

# Monday 27 June 2016 – Morning

# A2 GCE MATHEMATICS (MEI)

**4756/01** Further Methods for Advanced Mathematics (FP2)

## **QUESTION PAPER**

Candidates answer on the Printed Answer Book.

#### OCR supplied materials:

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Duration: 1 hour 30 minutes

### Other materials required:

• Scientific or graphical calculator

# INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer **Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

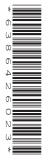
### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

# INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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#### Section A (54 marks)

- 1 (a) (i) Given that  $f(x) = \arctan x$ , write down an expression for f'(x). Assuming that x is small, use a binomial expansion to express f'(x) in ascending powers of x as far as the term in  $x^4$ . [3]
  - (ii) Hence express  $\arctan x$  in ascending powers of x as far as the term in  $x^5$ . [3]
  - (b) Find, in exact form, the value of the following integral.

$$\int_{0}^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}} dx$$
 [5]

- (c) A curve has polar equation  $r = \frac{a}{\sqrt{\theta}}$  where a > 0.
  - (i) Sketch the curve for  $\frac{\pi}{4} \le \theta \le 2\pi$ . [2]
  - (ii) State what happens to r as  $\theta$  tends to zero. [1]
  - (iii) Find the area of the region enclosed by the part of the curve sketched in part (i) and the lines  $\theta = \frac{\pi}{4}$  and  $\theta = 2\pi$ . Give your answer in an exact simplified form. [4]
- 2 (a) (i) Express  $2\sin\frac{1}{2}\theta\left(\sin\frac{1}{2}\theta j\cos\frac{1}{2}\theta\right)$  in terms of z where  $z = \cos\theta + j\sin\theta$ . [3]
  - (ii) The series C and S are defined as follows.

$$C = 1 - \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta - \dots + (-1)^n \binom{n}{n}\cos n\theta$$
$$S = -\binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta - \dots + (-1)^n \binom{n}{n}\sin n\theta$$

Show that

 $C + \mathbf{j}S = \left\{-2\mathbf{j}\sin\frac{1}{2}\theta\left(\cos\frac{1}{2}\theta + \mathbf{j}\sin\frac{1}{2}\theta\right)\right\}^n.$ 

Hence show that, for even values of *n*,

$$\frac{C}{S} = \cot\left(\frac{1}{2}n\theta\right).$$
 [8]

(b) Write the complex number  $z = \sqrt{6} + j\sqrt{2}$  in the form  $re^{j\theta}$ , expressing *r* and  $\theta$  as simply as possible. Hence find the cube roots of *z* in the form  $re^{j\theta}$ .

Show the points representing *z* and its cube roots on an Argand diagram. [7]

3 (i) Find the eigenvalues and eigenvectors of the matrix M, where

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Hence express **M** in the form  $\mathbf{PDP}^{-1}$  where **D** is a diagonal matrix.

(ii) Write down an equation for  $\mathbf{M}^n$  in terms of the matrices  $\mathbf{P}$  and  $\mathbf{D}$ .

Hence obtain expressions for the elements of  $\mathbf{M}^{n}$ .

Show that  $\mathbf{M}^n$  tends to a limit as *n* tends to infinity. Find that limit. [6]

(iii) Express  $\mathbf{M}^{-1}$  in terms of the matrices **P** and **D**. Hence determine whether or not  $(\mathbf{M}^{-1})^n$  tends to a limit as *n* tends to infinity. [4]

#### Section B (18 marks)

4 (i) Given that  $y = \cosh x$ , use the definition of  $\cosh x$  in terms of exponential functions to prove that

$$x = \pm \ln(y + \sqrt{y^2 - 1}).$$
[5]

(ii) Solve the equation

$$\cosh x + \cosh 2x = 5,$$

giving the roots in an exact logarithmic form.

(iii) Sketch the curve with equation  $y = \cosh x + \cosh 2x$ . Show on your sketch the line y = 5.

Find the area of the finite region bounded by the curve and the line y = 5. Give your answer in an exact form that does not involve hyperbolic functions. [8]

#### **END OF QUESTION PAPER**

[5]

[8]



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