.

Question No. 10
(i) The gradient method was the most common. Most candidates showed how their gradients were obtained – it was not sufficient to quote 2 and – ½, since for proof, evidence was needed that the gradients were independently obtained. Some candidates did not obtain the final mark, merely saying that the gradients are perpendicular; a reason was needed, for instance stating that $2x - \frac{1}{2} = -1$, or stating that the gradients are negative reciprocals of each other. The other popular method was to calculate the lengths of the sides of triangle ABC and use Pythagoras's theorem, which was usually well done, although some candidates sadly confused squares and square roots, such as stating $\sqrt{20} + \sqrt{80} = \sqrt{100}$.

(ii) The centre was usually found correctly. The radius caused more problems with some calling AC the radius instead of the diameter, and others reaching $\sqrt{100}/2$ but then making an error. The form of the circle equation was not always correct, with sign errors seen on the left-hand side as well as the right-hand side sometimes being r, d, or d^2 instead of the correct r^2.

(iii) Finding the gradient of the tangent was generally well done, although some used the gradient of AB or the perpendicular to AB as the gradient of the tangent. Some lost the final mark by not writing the equation in the requested form with integer coefficients. A few weak candidates had no idea how to proceed with this part and omitted it.

Question No. 11
(i) Many candidates earned all 3 marks in this part. Some forgot to find the y-intercept. A few used the quadratic formula or completed the square, perhaps not realising that factorising was possible.

(ii) The majority of candidates chose the straightforward approach and equated the line and curve given. These candidates most often simplified correctly to a required form and applied the formula. This was often very well carried out. A very good number of candidates earned 3 marks using this approach. Some attempted to complete the square. Those who, sensibly, divided through by 2 before doing so were usually successful – those who did not were less successful.

Most candidates struggled to find and simplify the *y*-coordinates. Some simply omitted them and the many who attempted them often just wrote the *x* coordinates '+ 3' or failed to convert the 3 being added to a fraction of a common denominator to add to the *x*-coordinate. Some candidates made their solution unnecessarily complicated by rearranging the equation of the line and substituting for *x*. These candidates often omitted to take the solitary *y*-term into account and mostly scored no more than the first two marks.

(iii) Some candidates were unable to cope with the constant of the equation they had formed being in terms of *k*. Many equated the line and curve, as before, and found $2x^2 - 6x - 3 - k = 0$ and then, rather than applying $b^2 - 4ac < 0$, they wrote $2x^2 - 6x - 3 = k$ and tried to apply $b^2 - 4ac < 0$ to the left hand side. Those who did work with $2x^2 - 6x - 3 - k = 0$ were almost always successful. Some candidates made sign errors through carelessness. Some introduced wrong brackets into their equation in an attempt to group the *c* term, such as $-(3 - k)$. Some candidates correctly substituted into $b^2 - 4ac < 0$ but were unable to multiply out correctly. The result $36 - 8 - (3 + k)$ was not uncommon amongst these candidates. Other candidates used trials on $2x^2 - 6x - 3 - k = 0$ to find the boundary value ie the constant that gave $b^2 - 4ac = 0$. These often scored 3 marks, but sign errors usually resulted in the loss of the final 2 marks. A very few candidates used a calculus approach. In most cases, once $y' = 4x - 5$ had been found, it was equated to 0 and the minimum point established, thinking that this would be helpful, then no further progress was made. Candidates' setting out in this question was often poor and difficult to make sense of – particularly if they had had several attempts or had used trials. Some candidates lost marks as they restarted several times, with each time being worth less than the previous attempt! Candidates should take care in this regard – and indicate which of their attempts they intend to be taken as the answer in these cases.

4752 Concepts for Advanced Mathematics (C2)

General Comments:

The paper was accessible to most candidates, but a small number were clearly ill-prepared and scored very poorly. A significant minority of candidates demonstrated a fair degree of understanding of Core 2 material, but failed to do themselves justice in the examination because of poor (GCSE level) algebra (bracket errors were especially common), careless arithmetical slips and failing to read the question correctly.

Most candidates presented their work neatly and clearly, but in some cases work was very difficult to follow, and candidates should understand the importance of presenting a clear mathematical argument, especially when there is a "show that" request in the question.

Centres are advised that using a graphical calculator to avoid a demand to use calculus, for example in question 9(iv), or to solve an equation for example in question 8(ii) will earn no credit unless the relevant working is presented.

Comments on Individual Questions:

Question No. 1
Part (i)
This was done well. A small minority of candidates failed to score: most problems were caused by a failure to put the original function into index form correctly. Occasionally $3^{-½}$ was seen as a final answer.
Part (ii)
A few candidates differentiated or tried to integrate both the numerator and the denominator independently, but most knew what to do here and went on to score 2 or 3 marks. A significant minority of candidates neglected to add "+ *c*", thereby losing an easy mark.

Question 2
Many candidates had difficulty with this question. In some cases it would seem that this was due to a failure to read the question properly, but it was also apparent that a significant minority did not understand how to generate the terms of the sequence. Even many of those who did generate the terms successfully then either ignored the sigma notation or summed an incorrect number of terms.

Question 3
This was done very well indeed, with many candidates scoring full marks. A few slipped up with the arithmetic and lost the accuracy marks, but the method was very well understood.

Question 4
This was very well done; the majority of candidates obtained full marks and almost all achieved at least 4 marks. A few worked with rounded numbers and then over-specified their final answer, thus losing the final accuracy mark, and a few left their calculator in radian mode and usually lost both accuracy marks.

Question 5
Part (i)
One or two easy marks were lost in a surprising variety of ways. Many candidates gave the answer as $y = \sin x$, $y = 2\text{sinx}$ or $y = \sin ½x$ and some omitted "$y =$".
Part (ii)
Only a few candidates presented good sketches with the key points clearly identified. Too much was often left to the imagination of the marker. Candidates are reminded of the need to indicate amplitude, period and centring by clear scales and labelling. Unnumbered strokes on the axes, for instance, are insufficient.

A variety of misunderstandings was evident. $y = \sin(x - 3)$ was a common error, and occasionally $y = 3\sin x$ or $y = -\sin x$ were seen.

Question 6
Part (i)
This was generally very well done, but some candidates gave the area of the triangle as $\frac{1}{2}a^2$ and a few gave the area of the sector as $r\theta$.
Part (ii)
A significant minority were unable to make progress with this part due to incorrect work in part 9(i). Many others set the area of the sector equal to the area of the triangle and failed to score. A few needlessly converted to degrees, and often went wrong either by losing the accuracy mark or making a method error in the formula for the sector.
A surprising number of candidates ignored their correct work in part (i) and began again with incorrect expressions.

Question 7
Part (i)
A significant minority of candidates chose to work backwards, but few were successful. Many candidates "started at both ends" and tried to meet in the middle – sometimes a method mark was achieved.
A good number of candidates earned the first method mark with one of the correct substitutions, but either failed to complete the argument or tried to show something else.

Part (ii)
Most candidates solved the quadratic successfully and went on to find 14.5 and 166. A surprising number omitted one or more of the three other roots, however.

Question 8
Part (i)
Most candidates achieved a method mark from $\log_a 1 = 0$, but were often unable to resolve the second term. Surprisingly, a few candidates dealt successfully with $\log_a (a^m)^3$, but not with the first term.
Part (ii)
This was done very well indeed. A small number of candidates slipped up in making x the subject, and a few lost the final mark by giving the answer correct to three decimal places.

Question 9
Part (i)
Most candidates used the Trapezium rule correctly and went on to score full marks. A few made bracket errors or misplaced the y-values. Even fewer successfully found the correct value for the area by splitting the area into separate triangles and rectangles. This approach is not recommended – most go wrong and fail to score.

Part (ii) (*A*)
A minority worked out what to do here and used a correct value of x to find y, which was usually correctly compared with 4.4. However, many candidates misunderstood what was required, substitution of 3 or 3.6 were common errors. A few unsuccessfully tried to compare cross-sectional areas.

Part (ii)(*B*)
Most candidates integrated successfully and substituted the correct limits to find the correct area. However, some made an error in one of the terms, and some omitted the factor of ${}^{5}/_{81}$, which cost the later accuracy marks A few candidates lost marks by substituting incorrect limits.

Question 10

Part (i)
The majority of candidates gained full marks on this question. A significant minority differentiated and substituted in the midpoint, or the endpoints of the chord and found the mean. Whilst these approaches do achieve the correct numerical answer, they nevertheless went unrewarded.

Part (ii)
Many candidates clearly didn't understand the notation, and either produced expressions involving x and h, or "expanded brackets" and worked with $5f + fh$.
A good number of candidates did understand what this question was about, and successfully substituted to obtain correct expressions. Some made sign errors or slips in arithmetic: $h + 12$ was a common wrong answer, and a few knew what the answer was supposed to be and "back-engineered" their incorrect work accordingly.

Part (iii)
Only a few candidates used the correct terminology or notation here. Some worked with $h = 0$ and a good number ignored part (ii) and differentiated. Neither approach scored.

Part (iv)
Many candidates found the correct equation and went on to achieve full marks. Some didn't read the question carefully and used $(5, 15)$ with $(3.125, 0)$. A small number of candidates found the equation of the normal and were thus only able to access two method marks.

Question 11

Part (i)
Many scored full marks in this part, but of those who derived the equation, a significant minority did so incorrectly, thus losing the first mark. "bt" was sometimes quoted as the gradient, and "a = intercept" was a common error. Some candidates failed to state the gradient or the intercept, simply drawing lines to their equation or linking with $y = mx + c$. This is insufficient.

Part (ii)
Most completed the table successfully, and went on to plot the points and draw a suitable line of best fit. A few lost an easy first mark through poor calculator skills (2.34 instead of 2.37 was quite common) and some rounded to 1 decimal place. A few candidates drew a curve of best fit, or failed to use a ruler.
Most were able to find the gradient of the line for an easy mark, but many failed to link this to b. Similarly, the instruction to find the value of a was often disregarded. Surprisingly, many candidates simply stopped when they had found a and b, thus losing the last two marks.

Part (iii)
The majority of candidates successfully obtained the correct value, but a significant minority lost an easy mark by failing to give the answer in context as an integer.

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments:

There was a very pleasing standard of work produced on this paper. The majority of candidates were clearly well prepared, and there were many excellent scripts, with a fifth of the candidates scoring over 65 marks, and 90% scoring over 30 marks. There appeared to be an improvement in performance on some topics, such as the modulus function, implicit differentiation and inverse trigonometric functions. There was little evidence of learners running out of time. Standards of presentation were as variable as ever, but many scripts were well presented and clearly argued.

Candidates sometimes offer repeated attempts at questions. Under these circumstances, learners should be told to cross out the ones which they do not wish to be marked. Otherwise, we mark the final complete attempt, notwithstanding if it scores fewer marks than previous ones!

Comments on Individual Questions:

Section A

1. This proved to be a straightforward starter question, with 80% of candidates scoring full marks. Some candidates stopped at $\pi/2+2\sin\pi/4$, presumably because they did not appreciate that 'value of' means numerical. A few weaker candidates confused differentiation and integration, either giving the wrong coefficient or sign for the $\sin x/2$ term.

2. Virtually all candidates formed the composite function in the correct order to obtain $fg(x) = \ln(2+e^x)$. A few then simplified this to $\ln 2 + x$ and therefore made no further progress. Of those who did correctly proceed to $2 + e^x = e^{2x}$, a substantial minority then incorrectly took logs of each side to reach $\ln 2 + x = 2x$. Of those who correctly rearranged the equation into a quadratic in e^x, nearly all then gained full marks, correctly rejecting the $e^x = -1$ solution.

3. Integration by parts was well understood, with just under half candidates scoring full marks for this question. Very occasionally, candidates took $u = x^{-1/2}$ and $v' = \ln x$, and were unable to score any marks. With u and v correct, the next hurdle is to simplify the $2x^{1/2}.1/x$ integrand, and some failed at this stage, and attempted to integrate the product term by term. Having negotiated this successfully, most got full marks, though very occasionally the final answer was spoiled by using $4\ln 4 = \ln 16$.

4. Sketches of the modulus function with $y = -x$ were generally well done, though quite a few lost a mark for neither clearly indicating the intercepts nor making a clear statement that there were two of them. The roots were then usually found correctly, with less evidence of faulty modulus algebra than in recent years.

5. This question was extremely well answered, with the majority of candidates scoring full marks.

5(i). The chain rule on V was successfully negotiated by over half the candidates, and then correctly evaluated at $x = 2$.

5(ii). Virtually everyone who scored 4 for part (i) went on to apply the chain rule $dV/dt = dV/dh \times dh/dt$, or some variation of it, to get full marks here. The rest usually earned the first two of the three marks.

6. This question as also very well done, with half the candidates scoring full marks.

6(i). The implicit differentiation was well understood, though there were the usual blemishes from mixing up the derivative and integral formulae for sin 2y. A few candidates re-arranged the equation to get x in terms of y, then found dx/dy, and then the reciprocal dy/dx.

6(ii). Re-arranging the given implicit equation to give $y = \frac{1}{2} \arcsin(x - 1)$ was well understood, and the transformations were usually accurately described. Note that the preferred terms here are 'translation' and 'one-way stretch'.

7. The first B1 for factorising $x^{2n} - 1$ was well done, but convincing proofs of the divisibility of $2^{2n} - 1$ by 3 were few and far between. We awarded M1 if candidates recognised that either 2^n-1 or 2^n+1 were divisible by 3, and two 'A' marks for proving this. The next 'A' mark was gained for stating that the consecutive numbers 2^n-1, 2^n and 2^{n+1} must include a multiple of 3, and the final mark for stating that 2^n is **not** divisible by 3; however, many candidates wrongly stated that 2^n was even and therefore not divisible by 3, or that two consecutive odd numbers must include a multiple of 3. The most elegant alternative solution seen was:

$x^{2n} - 1 = (x^2 - 1)(x^{2n-2} + x^{2n-4} + \ldots + 1) \Rightarrow 2^{2n} - 1 = (2^2-1)(2^{2n-2} + 2^{2n-4} + \ldots + 1) = 3m$, where m is an integer.

The language used by candidates in their explanations was often rather imprecise. In particular, the terms 'factor' and 'multiple' were often used incorrectly.

Section B

8. Most candidates scored well on this question, which covered calculus topics such as the product or quotient rule for differentiation and integration by substitution, which are generally well understood by learners.

8(i). The first three marks here were usually earned, though a minority of weaker candidates mixed up the product and quotient rules, for example using $v = (x+4)^{-1/2}$ in their quotient rule. The factorisation required to achieve the given result was less successfully done, but just over half the candidates still managed full marks here. There were a lot of repeated attempts at this, for example using the product rule when they got stuck with manipulating their quotient rule expression.

8(ii). This proved to be a straightforward 4 marks earned by over 70% of scripts. The asymptote and the gradient and equation of the tangent at the origin were usually correctly found, followed by the coordinates of Q.

8(iii). This 9-mark question required careful extended work from candidates, but there was a pleasing response, with just under half the scripts earning full marks. The first six of these were for finding the area under the function using substitution. Here, as usual, notation sometimes left something to be desired, with missing du's or dx's, integral signs, inconsistent limits, etc. Most of this we condoned, but we did require du/dx = 1 or its equivalent to be stated. The final three marks depended upon the correct coordinates for the point Q being found in part (ii). Occasionally the triangle area was found using $\int \frac{1}{2} x \, dx$.

9. The calculus here was not particularly demanding, requiring only the derivative and integral of e^{kx}; but the simplification of expressions using the laws of logarithms and exponentials proved to be quite testing and found out quite a few candidates.

9(i). This was an easy write-down for virtually all candidates, except those few who did not know that $e^0 = 1$.

9(ii). The first two marks were pretty universally earned, but deriving $x = \frac{1}{4} \ln k$, together with the final 'A' mark for getting $2\sqrt{k}$, caused a few problems, with some inaccurate logarithm work. For example, $e^{1/2 \ln k} = \frac{1}{2} k$ was a commonly seen misconception.

9(iii). The integration was usually correct, but, thereafter, as in part (ii), the simplification to arrive at $k - 1$ proved to be tricky, with similar errors being made.

9(iv)(*A*). Most attempts correctly substituted $x + \frac{1}{4} \ln k$ for x in f(x) to gain the first M mark, but we needed to see clear evidence of how this simplifies to the given result. Often candidates seemed to be working backwards from this without really understanding the process.

9(iv)(*B*). The definition of an even function was well known, but sometimes the structuring of the proof was indecisively presented. Some used 'f' instead of 'g' (here, f is indeed **not** an even function!), and we required to see either a clear statement of the definition of an even function, or a clear conclusion that g is therefore even. The structure '$g(-x) = \ldots = \ldots = g(x) \Rightarrow$ g is even' is the most transparent formulation to use in such proofs, rather than starting them by stating that $g(-x) = g(x)$, viz the result they are trying to prove!

9(iv)(*C*). The argument here proved beyond most candidates, with only 20% getting full marks. Many stated that f was an even function, perhaps thinking that any line of symmetry sufficed. Sometimes it was indeed a little difficult to decide whether candidates were referring to f or g in their answers.

4754 Applications of Advanced Mathematics (C4)

General Comments:

The performance of candidates on Paper A was similar and comparable to recent papers and the standard of work in the majority of cases was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Paper B, the comprehension, was well understood and most candidates scored good marks here.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Question 2)
- Failure to include a constant of integration (Question 8(v))
- Poor anti-logging and rules of logarithms (Questions 8(iv) and 8(v))
- Failure to give clear descriptions in the comprehension paper (Questions 2 and 6)
- Inappropriate accuracy, for example in Question 4, candidates either gave insufficient accuracy (answers to the nearest integer) or they gave too much accuracy (answers to 2 or more decimal places) – candidates are reminded to give answers to 1 decimal place for questions involving trigonometry
- Failure to give exact answers when required (Questions 3 and 5(ii))
- Failure to give sufficient detail when verifying given results (Questions 5(i), 5(ii), 6, 7(ii), 8(i), 8(ii), 8(iii), 8(iv) and 8(v)).

Some candidates assume that showing that a vector is perpendicular to one vector in the plane is sufficient to show that it is a normal vector.

Quite a number of candidates failed to attempt some parts but there did not appear to be a shortage of time for either Paper.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper. Furthermore, centres are requested that Papers A and B are not attached to each other and they must be sent separately for marking.

Comments on Individual Questions:

Paper A

Question 1

The majority of candidates correctly worked out the values of R and α although some candidates lost the first method mark by not including R in the expanded trigonometric statements $R\cos\alpha = 1$ and $R\sin\alpha = 3$. Some failed to give α in radians and a small minority stated R as10 rather than the correct $\sqrt{10}$. Candidates were less successfully in showing that $\cos\theta - 3\sin\theta = 4$ had no solutions with many simply stating that $\theta + 1.249 = \arccos\left(\frac{4}{\sqrt{10}}\right)$ 'does not work' or gives a 'math error'.

Many candidates failed to explain or give an equivalent mathematical statement that the maximum value of $\cos\theta - 3\sin\theta$ is $\sqrt{10}$ which is less than 4 and so did not score the final mark in this question.

Question 2

The binomial expansion of $\left(1+\frac{x}{p}\right)^q$ was done extremely well by the vast majority of candidates with the most common error being the failure to correctly deal with the x^2 term with many giving the coefficient (of this term) as $\frac{q(q-1)}{2p}$ rather than the correct $\frac{q(q-1)}{2p^2}$. It was surprising how few candidates could go on to form the correct pair of simultaneous equations and fewer still who could solve this pair of equations accurately and successfully. Those candidates who correctly found the value of p usually went on to state the set of values of x for which the expansion was valid.

Question 3

The vast majority of candidates considered the correct integral (with correct limits and including the factor of π) for the volume of revolution generated by rotating the given curve about the y -axis and most went on to integrate $\sqrt{y}$ correctly. A number of candidates, however, misread the question and instead tried to calculate the volume or revolution generated by rotating the curve about the x -axis. It was the mention of a rotation of 180° that seemed to concern many candidates and a considerable number divided the correct answer of $\frac{16}{3}\pi$ by 2.

Question 4

The majority of candidates correctly replaced $\sin 2\theta$ with $2\sin\theta\cos\theta$ and $\cos 2\theta$ with one of $\cos^2\theta-\sin^2\theta$ or $1-2\sin^2\theta$ or $2\cos^2\theta-1$, although a minority of candidates made the costly mistake of forgetting the 2 in the latter two identities. While a majority of candidates correctly obtained $2\sin\theta\cos\theta=\cos^2\theta$ many then cancelled $\cos\theta$ from both sides of the equation instead of factorising to obtain $\cos\theta(2\sin\theta-\cos\theta)=0$ and so lost the solution to the equation $\cos\theta=0$.

Most candidates correctly simplified $2\sin\theta-\cos\theta=0$ to $\tan\theta=\frac{1}{2}$ and obtained the correct answer of 26.6° although a small minority gave an answer in radians or additional answers both inside and outside of the given range. A number of candidates, after applying suitable double angle identities, squared their equation leading to either a quadratic equation in either $\sin^2\theta$ or $\cos^2\theta$. These attempts usually contained sign and/or algebraic errors or even, when the correct disguised quadratic was obtained, and solved, additional incorrect solutions (coming from the earlier squaring of the single angle equation) were given.

Question 5

This question provided a certain amount of discrimination between candidates with some producing clear, concise arguments for why $\sec^3\theta=2$ and why the ratio of the lengths ED to CB was $2^{\frac{2}{3}}:1$ while a significant number left both parts of this question blank or scored no marks. The majority of candidates, however, scored at least one mark in (i) for starting that $\mathrm{AC}=x\sec\theta$ (or equivalent) or that $\mathrm{AD}=2x\cos\theta$ but many failed to find corresponding expressions for either AD and AE or AC and AB in terms of x and one of $\sec\theta$ or $\cos\theta$. Examiners noted that many candidates did not make it clear which expression corresponded to which side of the three triangles given in the question making it almost impossible for examiners to award any marks.

In part (ii) many candidates scored at least two marks for stating that $\mathrm{ED} = 2x\sin\theta$ and $\mathrm{CB} = x\tan\theta$ although many then substituted in the angle from part (i) and tried to derive the exact value of $2^{\frac{2}{3}}$ using approximate values for these two lengths. Candidates who correctly found that $\dfrac{\mathrm{ED}}{\mathrm{CB}} = 2\cos\theta$ usually went on to obtain the correct ratio although many did not show sufficient steps of working to explain how they obtained the given answer.

Question 6

Candidates found this unstructured question on parametric equations quite demanding with the modal mark scored being 2 out of a possible 7. Most candidates scored these two marks by correctly obtaining $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{t^2}$ although the vast majority failed to make any further progress worthy of merit with many trying to argue that the area of triangle OQR is independent of *t* without any further working of a mathematical nature. Of those that failed to obtain the correct derivative a number incorrectly stated that $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 2\ln t$ or implied that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \times \dfrac{\mathrm{d}x}{\mathrm{d}t}$. All that was required from those candidates who had obtained the correct gradient function in terms of *t* was to write down the equation of the tangent $\left(y - \dfrac{2}{t} = -\dfrac{1}{t^2}(x - 2t) \right)$, substitute $x = 0$ and $y = 0$ to obtain $\mathrm{R}\left(0, \dfrac{4}{t}\right)$ and $\mathrm{Q}(4t, 0)$ respectively and hence calculate the area of the triangle as 8 $\left(= \dfrac{1}{2} \times 4t \times \dfrac{4}{t} \right)$ which is clearly independent of *t*. A number of candidates began by finding the Cartesian equation of the curve and correctly obtaining $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{4}{x^2}$ but they then incorrectly used this gradient function in their equation of the tangent.

Question 7

Nearly all candidates correctly obtained the length of the diagonal AG in part (i) with only a small minority stating only the direction vector $\overrightarrow{\mathrm{AG}}$.

In part (ii) a number of candidates seemed to think that just showing one direction vector was normal to the plane was sufficient and some candidates showed all three. It was unfortunate that so many lost marks by not showing the evaluation of the scalar product(s) even though this was a ‘show that’ question and so examiners had to be convinced that the candidates were indeed showing the required results and not simply stating them. The vast majority found the correct Cartesian equation of the plane as $15x - 20y + 4z = 20$.

Most candidates correctly found the direction part of the vector equation of the line AG in part (ii) but very few candidates stated a correct vector equation which needed to begin with either $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ or $\mathbf{r} = \ldots$ The rest of this part was answered well with many correctly obtaining the value of the parameter and hence the coordinates of the point Q. The ratio of AQ : QG was often found correctly although a number of candidates stated this ratio as either 3 : 2 or 2 : 5.

Candidates’ attempts to find the acute angle between the line AG and the plane DPF in part (iv) were varied with the vast majority considering the correct direction vectors of $-4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ and $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ although a number attempted to find this angle using one of the direction vectors in the plane (rather than the normal to the plane). While many correctly obtained an angle of either 56.0 or 124.0 few failed to realise that these were the acute and obtuse angles between the line and the normal and so many did not subtract the relevant right-angle to obtain the correct answer of 34.0°.

Question 8

Nearly all candidates correctly showed the required result in part (i) although a few attempted to use partial fractions with varying degrees of success.

In part (ii) many candidates incorrectly verified that when $x = 0$, $t = 0$ rather than the required result of showing that when $t = 0$, $x = 0$. Those that did begin by setting $t = 0$ usually went on to score both marks in this part.

Part (iii) proved to be quite discriminating with many candidates unable to show that the rate of change of x was proportional to the given product. The most common method seen was to write t as $\ln(2+x)-\ln(2-x)$ and then to differentiate this expression with respect to x and obtain $\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{1}{2+x}+\frac{1}{2-x}$ and then use part (i) to show that $\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{4}{(2+x)(2-x)} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=\frac{(2+x)(2-x)}{4}$, and hence the constant of proportionality is clearly $\frac{1}{4}$. The most common error was a failure to differentiate *t* correctly with many retaining the negative between the two terms. A number of candidates attempted instead to derive the given result by starting with the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t}=k(2+x)(2-x)$ and attempting to solve this using the method of separation of variables. While a number were successful in obtaining the required constant many failed to deal with $\int\frac{\mathrm{d}x}{(2+x)(2-x)}$ correctly or forgot to include the required constant of integration.

Part (iv) was answered well with many correctly starting by either writing $\mathrm{e}^{t}=\frac{2+x}{2-x}$ or $\mathrm{e}^{-t}=\frac{2-x}{2+x}$, the latter of these two usually lead to the correct given answer while the former lead to $x=\frac{2\left(\mathrm{e}^{t}-1\right)}{1+\mathrm{e}^{t}}$ with the vast majority of candidates being unable to explain clearly why this would lead to the given result. As this was a show that question there needed to be a clear indication of how this result would leads to $x=\frac{2(1-\mathrm{e}^{-t})}{1+\mathrm{e}^{-t}}$. Finally in this part many candidates correctly stated that the long-term mass of the substance was 2 mg.

Part (v) was answered with varying degrees of success with the vast majority correctly separating the variables to obtain $\int\frac{\mathrm{d}x}{(2-x)(2+x)}=k\int\mathrm{e}^{-t}\mathrm{d}t$ - however, from this point it was all too clear that a number of candidates did not, as requested, show by integration the given result, but simply wrote down the given answer (or an answer only a single step away from the given answer) without clearly showing how either side of the given equation was obtained. In many cases candidates failed to include a constant of integration that needed to be found using the given initial conditions.

Part (vi) was answered extremely well with many candidates obtaining the correct answer of 0.811 which was achieved by setting e^{-t} equal to zero and substituting 1.85 for x. The most common error seen by examiners was to set $\ln\left(\frac{2+x}{2-x}\right)$ equal to 1.85 and solve for k with e^{-t} equal to zero.

Paper B

Question 1

Nearly all candidates correctly stated the hub height of the turbine although a number incorrectly found this height as 59.5 m (which came from using the diameter of the blade rather than the radius).

Question 2

While the majority of candidates correctly explained how the figure of 12 m was obtained with the most common method being via the calculation $\frac{0.8\times 99.5}{6.7}$ which lead to a value of 11.88… many did not give sufficient detail or tried to justify the figure of 12 without showing any calculation at all.

Question 3

Parts (i) and (ii) were nearly always correct although in part (iii) many candidates failed to use the value of 7.3 to show that when the photomontage was printed on A3 paper, the height of the wind turbine was consistent with the angle of elevation found in part (ii).

Question 4

The responses to this question were mixed with many candidates failing to find the height above A of the lowest visible point as 199.5 m with many incorrectly using a height of 219.5 m. Furthermore, many candidates failed to read the question carefully and stated that the distance AC was 140 m rather the horizontal distance from A to C.

Question 5

Nearly all candidates correctly stated that $\alpha = \arctan\left(\frac{72}{800}\right)$ although many assumed incorrectly that triangle TQB was right-angled. Of those that did realise that triangle TQB was not right-angled many took the slightly more long-winded approach of using the cosine rule to find β instead of realising that $\beta = \arctan\left(\frac{90}{800}\right) - \arctan\left(\frac{18}{800}\right)$. Finally, many candidates did not realise that if they are required to show that two answers agree to 2 significant figures then they must quote both value correct to 3 significant figures and so in this case examiners needed to see as a minimum of $\alpha = 5.14$ and $\beta = 5.13$ followed by 5.1.

Question 6

While many candidates made reasonable estimates of the percentages of participants many confused the two cases of ‘too large’ and ‘too small’, or they did not make the numbers used in their calculations clear. A number of candidates did not calculate any percentages but instead gave the total number of participants in the three different groups. The most common estimates were that those who opted for 70 mm and 80 mm would believe that 75 mm was ‘about right’, while those who opted for 50 mm and 60 mm would say ‘too large’ and finally those who opted for 90 mm, 100 mm and 110 mm would say ‘too small’. Finally many candidates did not state any of their assumptions clearly even though this was specifically asked for in the question.

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments:

There were many good scripts from candidates who were well prepared for this paper.
Most candidates made very good progress through the first section. There was some evidence that many candidates were pressed for time and there were a surprising number of part questions not attempted, even when results that could be used were already given. Some of the algebraic manipulation required was quite lengthy, and for candidates lacking confidence in this area possibly too daunting.

There were several instances of the careless placing of answers, making it difficult for the Examiner to give the correct mark in the correct section. If a result has been obtained elsewhere there should be a signpost in the correct answer space, so that the Examiner can find the solution and reward it.

There was also much poor presentation, with work placed out of sequence and very often difficult to read. Figures should be rewritten, not altered. Use additional answer sheets when work is spilling out of the space available. Make sure that the annotations on a diagram are not obscured, and make sure that pencil diagrams are strongly drawn as the scanning process cannot pick up faint lines or shading.

Comments on Individual Questions:

Section A

Question No. 1
(i) This straight forward start to the paper was usually correct. Occasionally $8 - 2p$ was seen as the determinant. More rarely the determinant was forgotten, or there was an incorrect rearrangement of the elements of the matrix.

(ii) This was successfully done by the majority of candidates who multiplied the original area by the determinant. Some laboured methods to find the area of the original triangle by trigonometry usually produced an inaccurate answer. Some candidates worked with the transformed triangle; a decent sketch would have helped to find an easy method for the area in this case. Not many were successful.

Question No. 2
(i) Many correct results were seen here. Very rarely the meaning of z* was not known. The most common error was to mismanage the multiplication in the denominator, forgetting to square 5 or sometimes forgetting to use $j^2 = -1$. Another fairly common mistake was 10j in the numerator, instead of 20j.
(ii) Most candidates achieved full marks in this section, follow-through accuracy being allowed here. Errors in copying figures and sign errors in solving $2-b=\text{Im}(z^*/z)$ were not uncommon.

Question No. 3
(i) The matrix row by column multiplications were usually correctly carried out. λ was usually correctly found, but an extremely common mistake was to obtain the value 25 from the matrix multiplication, but then to claim $\mu = \frac{1}{25}$.

(ii) The wrong μ in part (i) would then lose one of the marks in this part, where it would be incorrectly used to obtain the right inverse. To find $\mathbf{B}^{-1}$ the instruction in the question was "hence…"; direct work on **B** was not necessary and rarely successful.

Question No.4

(i) This part question was possibly the most successfully answered of all. There were those who were careless with brackets and produced ….+*p* in the final expression. A few candidates failed at the outset by not splitting the summation, attempting a multiplication $\sum r^2 \times \sum r$. Candidates who were unwise enough to multiply out to a quartic expression could easily take out a factor of *n* but then had to demonstrate the factorisation of the cubic. The final result was given, so it was not enough to quote it at this stage, the factor $n+1$ needed justification.

(ii) It seemed that this section made algebraic demands beyond the abilities of many candidates. In trying to seek out the relevant terms by inspection many commonly failed to obtain both the ones in n^3. Several candidates thought that the problem could be solved by setting a value for *n*, sometimes zero, more often 1. A number did not attempt it at all.

Question No. 5

(i) This was well done by a majority of candidates. The circle was recognised to be a 'circle' but many 'circles' were extremely poorly drawn. Its placement was quite often centred on 3 – 4j. The half line was then also given a start point at 3 – 6j. The radius of the circle with either of these centres was such that the circle should have passed through the Origin. Quite a lot of candidates ignored this detail, and in the absence of a clear radius shown, only earned this mark if other identifiable points (e.g. -3 – j) were indicated on the diagram. Most candidates knew that a half line "straight up" was involved but sometimes annotations on the diagram were such that it appeared that the line started from the negative real axis, or from the circumference of the circle.

(ii) Given a correct diagram this was usually correct. Reverting to coordinates was penalised as the question specified that a complex number was required.

(iii) Most candidates, but not all, knew that the region required lay outside their circle, and most were able to indicate the other boundary and the region correctly. A fairly common mistake was to shade the inside of the circle between the two lines. Often the shading was difficult to make out on the screen after scanning. It should be strongly marked.

Question No. 6

An encouraging number of complete logical arguments were seen, but many candidates lost the final two marks for inadequate explanation. It is incorrect to claim the result is "true for *n* = *k* + 1" before both pointing out the structure and conditioning on the assumption. Using abbreviations such as "*n* = *k* + 1 is true" is also nonsensical and insufficient.

Section B

Question No.7

There were many routes possible to the solutions of the various parts to this question, but the examiners need to see the answers to each section in the right place and this was not always the case. Several candidates did the majority of the working for parts (i), (ii) and (iii) in part (i), by finding both quadratic factors, then using the sum and product of roots of the quadratic $2z^2+3z-2=0$ for part (i), solving the same quadratic for part (ii) and expanding the product of both quadratic factors to obtain the answers for part (iii). This was a very efficient method. In finding the product $\alpha\beta$ two common mistakes were to equate the product of the four roots to either $\frac{26}{2}$ or to -26. After this, quite a number of candidates gave up in (ii). Starting from the two correct equations for $\alpha+\beta$ and for $\alpha\beta$ some candidates were able to deduce the values of α and β by inspection.

Part (iv) was omitted by many and there were few correct answers. For the rest a common mistake was to divide the roots by j. It was evident that a fairly high proportion of candidates found the demands of this question too much to cope with and quite a number of scripts returned with no response in one or more parts.

Question No. 8

By contrast this question was probably the most successfully answered of all questions in the paper.
(i) Only a small number of candidates failed to write the asymptotes as the equations of lines. Three distinct equations are required. To write $x = 1, - 4$ is ambiguous. Rather than $x = 1$ or $x = - 4$ The asymptotes are both $x = 1$ and $x = - 4$.

(ii) The majority of candidates showed the calculations for large positive and negative values of x in full. The results are wanted to complete the justification and should not be omitted.

(iii) The curve was usually clearly and carefully drawn, with asymptotic behaviour usually well shown. Annotations are needed on the diagram to show the asymptotes' positions and the intercepts on the axes. These should not be left as working on the side, it is the sketch which is under scrutiny.

(iv) Most candidates scored all marks in this part, but there were some errors in the inequality signs used.

Question No. 9

There was evidence here from untidily rushed or incomplete work that candidates found that time was running out.

(i) Most candidates achieved the first four marks but there were quite a lot of errors, even in the initial terms. Sensible setting out of the work helped to generate the cancellation pattern of the fractions. Three consecutive values of r were needed to show this. The answer was given, and it was evident in many cases that the result was prematurely anticipated, either insufficient terms being shown or mistakes made which invalidated the work.

(ii) This question could be correctly answered from the given result to part (i), but even so, many wrong limits were chosen, the most popular being zero, and there were many omitted the part altogether.

(iii) Here there were many half-finished solutions and also no responses made. For the limits of the summation, 45 and 105 were often used. Using the result of (i) some candidates subtracted the sum from $r = 1$ to 20 instead of the sum from $r = 1$ to 19. In some cases premature approximation lost the final mark.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments:

Most candidates appeared to have sufficient time to complete the paper, and were able to demonstrate a sound understanding of the topics being examined. Q.1 (on inverse circular functions and polar coordinates) was the best answered question, and Q.2 (on complex numbers) was the worst answered.

Comments on Individual Questions:

Q.1(a)(i) Almost all candidates wrote down f '(x) and obtained the binomial series correctly. The only common errors were incorrect signs.

Q.1(a)(ii) The series was usually obtained by integrating the series from part (i), but most candidates did not score full marks on this part. Very many candidates did not mention the constant of integration at all, and many that did left + c in their answer, omitting to show that the constant was zero.

Q.1(b) This integration was very well done. Errors such as arcsin($4x/3$) instead of arcsin($2x/\sqrt{3}$), and omitting the factor ½, were fairly common.

Q.1(c)(i) The curve was usually drawn well, although some continued the curve beyond the domain required.

Q.1(c)(ii) Most candidates correctly stated that r tends to infinity; although some just wrote 'r increases', which was not an adequate answer. Many candidates thought that r tends to zero.

Q.1(c)(iii) The enclosed area was usually obtained correctly.

Q.2(a)(i) Candidates who used the half-angle formulae quickly obtained $1 - \cos\theta - \text{j}\sin\theta$ and hence $1 - z$. However, many candidates chose to express everything in terms of $z^{½}$, and this approach was much less successful, with many sign errors and missing j's; for example, $z^{½} - z^{-½} = 2\sin½\theta$ was a common starting point.

Q.2(a)(ii) Most candidates scored 3 marks or fewer (out of 8) on this part. $C + \text{j}\,S = (1 - z)^n$ was commonly obtained, and this was quite often rearranged into the given form, using part (i) or otherwise. Candidates were then expected to give explicit expressions for C and S by taking real and imaginary parts. It was crucial to state that j^n is real when n is even, but most candidates did not do this. Also, $(\sin½\theta)^n$ often became $\sin½n\theta$.

Q.2(b) Most candidates understood the exponential form of a complex number, and knew how to obtain the cube roots. There were some careless slips such as omitting j or π from the exponent, and some candidates did not divide the argument by 3 when finding the cube roots. The modulus of the cube roots was sometimes left as $8^{1/6}$ or $(2\sqrt{2})^{1/3}$ without being simplified to $\sqrt{2}$. The cube roots were usually indicated on the Argand diagram correctly, but a very large number overlooked the request to show z on the diagram.

Q.3(i) The methods for finding eigenvalues and eigenvectors was very well understood, and most candidates scored full marks on this part.

Q.3(ii) Most candidates wrote down the correct expression $\mathbf{M}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$, but the element $(-1/6)^n$ in $\mathbf{D}^n$ very often became $-(1/6)^n$ leading to incorrect evaluation of the elements of $\mathbf{M}^n$. The limiting value as n tends to infinity was very often found correctly.

Q.3(iii) Many candidates wrote $\mathbf{M}^{-1} = \mathbf{P}^{-1}\mathbf{D}^{-1}\mathbf{P}$ instead of $\mathbf{M}^{-1} = \mathbf{P}\mathbf{D}^{-1}\mathbf{P}^{-1}$. Candidates were expected to argue, in a similar way to part (ii), that the elements of $(\mathbf{M}^{-1})^n$ contained $(-6)^n$ and so did not tend to a limit. However, the explanations were very often unclear, and most candidates scored no marks or 1 mark (out of 4) in this part.

Q.4(i) Most candidates showed that $x = \ln(y \pm \sqrt{(y^2 - 1)})$, but very many could not prove the final step $\ln(y - \sqrt{(y^2 - 1)}) = -\ln(y + \sqrt{(y^2 - 1)})$. The solution $x = \ln(y - \sqrt{(y^2 - 1)})$ was often rejected as being undefined.

Q.4(ii) Most candidates obtained a quadratic equation in $\cosh x$ and completed this successfully. Some wrote the equation in exponential form, but these rarely made much progress.

Q.4(iii) The curve was usually sketched correctly, although the y-intercept was often missing or incorrect. Finding the area under the curve in an exact simplified form caused many difficulties. The simplest way was to write the integrated expression as $\sinh x(1 + \cosh x)$ and then substitute $\sinh x = \pm\frac{1}{2}\sqrt{5}$ and $\cosh x = 3/2$; but most candidates changed it into exponential form, which made the substitution much more complicated. Many candidates omitted the final step of subtracting the area under the curve from the area of a rectangle to obtain the area of the specified region.

4757 Further Applications of Advanced Mathematics (FP3)

General Comments:

Most candidates for this paper were able to produce substantial attempts at all three of their chosen questions. Q.2 (on multi-variable calculus) was the most popular question, chosen by over 80% of the candidates. Q.1 (on vectors), Q.4 (on groups) and Q.5 (on Markov chains) were each chosen by about 60% of the candidates. The least popular question was Q.3 (on differential geometry), which was chosen by fewer than 40% of the candidates.

Comments on Individual Questions:

Q.1(i) This was very well answered, with most candidates using the standard formula involving the magnitude of a vector product.

Q.1(ii) Most candidates knew how to find the shortest distance between the two paths, almost always using a scalar triple product.

Q.1(iii) Candidates used a variety of methods to find the points where the shortest distance occurred. Some applied scalar products of the general chord with the directions of the two paths to obtain two simultaneous equations. Some put the general chord parallel to the common perpendicular, which had already been found in part (ii); this was particularly efficient in this case, as the z component was zero. Another approach, quite often used successfully, was to take the general point on one path and put its shortest distance from the other path equal to $\sqrt{5}$, applying the formula from part (i). However, only a minority of candidates succeeded in finding the two points. Many tried putting the length of the general chord equal to $\sqrt{5}$, but the resulting equation proved too difficult to solve. Several candidates found an expression for the distance between the two aeroplanes in terms of time; this is a valid approach for showing that the aeroplanes never came as close as 2.24 km, but it does not answer the question asked and could not earn more than 2 marks (out of 7).

Q.1(iv) The simplest approach was to find expressions for the position vectors of Q and R at time t and then show that these position vectors were equal when $t = 0.1$. A more common approach was to show that the paths of Q and R intersect, either by evaluating a scalar triple product or by equating the components of general points on the two lines. Very many of the latter group omitted to check that the three equations were consistent, and so had not actually shown that the lines did intersect, even though they had found the correct point of intersection. It was then necessary to show that both aeroplanes reached the point of intersection at the same time; but many candidates omitted this step.

Q.2(i) The section was usually drawn correctly. For the line of symmetry, many candidates gave an answer which was not recognisable as the three-dimensional vector equation of a line. A common error was to give the normal line to S, rather than the line of symmetry of the section.

Q.2(ii) Most candidates found the partial derivatives and the stationary points correctly. As the answers were given it was necessary to show sufficient working, and many candidates lost a mark by not showing the calculation of the z coordinate of P.

Q.2(iii) This involved substituting into the equation of the surface and rearranging, which was usually done correctly. Some candidates tried to use an approximate result for small changes using the partial derivatives, although an exact result was required here. Many candidates did not then show convincingly that $\lambda > -4$ for all small values of h and k; some omitted this part, and some resorted to substituting numerical values.

Q.2(iv) Most candidates tried to show that z can take both positive and negative values close to O, but few produced a convincing argument which applied arbitrarily close to O. Many candidates wrote '$3x^2$ is always positive, $6xy$ and y^3 can be positive or negative, therefore $3x^2 + 6xy + y^3$ can be positive or negative', which was not quite sufficient. The simplest way was to consider the section given by $x = 0$, which was $z = y^3$.

Q.2(v) Candidates who started with $\partial z/\partial x = \partial z/\partial y = 18$ were usually able to complete this successfully.

Q.3(i) Almost all candidates obtained the given values of t correctly. However, many candidates ignored the request to give the y-values.

Q.3(ii) Most candidates knew how to apply the parametric version of the formula to find the radius of curvature, and this was very often carried out accurately. The centre of curvature was sometimes omitted, and very often incorrect. A lot of candidates were unable to find the correct unit normal vector, and in particular there were very many sign errors.

Q.3(iii) Most candidates found the arc length correctly, although some were unable to simplify $\sqrt{((\mathrm{d}x/\mathrm{d}t)^2 + (\mathrm{d}y/\mathrm{d}t)^2)}$ to an form that can be integrated.

Q.3(iv) Most candidates selected the correct formula for the curved surface area, and the given answer was frequently obtained correctly.

Q.4(a)(i) Most candidates understood what was meant by associativity, although many demonstrated associativity of ordinary multiplication rather than multiplication modulo 16. Showing that P formed a group was done well, although those who did not exhibit the composition table often lost marks by not doing enough to demonstrate closure.

Q.4(a)(ii)-(iv) These parts were answered very well by most candidates.

Q.4(b) Most candidates gave a correct composition table; some gave a group with only the three elements e, a and b, and some gave a table which included more than four different elements. Most candidates explained that the group was non-cyclic because all the elements were self-inverse.

Q.4(c) Most candidates showed that **X**, **Y** and **Z** were self-inverse (and often completed the composition table) and deduced that G was isomorphic to the group in part (b). Most were able to specify an isomorphism, although some omitted to do so.

Q.4(d) Candidates were expected to explain why none of the four elements could be the identity element. Many did this well, although some lost marks for insufficient working, for example, simply stating that neither p nor q is the identity without further explanation. Several candidates based their argument on known group properties such as 'all groups of order four are abelian'. This was not given any credit as the question required reference to the group axioms.

Q.5(i) The transition matrix was almost always given correctly.

Q.5(ii) Most candidates used the fact that all the equilibrium probabilities were 1/3 to find the correct value $p = 0.25$.

Q.5(iii) This was very well understood, with just a few candidates using the fifth power of the transition matrix instead of the fourth.

Q.5(iv) Most candidates realised that the tasks chosen on day 7 and on day 4 were not independent, and therefore used the probabilities for day 4, together with the diagonal elements from the third power of the transition matrix, to obtain the required probability.

Q.5(v) The expected run length was very often found correctly.

Q.5(vi) Most candidates found the equilibrium probabilities successfully, some using the equilibrium equation, but most by evaluating high powers of the transition matrix. Some of those who obtained the limiting matrix did not then write down the equilibrium probabilities for tasks A, B and C.

4758 Differential Equations (Written Examination)

General Comments:

Candidates performed well on this paper and the majority of the responses were of a high standard. The level of accuracy displayed by most candidates was commendable. The methods required to solve the second order differential equations in Questions 1 and 4 were known by almost all candidates and these two questions were attempted by the majority of the candidates. Question 3 was the least popular question. Very few candidates attempted more than three questions.

Comments on Individual Questions:

Question No. 1

Second order linear differential equations

(i) All candidates were familiar with the method of solution required in this part and there were very few arithmetical errors.

(ii) The majority of the candidates applied the initial conditions accurately and scored full marks.

(iii) Almost all candidates differentiated their solution from part (ii) and equated their derivative to zero. The resulting equation involved exponential terms and it was pleasing to see that most candidates knew how to solve this equation.

(iv) Most candidates recognised the form of the particular integral that was required to solve this second differential equation and worked accurately to find the particular solution. A minority of the candidates did not use the product rule when differentiating the correct particular integral.

Question No. 2

First order differential equations

(a)(i) Almost all candidates recognised that the given differential equation required the application of the integrating factor method and most began correctly by dividing through by x, the coefficient of $\frac{dy}{dx}$. Most candidates found the correct integrating factor and worked through accurately to find the general solution.

(ii) All candidates applied the initial condition to their solution in part (i).

(b)(i) Almost all candidates recognised that the given differential equation required the application of the separation of variables method. This led directly to a solution for y^{-1}. A common error when inverting their expression to find y was to invert term by term, rather than treating the expression as a whole.

(ii) Candidates who found the maximum value of y by differentiation usually scored full marks. Those candidates who attempted to argue the result based on the possible values of the sin x that was present in the denominator of a fraction often produced incomplete solutions. A convincing explanation of how the maximum value of sin x led to the maximum value of y was required.

(c) Almost all candidates rearranged the given differential equation into the form required to apply Euler's method and many scored full marks. A minority of the candidates gave a list of numbers, none of which related to the correct ones, and it was not possible to award any marks. Sight of either 0.5403 or 0.537(2) or equivalent was required as evidence that the method was being applied correctly.

Question No. 3

First order differential equations

This was the least popular choice of question, but the candidates who opted for it almost always scored the majority of the marks.

(i) This was a straightforward application of Newton's second law.

(ii) This part required the application of the method of separation of variables resulting in a logarithmic expression involving *v*. The majority of the candidates worked accurately and found the given expression for v^2 in terms of *x*.

(iii) Most candidates produced a correct sketch showing an increasing curve from the origin to a horizontal asymptote. The second mark was awarded for identifying the value of *y* corresponding to the asymptote.

(iv) This was a simple numerical substitution from given information.

(v) Most candidates were able to write down a correct differential equation obtained by applying Newton's second law. This differential equation could be solved by separating the variables, by using an integrating factor or by finding a complementary function and a particular integral. The latter was the most elegant approach and gave an expression for *v* in terms of *t*, without any further rearrangement being required. Most candidates separated the variables and errors often appeared in the subsequent algebraic manipulation.

(vi) Most candidates integrated their answer to part (v), but many did not include a constant of integration.

Question No. 4

Simultaneous second order linear differential equations

(i) There were many excellent responses to this part and the majority of the candidates scored full marks. There were occasional numerical slips when finding the coefficients in the particular integral.

(ii) Almost all candidates gained the two method marks and the majority also gained the accuracy mark.

(iii) All candidates made a good attempt at this part and most produced accurate solutions.

(iv) Most candidates realised that they needed to equate their expressions for *x* and *y*. Those candidates who had worked accurately up to this point were able to simplify their expression to the trigonometric equation $\cos 2t = \sin t$. Solving this equation posed very few problems. Candidates who had been inaccurate in the earlier parts of the question were not usually able to solve the equation that they obtained by equating their expressions for *x* and *y*.

4761 Mechanics 1

General Comments:

This paper was well answered. Almost all candidates found questions that allowed them to demonstrate their knowledge and the techniques with which they were confident. There were very few really low marks.

There was a noticeable improvement over previous years in certain particular areas: 2-stage motion; connected particles; extracting the cartesian equation of the path of a particle from its position vector at a general time.

Comments on Individual Questions:

Q1(i) In this question candidates were asked to draw a force diagram and many did this successfully. The most common mistake was to reverse the direction the thrust applied to the block, making it into a tension instead; a few candidates omitted the normal reaction. Some candidates replaced the thrust by its vertical and horizontal components and that was entirely acceptable provided that they were not presented as extra forces in addition to the actual thrust.

Q1(ii) In part (ii) candidates were expected to apply Newton's 2nd law to the block and to deduce the frictional force acting on it. Most candidates got this right but there were a few sign errors.
Answer 12.5 N

Q2(i) This question was about the movement of a particle along a straight line with constant acceleration. In part (i) candidates were asked to use the given information to find two equations for the initial velocity and the acceleration and to solve them. Most candidates got this right. Only a few failed to find the equations and there were also some careless mistakes when it came to solving them.
Answer $u = 8$, $a = -2$

Q2(ii) The question then went on to ask for a distance AB where B was the point where the particle was instantaneously at rest. Most candidates successfully found the distance OB but a common mistake was to fail to subtract OA to find the distance requested.
Answer 4 m

Q3(i) Question 3 involved two connected particles in two different situations. In part (i) candidates were asked to write down the equation of motion of each particle. Most did this correctly but a common mistake was to introduce the weight of the block that was on a smooth horizontal table as an extra force.

Q3(ii) The question then went on to ask candidates to solve the equations to find the acceleration of the system. Those who got the right equations in the previous part were almost entirely successful. By contrast those who made a mistake on one or both equations in part (i) were almost entirely unsuccessful. No follow through was allowed from wrong equations in part (i).
Answer 3.27 $\mathrm{m\,s^{-2}}$

Q3(iii) In part (iii) the table was titled and the system was in equilibrium. Candidates were asked to find the angle of the table. There were many correct answers. The most common mistake was to try to work with the weight of the block on the table rather than its resolved component down the slope. A few candidates lost a mark by missing *g* out altogether.
Answer 30°

Q4(i) In this question the position of a particle at time t was given as a column vector. In part (i) candidates were asked to write down **u** and **a** as column vectors. Most were successful in this but a common mistake was to give **v** instead of **u**.

Answer $\begin{pmatrix} 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ -8 \end{pmatrix}$

Q4(ii) In the next part candidates were asked to find the speed at a certain time and this was well answered with many recovering from errors in part (i). Follow through was allowed for the values of **u** and **a** that they found in part (i). Common mistakes were sign errors and not distinguishing between speed and velocity.

Answer $\begin{pmatrix} 2 \\ -10 \end{pmatrix}$, 10.2 m s^{-2}

Q4(iii) In the final part candidates were asked to show that the position vector at time t led to a given cartesian equation for the path of the particle. This was answered confidently and almost entirely successfully.

Q5 This question was on projectiles. It involved Mr McGreggor throwing a stone at a pigeon, missing it and hitting the window of his house instead. It was extremely well answered.

Although presented as a single question for 7 marks, it actually broke down into two parts: showing that the stone did not go high enough to hit the pigeon and then showing that it did hit the window. Most candidates found the maximum height of the stone and showed that it was less than the height of the pigeon. However, a considerable number substituted the height of the pigeon in the quadratic equation for the height of the stone at time t and then showed that this equation had no real roots; this showed considerable mathematical understanding. Full marks were available for either method and for any correct variant on them, for example working with the equation of the stone's trajectory.

Most candidates found the correct height of the stone when it reached the house but many lost a mark by failing to give a convincing argument that this height was within the interval for the window.

Answers Max height of stone = 3.27 m, Height at the house = 0.975 m

Q6 Question 6 was about modelling. It involved building up a model in three stages of increasing sophistication. At each stage candidates were asked to comment on which aspects of the model had changed and which had remained the same. The context was estimating the depth of a mine shaft from the time it took a stone to reach the bottom. Throughout the model was checked against local records. This question was very well answered.

Q6(i) The question started with applying a simple model given by a formula and comparing the depth it gave to local records. It then went on to ask for an explanation of the model. Almost all candidates answered this fully correctly.
Answer 320 m

Q6(ii) The question then moved on to the second model which was given by a velocity time graph. Nearly all candidates obtained the correct distance but many lost a mark by not making a numerical comparison of their result with the local records.
Answer 275 m

Q6(iii) In this part candidates were asked to identify one respect in which the two models (so far) were the same and one in which they were different. Many candidates gave good answers. In both models the stone has acceleration of g for the first 5 second but then in model B it has constant velocity while in A it continues to accelerate. No marks were given for answers that referred to the conditions given in the question, such as that it takes 8 seconds, nor for answers that compared the mathematical presentation, for example algebra against a graph.

Q6(iv) The question then moved on to Model C where there was variable acceleration and so calculus had to be used. This was very well answered. Only a handful of candidates tried to use constant acceleration formulae. Most carried out the integration and did the appropriate substitution to find the distance covered in the first 5 seconds successfully, and then went on to add on the distance covered at constant velocity. All but few candidates handled the two stage motion correctly.

The final mark required the distance found to be related to the local records and in this case it was necessary to identify the interval within which it lay. Many candidates did not do so and so scored 5 out of 6.

Answer $158\frac{1}{3}$ m

Q6(v) This part was similar to part (iii) asking about how the model had developed. Those who had done well in part (iii) tended to do well here too. Both models involved terminal velocity but its value was different. In the new model the acceleration was variable for the first 5 seconds whereas it had been constant in the previous model.

Q7 This was the second of the long questions on the paper, worth 18 marks. It was set in the context of raising an unexploded bomb from a hole on a building site. This question was quite challenging and many candidates were unsuccessful on the later parts.

Q7(i) The question started with a straightforward piece of trigonometry for 1 mark, and almost all candidates were successful.
Answers 0.8 and 0.6.

Q7(ii) The question then went on to consider the horizontal and vertical components of acceleration in a particular situation. The first demand was to show that the horizontal component is zero. Most candidates got this right but some lost a mark by failing to take the step of going from zero resultant force to zero acceleration; this was a given result and so a high standard of argument was expected. The question then went on to find the vertical component of acceleration and this elicited many good answers. A few candidates failed to convert the weight of the bomb to its mass, and some missed it out completely.
There were fewer sin-cos interchanges than might have been the case a few years ago.
Answer 3.27 m s^{-2}.

Q7(iii) The question then went on to consider a general situation during the lift. The first request was derive a given result for *T*. Many candidates lost marks here by not relating it to the horizontal direction. Some candidates may not have been aware that because this was a given result a high standard of argument was expected.

Candidates were then asked to show that the given result for *T* could be written in a different form. While there were plenty of correct answers, there were also many that appeared to conjure the given result out of incorrect working.

Q7(iv) In this part of the question, the bomb was at the height at which its vertical acceleration was zero and candidates were expected to use the result given at the end of part (iii) to discover this. Only the stronger candidates were successful. Many of those who attempted to find the equation of motion used the wrong angles or the wrong tensions, or forgot about the weight completely.
Answer Acceleration = 0 m s^{-2}

Q7(v) In part (ii) the bomb was in a position where it was accelerating upwards. In part (iv) it was in a position where equilibrium was possible but there could be no upwards acceleration. The final part of the question considered the hypothetical situation where the bomb was at the top and so was at the same level as the winches. Candidates were asked to explain why equilibrium was impossible in this situation and to state the acceleration. While there were some excellent explanations many were garbled or wrong. Many candidates said there would be zero acceleration. Answer $g\,\text{m}\,\text{s}^{-2}$ vertically downwards.

4762 Mechanics 2

General Comments:

The standard of the solutions presented by candidates was pleasing. Most candidates were able to make a reasonable attempt at most parts of the paper. There was some evidence that candidates felt rushed towards the end of the paper.

As always, candidates should draw clear and labelled diagrams and these are always appropriate when dealing with forces or velocities. A lot of potentially very good work was marred by sign errors that, perhaps, could have been avoided by having a clear diagram.

Comments on Individual Questions:

Question No. 1

Momentum and Impulse

Candidates showed that they were able to write down equations using the principle of conservation of linear momentum and Newton's experimental law, but these equations were often inaccurate in the detail. This appeared to be due to a lack of understanding of some of the situations described in the question.

(a)(i) Most candidates scored full marks.

(ii) The best approach was to find the resulting speed of Q in this new situation and show that it led to an invalid value for the coefficient of restitution. A significant number of candidates did not realise that this part of the question had to refer to a different situation to that in part **(i)**. As a consequence, they carried forward the values of the coefficient of restitution and the speed of Q found in part **(i)**.

(iii) There were some good, well-reasoned answers to this part, leading to the conclusion that the speed of Q was unchanged and equal to the value found in part **(i)**. A common error was to attempt to find a new value for the speed of Q. Other candidates realised that the speed of Q was unchanged but were not able to support their realisation with a convincing argument involving the absence of an external force or conservation of momentum for the truck.

(iv) Only a minority of candidates earned more than a single mark. Most candidates did not appreciate the significance of the fact that the speed of the object was given *relative* to P. This relative speed was often used as the actual speed.

(b) The majority of candidates considered the horizontal components of the velocity before and after impact and concluded that since these components were not equal, the plane could not be smooth. A common error was to interpret an elastic collision as one in which the coefficient of restitution must be equal to one. This led them to compare the vertical components of the velocity before and after impact.

Question No. 2

Work, Energy and Power

As is often the case in problems on the topic of work and energy, an alternative method of solution using Newton's second law and *suvat* equations is available. In parts of the question where no method is specified, either approach is acceptable, but when there is an instruction to use a method involving a particular part of the specification such as "use an energy method" candidates should follow the instruction.

(a)(i) Almost all candidates scored full marks.

(ii) Most candidates used linear momentum to find the speed of the block. Some candidates did not include the mass of the bullet, using 3.96 kg instead of 4 kg as the final mass.

(iii) The majority of candidates followed the instruction to consider mechanical energy, and equated a loss in kinetic energy to a work done against the resistance. A common error was to use incorrect masses in the kinetic energy terms, neglecting to include the mass of the bullet embedded in the block.

(b)(i) Most candidates used an energy method, as instructed, but a significant minority of candidates used Newton's second law and *suvat* equations. The energy equation required 5 terms: two kinetic energy terms; a gravitational potential energy term; two work done terms, one involving the tension and the other involving the frictional force. The common errors were

- omitting the work done by the tension
- including the weight component term twice, once as a work done term and once as a gravitational potential energy term
- omitting one of the kinetic energy terms
- using forces instead of work done terms in an energy equation.

(ii) Most candidates multiplied their frictional force from part (b)(i) by the given speed and gained both marks.

Question No. 3

Forces and Equilibrium

Candidates appeared to be confident with the content of this question and there were many very good, well-presented solutions.

(i) Almost all candidates produced clear and concise moments equations followed by accurate working to give the coordinates of the centre of mass of the inn sign.

(ii) Again, almost all candidates scored full marks.

(iii) The single mark available in this part required the labelling of the internal forces to all of the rods and the external forces at B and C. The internal force in each rod should be marked with a pair of arrows and a label such as T_{AB}. This would help candidates to produce accurate equations when evaluating the magnitudes of the forces.

(iv) Most candidates took moments about A and used the results from part (ii) to find X_D.

(v) There were many fully correct solutions to this part. The clear and logical presentation of most of these solutions was very pleasing and demonstrated an ability to work out a strategy before embarking on writing down equations. A minority of candidates resolved horizontally and vertically at A, B, C and D and then faltered in trying to evaluate the forces that were required. Apart from numerical and sign errors, the most common errors were to assume that, because triangle ACD is isosceles, $Y_A = Y_D$ and the internal force in AC = the internal force in CD.

Question No. 4

Centre of mass

There were many pleasing responses to this question, with good diagrams and logically presented work. When a given answer has to be shown, it is important to remember that clear explanations are required. The majority of candidates produced moments equations that were dimensionally correct, but a minority of candidates omitted the distance part of at least one of the terms in their equations.

(i) Most candidates knew the method of approach to use, but there were two common errors:

- the ratio of the densities of the two materials was reversed or ignored
- the formula used for the surface area of the cylinder was incorrect.

Many candidates did not address the fact that they needed to show that the centre of mass of the cylinder was on OC. A simple statement that this was true by symmetry was sufficient.

(ii) Candidates who drew a clear diagram usually gained full marks in this part. The significant number of candidates who did not draw a diagram often had the expression for tan α upside down.

(iii) Almost all candidates attempted to take moments about an appropriate point. Most candidates attempted to resolve T into two components. A very common error was to consider only one of the components of the force T. Some candidates worked with T and attempted to find the perpendicular distance from the tipping point to the line of action of T. This was rarely accomplished successfully.

4763 Mechanics 3

General Comments:

Most candidates were able to demonstrate a good working knowledge of the topics being examined. The first three questions (on dimensional analysis, elasticity, centres of mass and simple harmonic motion) were well answered; but the last two items on the paper, Q.4(iii) and Q.4(iv), were found to be considerably more challenging.

Comments on Individual Questions:

Q.1(a)(i) The method for finding powers in a formula was very well understood; although some candidates started with the wrong dimensions for velocity or weight. There were a few slips in solving the equations.

Q.1(a)(ii) Most candidates used the first set of values to obtain an expression for k, then used this with the new values of V and r to calculate the mass. This was very often carried out accurately; but careless errors such as forgetting to change one of the variables, or omitting g from the final equation, were fairly common. The answer was sometimes given as 0.05 kg instead of 0.05 grams. The more efficient approach, noting that m is proportional to Vr, was adopted by some candidates.

Q.1(b)(i) This application of the conservation of energy was well answered, although many candidates gave the distance travelled (12.5 m) instead of the required AB (17.5 m).

Q.1(b)(ii) This was also answered well, using conservation of energy. Just a few candidates assumed that the jumper was in equilibrium at the lowest point.

Q.2(i)-(ii) The use of integration to find centres of mass was well understood, and most candidates obtained the given results correctly.

Q.2(iii) Most candidates knew how to find the centre of mass of the composite body. A very common error was to take the distance of the centre of mass of S from the vertex to be 0.625 cm instead of 8.625 cm.

Q.2(iv) Most candidates realised that the centre of mass was vertically below the point of suspension, although very many found the complementary angle (17.4° instead of 72.6°).

Q.3(i) Candidates were expected to show explicit expressions for the tension in each string and to form the equation of motion. This was quite well done and the given result was very often obtained convincingly.

Q.3(ii)-(iii) These parts were usually answered correctly.

Q.3(iv) Most candidates formed a displacement-time equation. Many used x to represent a different quantity in this part (such as the displacement upwards from the centre of motion); this in itself was not penalised, although it is of course not a practice to be recommended. A fairly common error was to put ω equal to the period (even when $\omega = \sqrt{10}$ had been used correctly in the previous part). Those who differentiated their equation to obtain the velocity were usually able to find the speed and direction of motion correctly. Those who calculated the displacement first and then used $v^2 = \omega^2(A^2 - y^2)$ often failed to determine the direction of motion.

Q.4(i) Most candidates used the conservation of energy correctly in this part.

Q.4(ii) Most candidates formed a radial equation of motion, with zero tension in the string, and obtained the given result correctly.

Q.4(iii) There were three stages to consider: using conservation of energy to find the speed immediately before the collision; using the result from part (ii) and conservation of energy to find the speed immediately after the collision; using conservation of linear momentum in the collision. Unfortunately this was rarely seen. A very common strategy was to use conservation of energy from the single particle at A (or B) to the combined particle at A, in effect assuming that energy was conserved in the collision. Most candidates scored 2 marks or fewer (out of 9) in this part.

Q.4(iv) This was omitted altogether by about one fifth of the candidates, and most scored 2 marks or fewer (out of 4). The tension in the string immediately before the collision was quite often found correctly, but previous errors usually prevented a successful conclusion.

4766 Statistics 1

General Comments:

As last year, the majority of candidates coped very well with this paper and a large number scored at least 60 marks out of 72. There was no evidence of candidates being unable to complete the paper in the allocated time. Most candidates had adequate space in the answer booklet without having to use additional sheets. Candidates who did need additional space often used the last page of the answer book, but a number did not, presumably not realising that it was available. It is also pleasing to report that losing marks due to over-specification was less of a problem than in previous years. Some candidates lost a mark in question 6 part (i) due to giving their answer to the mean and/or the standard deviation to 5 significant figures. A very small number gave probabilities to more than 5 significant figures thus again losing a mark.

Candidates usually scored very well on question 4 on frequency distributions, question 6 parts (i) and (iii) on data measures and histograms and on question 7 parts (i) on the binomial distribution. Even part (ii) of question 7, where candidates had to state hypotheses and define *p*, was very well answered. This is very pleasing, as up until recently, this topic has caused many candidates problems.

Questions on which candidates did not score so highly included question 3 parts (iii) and (iv) on probability, question 5 part (iii) on conditional probability and the latter parts of question 7 on hypothesis testing. In question 1, a surprisingly large number of candidates did not know how to find the quartiles correctly from a stem and leaf diagram, although most then knew the definition of outliers, so were able to gain marks in part (ii). Questions 2 and 3 caused difficulties to some candidates who scored highly in other areas of the paper. These two questions appeared to be a little less routine and required engagement with the scenario.

Candidates sometimes did not read the question carefully enough, so explanations did not always answer what was asked or explanations were missed out entirely. Candidates have, in most cases, been well prepared for calculations required in the paper but less so for analysing their findings. Although many candidates gave well written explanations, the poor handwriting, grammar and/or use of English made it difficult to work out what some others were trying to say. There were also problems with fours that looked like sixes or nines and ones and sevens that were difficult to tell apart.

Comments on Individual Questions:

Q1(i) The vast majority of candidates found the median correctly. A small minority misread/ignored the key to the stem and leaf diagram and gave an incorrect answer of 290. However under half of candidates found the quartiles correctly, with many using 5th and 15th values, which was penalised.

Q1(ii) Most candidates gained full marks, often on follow through from quartiles which were slightly out. The most common error was to use the median in calculations. A few candidates started from scratch and calculated mean and standard deviation. Some managed this successfully, but others made errors in their calculations, or incorrectly used a combination of both methods such as mean $\pm$ 1.5 × interquartile range.

Q2(i) Many candidates thought that not losing meant winning, and hence gave the common wrong answer of $0.5^3 = 0.125$. Others tried to consider combinations of wins and draws, often without success. The fact that the question part was only worth 1 mark should have been a clue to the fact that there was an easier approach.

Q2(ii) This was generally well answered although a few candidates interpreted this as ‘find three separate probabilities which they did, and listed them but with no addition thus scoring zero.

Q2(iii) Only a small proportion of candidates used the most elegant approach (the first method in the mark scheme) and of those who did, many forgot to multiply by 6. Most candidates gave lists or tree diagrams to show P(WWL) etc., but many then did not multiply by 3, so the most common answer was 0.22, rather than the correct 0.66.

Q3(i) This was generally well answered.

Q3(ii) Again this was generally well answered, although some candidates truncated their decimal rather than correctly rounding. The use of fractions was preferable here.

Q3(iiii) Only around two thirds of candidates scored the mark here. The most common error was to use combinations.

Q3(iv) Again about two thirds of candidates scored both marks, with many scoring the marks for their answer to part (iii), correct or not, multiplied by 6, rather than for the fairly simple $^{3}/_{5} \times {}^{2}/_{4} \times {}^{1}/_{3}$.

Q4(i) Generally the solution of the equation to find *k* was carried out well, although not always entirely efficiently. Most candidates seemed to know that the sum of the probabilities should be 1, although not all thought it necessary to say so. However some did not include addition signs and so lost the marks, and others seemed to have no idea about addition of fractions, thinking that the sum of the probabilities was $^{5k}/_{70}$. Those who used $k = 1.2$ and verified the sum got some very easy marks, although not all were convincing enough to get full credit. The vast majority used the correct probabilities in part (ii), although not all tabulated them in a table to get the third mark in part (i).

Q4(ii) This part was very well answered with about 80% of candidates scoring full marks. A few candidates only found $E(X^2)$ and a few used spurious division. Very few candidates attempted to find $E(X- \mu)^2$ and those who did were rarely successful.

Q5(i) This part was very well answered but a considerable number of candidates assumed that the probabilities were independent and calculated $P(R) \times P(S)$. Some were more confused about the correct formula to use and calculated $P(R) \times P(R \cup S)$.

Q5(ii) The idea of the Venn diagram was well understood, and most candidates produced a fully correct solution (often following through from an error in part (i)). Very few noticed the contradiction produced by their wrong answer, which gave the outer zone as 0.0936 instead of 0.06 from the question.

Q5(iii) Among those who had not made the independence error in part (i), the correct answer was quite common. The explanation of what the probability means was usually correct but sometimes lacked sufficient detail. There were a few candidates who ‘reversed’ the statement and gave an explanation of $P(S|R)$.

Q6(i) This part was fairly well answered with over half of candidates gaining full credit. A few had no idea how to proceed, but most used correct midpoints, although some made slips with them or occasionally used figures such as 25.5, 45.5, etc. The standard deviation proved more difficult for a number of candidates with a variety of wrong methods seen. Very few used the statistical functions on their calculator to do this question, despite this being the recommended method. A few candidates over-specified either or both of their final answers and so lost a mark.

Q6(ii) Candidates found this part rather more challenging, although almost half scored full marks. Trying to establish the proportion they were after was the biggest stumbling block. However, some were then unsure what to do with the figure of 145.83 once they had found it. Some rounded down to 145 (probably the most common mistake of those who understood what they needed to do) and others failed to finish by finding the percentage, just giving the final answer as a decimal 0.157.

Q6(iii) This part was again well answered with around 80% of candidates gaining at least 4 marks out of 5. Various errors were seen, but none very commonly. The most frequently seen were: using frequency rather than frequency density, using a non-linear scale on one of the axes (usually the horizontal axis), stopping the horizontal axis at 120, and labelling the horizontal axis ‘Class width’.

Q6(iv) Over 90% of candidates scored the one mark available here.

Q6(v) A good number of candidates achieved full marks, and the question was answered better than question 6 part (ii) which is a similar calculation. Of those who got the calculation incorrect most started with 240/990 or 20/990, rather than 200/990. The explanation over certainty was well answered with most candidates achieving this mark, whether or not they got the first 2 marks.

Q6(vi) Although this is essentially a simple question, almost a third of candidates scored zero. Candidates struggled to provide acceptable comparisons, with many relying on terms such as “central tendency” when comparing the means, and relatively few discussing averages. A more encouraging proportion of candidates were able to provide a good interpretation for the differences in the standard deviations. Some thought that central tendency was something to do with variation. A number of candidates were unable to construct a proper, legible, grammatically correct sentence.
Q7(i)(*A*) This was very well answered.
Q7(i)(*B*) Again this was well answered, usually by use of tables, although some candidates did calculate the three probabilities, add and subtract from 1. A few candidates forgot to subtract from 1, and a few just subtracted $P(X = 2)$ from 1.

Q7(i)(*C*) The majority of the candidates found this part straightforward, but a small minority lost the mark when they rounded their final answer to 1 or 2.

Q7(ii) As in recent years, candidates did well on this part, with over 80% gaining at least 3 marks out of 4. Most candidates scored the first two marks for the hypotheses, with many knowing that they needed to define p, thus scoring the third mark. A valid explanation of the reason for the form of the alternative hypothesis was usually given, even if not always very well worded.

Q7(iii)(*A*) Only about half of the candidates scored any marks here at all. Many candidates did not use any numbers so could not gain all the marks, but were awarded special case 2 if they gave a very convincing explanation. Some of those that did state that $P(X \leq 0) = 0.1216$ then failed to show a comparison with 10% or 0.1 and so only scored 1 mark.

Q7(iii)(*B*) Those candidates who had 0.1216 in the previous part usually gave the correct answer of 13%. Some who did not get marks in the previous part did give the correct answer, so they probably simply did not know how to verbalise the previous answer. However under half scored the mark, some by simply failing to round to an integer.

Q7(iv) There were many good, clear answers to this part of the question but there were still a good proportion of candidates that were tempted to use point probabilities. A significant number who did use the correct probability (or probabilities if using a critical region method) failed to give the conclusion of the test in context. Some lost the final mark for commenting that the proportion had not changed instead of had not reduced and some gave a conclusion which was too assertive.

4767 Statistics 2

General Comments:

The vast majority of candidates appeared to be well prepared for this examination. The overall performance was very good and the average score is once again very high. In hypothesis tests, most candidates provided appropriately worded hypotheses and conclusions. Most candidates were able to complete required calculations correctly and with suitable working provided. Over-specified answers were present though many candidates managed to choose suitable degrees of accuracy for their final answers. Most of the candidates with access to more advanced calculators managed to provide sufficient detail in their solutions to be awarded full credit.

Comments on Individual Questions:

Question No. 1

(i) The scatter diagram was well drawn by many, with some choosing more manageable scales than others.

(ii) The key points concerning the absence of any discernible elliptical shape and the corresponding questioning of the underlying bivariate Normal population were handled well by many. Though most candidates managed to comment on the lack of an elliptical spread of points, often with poor spelling of 'ellipse', there were still many candidates who struggled to differentiate between data and population.

(iii) Many achieved full marks here. Only a few candidates reversed the ranking of one of the sets of values. Errors tended to involve mistakes in ranking, in adding the squares of the differences or in rounding the final answer. Very few candidates failed to rank the data.

(iv) This part proved to be challenging for many candidates, especially in the identification of the underlying population of countries involved. Inappropriately worded hypotheses, using 'correlation' in place of 'association', were seen in a few cases. Many realised the one-sided nature of the test which investigated the possible negative association and went on to use the appropriate critical value to obtain a suitable conclusion in context. Few candidates expressed the conclusion in terms of the null hypothesis.

(v) There were many correct answers to this part of the question, either in terms of incorrectly rejecting H_0 or one of the equivalents. Many answers referring to 'accuracy' or 'reliability' were seen.

(vi) Few candidates appeared to understand that for this test no modelling assumptions about the underlying distribution are required.

Question No. 2

This proved to be a very straightforward question with most candidates scoring high marks.

(i) Many candidates succeeded in defining 'random' and 'independent' but many failed to define independence in terms of probability.

(ii) Well answered. A minority of candidates used a Poisson calculation here.

(iii) Though most candidates identified the usual 'n is large and p is small', very few explicitly related these values to the binomial distribution.

(iv) (*A*) Well answered.

(*B*) Well answered.

(v) Well answered. Common errors tended to involve either use of an incorrect standard deviation or omission of the required continuity correction.

Question No. 3

There were many good responses to this question. Spurious continuity corrections were rarely seen. It helps when candidates provide sketches for questions involving the Normal distribution.

(i) Well answered. Errors caused by lack of accuracy reading Normal tables were seen fairly regularly. Most candidates used the correct probability structure with their z values.

(ii) This was well done on the whole though many candidates did not provide the required comparison to justify their conclusion. In most cases the working provided was clear – diagrams were helpful to examiners in conveying the candidates' intentions – often more successfully than their wording.

(iii) Many correctly identified the z value of -3.09 and went on to find the appropriate value for h, rounded to a suitable level of accuracy.

(iv) There were some pleasing attempts at this question, marred only by an inappropriate degree of accuracy for the final answers. It was good to see that once the initial equations had been established with the correct z values, many could still solve the simultaneous equations. A few candidates failed to identify, for the given probabilities, the z values needed for the simultaneous equations.

(v) This was answered well, though many candidates could have made a greater effort to include symmetry in their sketches and to pay more attention to the asymptotic nature. Spurious labelling of axes was seen but only rarely.

Question No. 4

Most candidates scored well on this question. Pleasingly, overly-assertive conclusions to the hypotheses tests were rarely seen.

(a) (i) Most candidates found this to be very easy.

(ii) Well answered. Some candidates failed to word their hypotheses and conclusion in terms of 'association'. Most stated the correct number of degrees of freedom and critical value. Most candidates were able to finish off with appropriately worded conclusions.

(b) With a little more background work to be done, finding sample mean and standard deviation, candidates found this part of the question more difficult than part (a). The calculation of sample standard deviation caused problems for many. Issues with premature rounding of sample mean and standard deviation, leading to inaccuracy in the calculation of the test statistic, were quite common. A small number of candidates did not express their hypotheses in terms of μ. Definitions of μ as 'sample mean' were, thankfully, rare. Given that most candidates provided the correct alternative hypothesis it was disappointing to see many working with a negative test statistic and critical value – these were deemed inappropriate and thus penalised. It was again pleasing to see candidates taking care to word conclusions in an appropriately non-assertive manner.

4768 Statistics 3

General Comments:

This paper was generally very well answered. A vast majority of candidates attempted all the questions and were able to show what they could do.

The solutions to the hypothesis testing questions (2 and 4) were correctly structured, with both hypotheses and conclusions stated clearly and in the context of the question. Occasionally they lacked sufficient context, or the conclusions were overly assertive, but it was pleasing to see that such cases were in a minority. Learners should be reminded that it is good practice to state the hypotheses before carrying out any calculations, even though marks in the examination are awarded for the hypotheses seen anywhere within the solution.

Generally the solutions contained sufficient detail to make the methods clear, even when graphical calculators were used to find, for example, probabilities from the Normal distribution.

The notation for random variables was sometimes unclear, both in Question 1 and when explaining the Central Limit Theorem in Question 3(v). Insisting on clearer notation here might have helped candidates avoid confusion between a random variable and the corresponding sample mean, and enable them to score more marks.

It was pleasing to see that most candidates attempted to answer the 'wordy' questions. There were many good explanations, but sometimes the sentences were not clearly structured which made it difficult to follow the ideas. Bad handwriting on occasion made it difficult to award marks.

Candidates should be reminded to give their answers to an appropriate degree of accuracy, which is generally three or four significant figures. Over-specifying answers can lead to a loss of marks.

Comments on Individual Questions:

Question No. 1

This question was about combinations of Normal variables. Calculations using the Normal distribution were generally done well, with sufficient detail and to an appropriate accuracy.

The random variables in question were not always clearly defined. This was not penalised in itself, but often led to the wrong variance and hence the wrong answer. There were two common mistakes: using the variable instead of its sample mean (and thus failing to divide the variance by n); and writing, for example, $4M$ when what was meant was $M_1 + M_2 + M_3 + M_4$ (which would lead to the variance $16\sigma^2$ instead of $4\sigma^2$). Teachers are therefore advised to insist on correct notation for random variables in this type of question.

In part (i) a large number of candidates divided by 20 instead of 10 in the standardisation. In parts (ii) and (iii) careless notation, as described above, often led to the incorrect variance. Where this was avoided, part (ii) in particular was very well done. In part (iv) many either misinterpreted the question ($M - F - 25$ was often seen) or forgot that it was about the means.

Question No. 2

This question consisted of two parts, the first requiring a chi-squared test and the second a Wilcoxon test. Both parts were very well done overall, with the calculations mostly being correct, hypothesis and conclusions being given in sufficient detail, and critical values correctly identified.

In part (a), it was good to see that the hypotheses were generally stated correctly (in the past we saw 'data fits model' much more often). Candidates are expected to understand that 'result insignificant' means that we have insufficient evidence to reject H_0; although this was not always clearly stated, a majority managed to score full marks by correctly interpreting the conclusion in context.

In part (b), the most common loss of marks was for forgetting to include 'population' in the definition of the parameter in the hypotheses, and concluding that the South American fruit flies were larger than the European ones without referring to the 'median' or 'average' length. A more subtle error was to omit the value 2.5 from the hypotheses (stating only that 'the median length of the South American flies is larger than the median length of the European ones'); the value 2.5 is in fact needed in testing the hypotheses. However, it was good to see that most hypotheses included a definition of the parameter and that the conclusion was nearly always contextualised.

Question No. 3

Early parts of this question were usually well done. In part (ii) a few candidates drew a triangle instead of a curve, and some parabolas had "tails" (looking more like a normal distribution curve).

In part (iii), since E(X) = 0, we expected to see '-0^2 ' or '$-\mathrm{E}(X)^2$ ' in the calculation of the variance.

In part (iv) most candidates reached the correct 3-term cubic, although some stopped too soon. Some candidates equated an integral with limits 0 and q to 0.25 which is correct, but it was not clear that they were using the symmetry of the pdf. For the final part, most were content to demonstrate that substituting 0.35 gave an approximately correct right-hand side of the equation, rather than showing that the value of q was 0.35 to two decimal places. The latter requires looking for a sign change between 0.345 and 0.355.

Part (v) asked for an application of the Central Limit Theorem. The numerical answer was mostly correct, but the justification for the use of the Normal distribution revealed some misconceptions about the CLT. Many candidates seem to think that, when a sample is large, X itself can be approximated by a Normal distribution. Many correct references to the CLT forgot to mention the large sample, which is required for the application of the Theorem to be valid.

Question No. 4

In part (i) most candidates made at least one sensible comment. However, the language was often unclear so it was difficult to tell exactly what point was being made. For Procedure A there needed to be some mention of the two systems, rather than just the two sets of forms. Procedure B weakness was generally well explained, although some candidates resorted to the need for a larger sample.

Part (ii) was largely well done. In stating the hypotheses quite a few described μ as the difference between the two sets of times, omitting the word 'mean'. A minority described μ as the 'difference of the means', which is incorrect for a paired test. Again, the final conclusion often failed to mention 'average' reduction in time.

Part (iii) revealed that the conditions for using a t-test are not very well understood. Most knew that something needed to be Normally distributed, but many simply said 'the data' or 'the underlying variable', not specifying that it is in fact the population of differences. Most knew that the sample variance should be unknown. Many candidates stated a requirement for a small sample, which is not in fact necessary. On the other hand, the requirement of a random sample was often forgotten.

Finally, part (iv) asked for a confidence interval for the mean reduction in time. Some candidates used one of the single samples instead of the differences, and it was quite common to see the interval for the increase, rather than the reduction. Quite a high proportion used a wrong *t* value. Overall, however, most candidates knew the correct method and scored at least one mark on this question.

4771 Decision Mathematics 1

General Comments:

As always, many of the candidates for this paper exhibited poor communication skills. Communication skills go hand-in-glove with thinking skills.

Comments on Individual Questions:

Question No.1

The first question carried an obvious moral tale about gambling, which will perhaps be well used in future revision classes. Regrettably, the message will have eluded the many candidates who failed with part (iv).
Parts (i), (ii) and (iii) were answered well, which makes the repeated failures in part (iv) all the more surprising. Having considered what happens, and then simulated what happens, it should have been clear that the mean required in part (iv) was the mean monetary outcome. This shows that the expectation is for a substantial loss. Instead many candidates computed simulated probabilities of winning/losing. This misses the point when wins were always £100, but when losses in this scenario were £700 a time.

Question No. 2

This question was also well answered. The most common mistake in part (i) was to give only the updating probabilities, instead of using them in the updating calculation.
The other parts were answered well, with many candidates referring to unequal probabilities in their answers to (iv).

Question No. 3

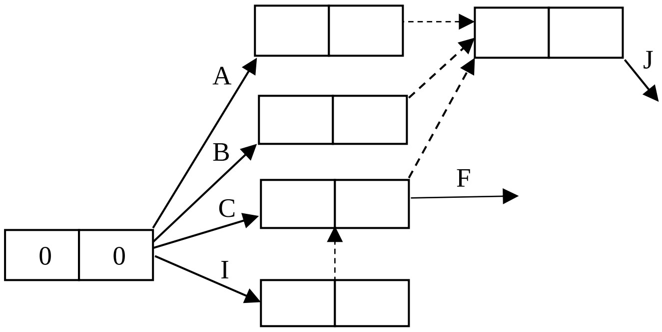
Parts (i) and (ii)(A) were answered well, but very few marks were scored in (ii)(B). It had been hoped that the solution for the cube, described in the question, would have pointed the way. Inevitably there were candidates who described a colouring, which does not answer the question.

Question No. 4

This question proved to be too difficult for nearly all candidates. Very few decent graphs were seen.
Most could do parts (i) and (ii), but thereafter confusion reigned. Candidates could not get to grips with the scenario, and it was common to see $x >$ (or $\geq$) 20 for (iii) and $y >$ (or $\geq$) 24 for (iv).
The instruction seemed clear enough in (v), but there were very few attempts to use $z = 20 - x - y$.
Given all that, few could make any coherent sense of where they had got to in terms of drawing the feasible region.

Question No. 5

This question started by requiring candidates to model written precedence expressions by a list of immediate predecessors. This made it more difficult than usual to produce the activity-on-arc network in part (ii). Creditable attempts were seen, but many candidates lost marks in their use of dummy activities. For instance, many candidates did not realise that the following logic is OK for F (immediate predecessors of C and I) but not for J (A, B and C), since it also has I as one of its immediate predecessors.



Question No. 6

Part (a) was answered well. In part (b) examiners needed to be convinced that Kruskal was being used in (i) and Prim in (ii). The safest way to do this is to show the order of including arcs for Kruskal, and the order of including vertices for Prim.

4772 Decision Mathematics 2

General Comments:

Candidates for this demanding paper present a wide variety of attainments. Some answers are very good indeed, with answers exceeding examiner expectations. Others struggled with the abstraction that is needed to apply maths to real problems.

Comments on Individual Questions:

Question No. 1

Decision Analysis represents a simple but powerful meta-tool, a methodology for structuring problem solving. Candidates were impressive in their mastery of it. Few put a foot wrong in the structuring of the problems, losing marks only in some details.

Parts (i) and (ii) presented very few difficulties. Some candidates worked in outcomes rather than losses, which was OK. Indeed, arguably it is better since it is consonant with applying a utility function. Less good was the practice seen on some scripts of incorporating the £20 cost of vaccination downstream from the relevant decision node. This is not a good idea. The total cost/benefit of each outcome needs to be evaluated at the end of the corresponding branch.

In part (iii) many candidates lost a mark by not starting from the decision to answer or not answer the questionnaire. The “not” branch does not need to repeat part (ii) - the outcome is sufficient. Very few candidates answered the question. They evaluated the cost corresponding to the “answer the questionnaire” branch, but they failed to then find, by subtraction, the value of the questionnaire.

A mark was reserved in part (iv) for establishing an “unpleasantness measure”, or at least recognising the need for it. Many failed to establish the outcomes of 1000u versus 0u, and 1001u versus 1u.

Question No. 2

This question, the logic question, was far more problematic. It was difficult to mark because many candidates who did understand what they were doing found it difficult to express that with sufficient clarity to distinguish themselves from candidates who were not scoring marks.

Part (a)(i) was surprising in the number of candidates it caught out. It seemed quite straightforward to note that Emelia needed Gemma to tell her about precipitation as well as about temperature, but many thought not. It is understandable that confusion can take hold in the stress of an examination, and tools might then help. Thus Emelia’s statement can be modelled as:

$(\sim d \vee \sim w) \Rightarrow \sim walk$, the contrapositive of which is:

$walk \Rightarrow (d \wedge w)$.

Some candidates pointed out that even if it is dry and warm, we don’t know if Emelia will walk, which is true – necessity versus sufficiency. But that comes after the need first for it to be dry as well as warm.

Most candidates collected both marks in part (b)(i), but proving their assertions in part (b)(ii) was a different matter. There were many ways to go about this. The most productive was to use the contrapositive, or the properties of the implication connective, which comes to the same thing.

Candidates reaching for their truth tables often failed to keep thinking, many doing no more that producing a truth table or truth tables for the given implication statement(s).

A full handle-turning, all-possibilities statement for a truth table would be:

$$\left(\left(\left(d \wedge h\right) \Rightarrow r\right) \wedge \left(\dim \Rightarrow h\right) \wedge d \wedge \sim r\right) \Rightarrow \left(\sim h \wedge \sim \dim\right)$$

This looks daunting, but it can be reduced to 4 lines and have the "$\wedge\ d \wedge \sim r$" dropped by restricting attention to d = 1 and r = 0. Even better to split it into two parts with two 2-line tables, as most did.

Part (c)(i) caused difficulties. Very few candidates were explicit in their identification of switches/up/down with statements/true/false, nor with current flowing or not flowing when the corresponding compound statement was true/false. Having said that there were some excellent solutions, arguably the best of which was to use Boolean algebra to show that $(A \wedge B) \Rightarrow C$ is equivalent to $\sim A \vee \sim B \vee C$, which is seen to be implemented in the circuit.

Question No. 3

This question was an attempt to demonstrate how LP is often used in practice – to explore options rather than as an all-encompassing model. Candidates seemed very comfortable with this.

Many candidates failed to score the first mark by offering answers such as "To have more expensive paint used than inexpensive paint". The correct answer was "He must paint the lower half of each wall in the more expensive paint".

In part (ii) most candidates scored the first mark, wall coverage. The preferred answer for the second mark was to keep it simple by avoiding two-stage Simplex. Here we departed from reality since this is dealt with easily in computing packages.

There was a very good level of skill shown in applying the algorithm in part (iii). Most candidates also scored the interpretation mark, which hung on them recognising that the numbers represented square metres of coverage. That also applied to part (iv).

At this point most candidates, probably tiring, lost sight of the objective. There was a collective assumption that Neil would wish to minimise cost, but that was never mentioned as an objective. It was stated that Neil wants to use as much of the more expensive paint as possible, within constraints on coverage and his budget … because it is easier to use. So most candidates missed this in (v) and did not model it in (vi). The mark scheme in part (vi) was kind to these candidates, and good drilling led to good marks for most.

Question No. 4

This question covered the usual mix of network algorithms but with an unusual balance, all applied to one given network.

Part (a) was the CPP, with 4 odd nodes. So 3 pairings were possible, and all needed to be seen. Most candidates were alert to what was needed. A few had no idea. Some lost marks on details. A surprising number of candidates tried to use nearest neighbour on this part.

Part (b)(i) was Floyd, and was done exceptionally well. Most also did well, but not as well, with the explanations in (b)(ii). Some gave answers rather than explanations, and some could only manage incoherent attempts at explanations

Part (b)(iii) was there to set up the remaining 2 parts. Part (b)(iv) was done very well. In part (b)(v) the minimum connector was often wrong, and the deduction was often incomplete..

4776 Numerical Methods (Written Examination)

General Comments:

Candidates seemed, for the most part, well prepared for this paper. Routine numerical work was generally done well, and the selection of methods was usually appropriate. Interpretation of results was less good, however, and where questions called for comments the answers given were quite often weak.

Poor presentation continues to be an issue, with illegible hand-writing and a propensity to scatter numerical work haphazardly on the page being far too common. There were several instances in which the work submitted simply couldn't be deciphered.

Comments on Individual Questions:

Question No. 1
Most candidates handled this question on absolute and relative errors well. In part (i), some candidates took the denominator in the relative error term to be the approximate value. In part (ii), some ignored the suggestion to use trial and error and attempted an algebraic approach; this proved far harder.

Question No. 2
Part (i) was a routine exercise in using the Newton-Raphson method, and it was generally done well. In part (ii) the approach asked for required the calculation of ratios of differences and the observation that they decrease rapidly. A fairly common error was to say that the ratio of differences in the Newton-Raphson method will be less than 0.5.

Question No. 3
Again, the straightforward numerical work in part (i) of this question was done well. In part (ii), most stated correctly that Simpson's rule is of order 4. However, using that fact to extrapolate to the next Simpson's rule estimate was found more difficult. In the final part, many candidates were overly cautious in stating the number of decimal places to which the integral is accurate.

Question No. 4
The orders of accuracy of the forward difference and central difference methods were usually stated correctly. In practice this means that the central difference method will usually converge more quickly than the forward difference method. In part (ii), the two methods give the same values as one another. Some candidates correctly observed that the function appears to have a point of inflection at $x = 0$, but this observation was not required for full marks.

Question No. 5
The difference table was constructed correctly by the vast majority, and they commented that the similarity of the second differences indicated that a quadratic would be a good fit. The forward difference interpolation formula was generally done well. It was encouraging to see far fewer examples than previously of candidates confusing the x and $f(x)$ values.

Question No. 6
This question proved to be a fairly straightforward exercise in fixed point iteration. In part (i) most candidates drew the two functions correctly. However it was rare to see any comment on how the graph of $y = kx$ depended on the value of k. In part (ii), some comment or discussion or explanation was required of the convergent and divergent processes. It was not judged sufficient simply to exhibit some iterates. Neither was it was sufficient to add the words 'converging' and 'diverging': the information is already in the question. The simplest approach was to note that the oscillating

iterates are getting closer in one case and further apart in the other. The algebra in part (iii) was done well, as were the two numerical calculations.

Question No. 7
This question was all about the relationship between convergence to an answer and agreement to a number of decimal places. Candidates carried out the numerical work correctly, but there were very few clear and correct comments on the underlying concepts. In part (i) it was evident from successive terms that the sequence S_n is converging slowly. In parts (ii) and (iii), the same sequence is explored, but now with adjacent terms combined. This gives the *impression* of convergence because of the agreement of decimal places. However, we know that the convergence can't be any faster because the sequence is essentially the same as in part (i). In part (iv) a correction term is added to improve the convergence. This gives agreement to 5 decimal places, but we should be hesitant in saying that the answer is correct to 5 decimal places as we have already seen that agreement does not imply convergence. Part (v) asks candidates to comment on convergence and agreement in the light of their earlier answers. Unfortunately, most candidates made incorrect statements here: despite the evidence to the contrary, they often said that agreement indicates that a sequence has converged.

Coursework

Administration

Administration can cause difficulties in the moderation process if not carried out efficiently and in accordance with the instructions from the board. In the vast majority of centres, however, administration was effective. Most coursework arrived on time if not early, there were few clerical errors and the vast majority enclosed the Authentication Form, CCS160. This all made the process of external moderation very much easier.
Centres are once again reminded that it is also a great help to have the cover sheets filled in properly. This means

- Full candidate name and candidate number,
- Marks given by criteria rather than domain,
- Comments to help the external moderator determine which marks have been awarded and which have been withheld,
- An oral communication report.

Assessors are asked not to tick work that they have not checked, but they are required to do some checking of calculations so we do expect to see some annotation in the body of the work.

The marks of candidates in most centres were appropriate and acknowledgement is made of the amount of work that this involves to mark and internally moderate. The unit specific comments are offered for the sake of centres who have had their marks adjusted for some reason.

These reports should provide a valuable aid to the marking process and we would urge all Heads of Departments to ensure that these reports are read by all those involved in the assessment of coursework. All that follows has been reported before!

Core, C3 - 4753

The marking scheme for this component is very prescriptive. However, there are a significant number of centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance.

<u>Change of Sign</u>

- A graph of the function being used does not constitute an illustration of the method. The graph should be very carefully annotated or at least two “zoom-in” graphs should serve to illustrate the method. Candidates could be encouraged to draw a “zoomed in” graph for every scale of x that they use.
- Trivial equations should not be used to demonstrate failure.
- If a table of values actually finds the root then the method has not failed.
- Graphs which candidates claim crosses the axis or just touch but don’t should be checked. A clue is the scale on the y-axis. A scale that goes from 0 to 10, for instance, could show that the graph just touches the x-axis but in fact does not meaning that there is no root to find.
- The root is given as an interval rather than an actual value with error bounds.
- Through a process of rounding it is possible to give error bounds which do not encompass the root.

<u>Newton Raphson</u>

- Candidates who use equations with only one root should not be credited the second mark.
- Iterates should be given for success and failure.

- A print out from "Autograph" is not sufficient. Candidates should derive the formula by differentiation and algebra. This should be for the equation being used rather than theory.
- Poor illustrations (for example, an "Autograph" generated tangent with no annotation or just a single tangent) should not be awarded the full mark.
- The graph used to demonstrate the method should match the iterates.
- Error bounds should be established by a change of sign.
- Failure should be demonstrated "despite a starting value close to the root". If the starting value is too far away from the root or too artificial then the mark should not be awarded.
- The roots in this domain should be given to 5 significant figures. Teachers should note that the default position of "Autograph" is only 4 significant figures so candidates need to make some adjustment before the use of the software is satisfactory.

Rearrangement

- Incorrect rearrangements are often not spotted and marked as correct.
- Graphs do not match iterates.
- The method should be explained by a graph which may need to be annotated.
- Weak discussions of $g'(x)$ are often given. Candidates should not just quote the criterion without linking it to their function.
- The criterion states that candidates should use a rearrangement of the same equation to demonstrate failure. This might mean using the same rearrangement to try to find a different root or a different rearrangement to try to find a different or the same root.

Comparison

- The methods need to be compared by finding the same root of the same equation with the same starting value to the same degree of accuracy.
- The discussions are often thin and omit references to the software actually being used.

Notation

- Equations, functions, expressions still cause confusion to candidates and teachers! Candidates who assert that they are going solve $y = x^3 + x + 7$ or that they are going to solve $x^3 + x + 7$ should be penalised.

Differential Equations - 4758

Although there was a range of coursework tasks presented, *Aeroplane Landing* and *Cascades* are still the most popular choices for many centres. In the case of the former, work is still being produced where still the initial model is rejected. on the basis of the motion in the first 9 seconds, without considering the effect of the braking motion. This should be penalised. Similarly, in the latter case the focus of *'Cascades'* has to be on the flow through the second container. The initial model should not be rejected on the basis of the flow through a single container. This should be used in order to calculate the necessary parameter.

As a general observation, when investigating Modelling tasks it should be appreciated that the criteria in Domains 2 and 4 only apply to the initial model. While there may well be, for example, variation considered for the revised model, this cannot be used to fulfil the criteria in these domains.

Curve fitting, although tempting, should be resisted since it does not rely upon the formulation and later modification of the assumptions. A particular example of this is assuming that the flow of water in cascades is proportional to some power of its height in the container, then finding what particular power produces the best fit. The choice of parameters, based on guesswork or shape of the data also often seems to occur in 'Interacting Species'.

It is also worth mentioning that, particularly as the Differential Equations are given for the *'Interacting Species'*, a very clear description of the parameters is expected.

Whilst not detrimental to the marking, it helps the narrative if, in modelling say *'Aeroplane Landing'* or *'Interacting Species*', the provided data are introduced at the beginning of the work.

The essential function of the coursework element of this module is to test the candidates' ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

Finally it is usually easier to agree with those markers that provide the most detailed signposting, through comments and references, of where and why marks were awarded.

Numerical Methods - 4776

The most popular task is to find the value of an integral numerically. The following comments are offered – it is to be hoped that those teaching and assessing will take note so that the problems do not continue to occur with such regularity!

Domain 1
A formal statement of the problem is required that has been chosen by the candidate. Please note that the function to be used does not have to be very complicated, merely one that cannot be integrated directly by standard techniques available to the candidate. There are a number of cases each year when the function is entered incorrectly into the spreadsheet. The consequence is that the answer is not that of the problem stated. In most cases this arises from an unnecessarily complicated function

Domain 2
A criterion in this domain is for candidates to say why they are using the techniques chosen. They are, however, usually good at saying what they are going to do.

Domain 3
Finding numerical values for one of the methods up to at least 64 strips is a requirement for a substantial application.

Domain 4
It is not enough to state what software is being used. A clear description of how the algorithm has been implemented is required, usually by presenting an annotated spreadsheet printout.

Domain 5
It is accepted that candidates might use a function that they are unable to integrate (because of where they are in the course) but which is integrable. However, it is not then appropriate to state a value found by direct integration.
Many candidates will state the value to which the ratio of differences is converging without justification from their values.. Other candidates use the "theoretical" value regardless of the values they are getting or even use these values without working the ratio of differences at all. These errors will lead to inaccurate or even incorrect answers; these should be noted by the Assessor and penalised.

Domain 6
Most of the marks in this domain are dependent on satisfactory work in the error analysis domain and so often a rather generous assessment of that domain led also to a rather generous assessment here as well. Teachers should note that comments justifying the accuracy of the

solution are appropriate here, but comments on the limitations of Excel are not usually creditworthy.

Introduction to Quantitative Analysis – G244

Administration, particularly from centres not entering candidates for MEI Structured Mathematics, was much improved this year. We still have problems outlined in the general section above and centres are encouraged to note the points made and the instructions distributed by OCR.

The standard of work was also much improved. There are still many reports, however, that do not meet the criteria and are rather too simple for a piece f coursework at this level.
Most of these were given the poor mark they deserved. There were still a number of centres, however, where the assessment was rather too generous requiring some scaling. Centres should note the comments below and also the specific centre report.

- Candidates should say why the investigation is worth doing
- The population should be clearly defined and the sampling procedure discussed. There are problems over this where the data are taken from an internet site where the details of the population are not given, but the marking criteria addresses those problems.
- A variety of displays should be used to describe the sample.
- Candidates should use a spreadsheet to carry out the calculations. A task where no calculations are done should obviously be avoided.
- Candidates should say why both the diagrams and calculations are appropriate.
- As commented last year, questions raised by the work should not be simply a discussion of what candidates might do instead or in addition to what has been done but questions that arise from the conclusions drawn.

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