# Section Check In – 1.02 Algebra and Functions

## Questions

1. Use the factor theorem to show that  is a factor of .

2. Simplify .

3.\* Solve the inequality , giving your answer using set notation.

4.\* Simplify .

5. (a) Express each of  and  in completed square form.

(b) Sketch the curves  and  on the same axes, and show that the shortest distance between the two curves is .

6.\* The functions  and  are defined by

 with domain ,

 with domain .

(a) Show that the range of  is .

(b) Explain why  has no inverse.

(c) Explain why the function  does not exist.

(d) Solve the equation .

7. Show that the straight line  meets the curve  in two distinct points for all values of the constant .

8.\* (a) Express  in partial fractions and hence expand  in ascending

powers of  up to and including the term in .

(b) Substitute a suitably small value of  in both the original rational expression and in your expansion. Comment on the two results obtained.

9. An object is projected vertically upwards from ground level and its height, metres, after  seconds is given by the formula

.

Show that the object is above a height of metres for approximately seconds.

10.\* The design of the slide in a playground is based on two curves

 for  and  for .

Units are metres and ground level is represented by .

The curve  is the result of transforming the curve  by the translation 

followed by a stretch in the direction with scale factor .

It is given that  for constants  and . The end of the slide (where )

must be at a height of metres above the ground.

Find formulae for  and .

**Extension**

A self-inverse function is one for which . By applying  to both sides, we see that this is equivalent to .

(a) Verify that the function defined by  for all real values of  is self-inverse.

(b) Find conditions on the constants  and  so that the function defined by  is self-inverse.

(c) Similarly investigate  and .

## Worked solutions

1. Let 



Since , by factor theorem  or  is a factor

2. Simplifying, 

3. Squaring both sides, 

Expanding and simplifying, 

Factorising,  and so  and  are critical values

Checking values  and , for example, satisfies the inequality and so the solution

consists of values to the left of  and to the right of 

Solution is 

Alternative solution:

Sketching  and 

*y*

1

4

*x*





Points of intersection given by 

and by , i.e. when  and 

From diagram,  is ‘above’  for the values 

4. By inspection or use of factor theorem,  is factor of the numerator and



Expression is 

Factorising the quadratic expressions, 

Cancelling factors common to numerator and denominator, expression is 

5. (a) 



(b)  has minimum at 

 has maximum at 

*y*

1

27

*x*









Shortest distance between curves is the distance between their stationary points.

Shortest distance is 

6. (a) Completing the square, 

For all values of ,  and therefore  for all 

(b) To have an inverse,  must be one-one so that  always has a unique value; but, for example,  and  are both equal to  and so  is not one-one and therefore has no inverse.

(c) For the composed function  to exist, the range of  must be part of, or the same as, the domain of  so that  can be found for all values of  in the domain of ; but   
 the range of  includes some negative values which are not included in the domain of   
; for example,  and this cannot be found

(d) The equation  leads to  and therefore  or

Factorising (or using a substitution ), 

Solving  or 

But  cannot be negative and so , giving  as the only solution

7. Substituting  in the equation of the curve, 

Expanding, 

Simplifying, 

Discriminant, 

For all values of ,  and therefore 

Since the discriminant is positive, the quadratic equation has two distinct roots and so the line meets the curve in two distinct points.

8. (a) Let 

Multiplying by  gives 

Substituting  gives  and therefore 

Substituting  gives  and therefore 

Expression is 

Using binomial expansions, expression is 

Simplifying, expansion is 

(b) Expansion is valid only if  so value chosen must be between these values

Choosing  to substitute: using calculator, value of  is 

and value of  is 

The values agree correct to 5 significant figures; agreement would be closer if more   
 terms had been used in the expansion or if a smaller value of  had been chosen (or   
 both); choosing a particular value in this way does not prove that the expansion is   
 correct but no obvious error has been revealed

9. Times when object is at a height of metres found by substituting 

Equation is  which simplifies to 

Using quadratic formula,  or 

Object at height of metres after seconds (going up) and after seconds (going down)

Object above this height for seconds, i.e. for seconds (to 3 significant figures)

10. A translation of  applied to the curve  gives the curve 

The stretch applied to  gives the curve 

The two sections of the slide must meet when  so that , giving the

equation 

At the end of the slide when , the height of the slide is , i.e.  giving

a second equation 

Solving the simultaneous equations for  and  gives  and 

Hence  and 

**Extension**

(a)  so that 

Since , this function  is self-inverse

(b) Now consider 

If , then  and so 

Rearranging gives 

For this to be true for all values of ,  and 

Therefore either  and  or  and  can take any value

The first option is just the trivial case 

The second option is , i.e. the function  defined by  is a   
self-inverse function for all values of the constant 

(c) For the function , applying  and simplifying leads to



This means  and  and  giving only 

Hence the function  defined by  is a self-inverse function for all values of the

non-zero constants  and 

A similar approach in the final case shows the function  defined by  is a self-inverse function provided that  and 

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