OCR
Oxford Cambridge and RSA

Wednesday 8 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **24** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Positions in space around an aerodrome are modelled by a coordinate system with a point on the runway as the origin, O. The x-axis is east, the y-axis is north and the z-axis is vertically upwards. Units of distance are kilometres. Units of time are hours.

At time $t = 0$, an aeroplane, P, is at $(3, 4, 8)$ and is travelling in a direction $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ at a constant speed of $900\,\text{km}\,\text{h}^{-1}$.

(i) Find the least distance of the path of P from the point O. **[4]**

At time $t = 0$, a second aeroplane, Q, is at (80, 40, 10). It is travelling in a straight line towards the point O. Its speed is constant at $270\,\text{km}\,\text{h}^{-1}$.

(ii) Show that the shortest distance between the paths of the two aeroplanes is 2.24 km correct to three significant figures. **[6]**

(iii) By finding the points on the paths where the shortest distance occurs and the times at which the aeroplanes are at these points, show that in fact the aeroplanes are never this close. **[7]**

(iv) A third aeroplane, R, is at position (29, 19, 5.5) at time $t = 0$ and is travelling at $285\,\text{km}\,\text{h}^{-1}$ in a direction $\begin{pmatrix} 18 \\ 6 \\ 1 \end{pmatrix}$. Given that Q is in the process of landing and cannot change course, show that R needs to be instructed to alter course or change speed. **[7]**

Option 2: Multi-variable calculus

2 A surface, S, has equation $z = 3x^2 + 6xy + y^3$.

(i) Find the equation of the section where $y = 1$ in the form $z = \mathrm{f}(x)$. Sketch this section.

Find in three-dimensional vector form the equation of the line of symmetry of this section. **[5]**

(ii) Show that there are two stationary points on S, at O (0, 0, 0) and at P (−2, 2, −4). **[4]**

(iii) Given that the point $(-2 + h, 2 + k, \lambda)$ lies on the surface, show that

$$\lambda = -4 + 3(h + k)^2 + k^2\,(k + 3).$$

By considering small values of h and k, deduce that there is a local minimum at P. **[5]**

(iv) By considering small values of x and y, show that the stationary point at O is neither a maximum nor a minimum. **[3]**

(v) Given that $18x + 18y - z = d$ is a tangent plane to S, find the two possible values of d. **[7]**

Option 3: Differential geometry

3 Fig. 3 shows the curve with parametric equations $x = t - 3t^3,\ y = 1 + 3t^2$.

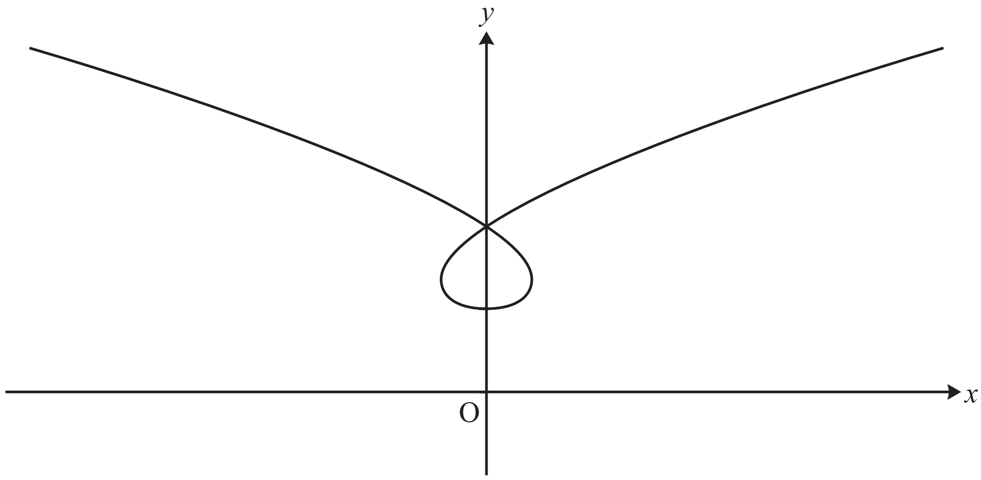

Fig. 3

(i) Show that the values of t where the curve cuts the y-axis are $t = 0, \pm\frac{1}{\sqrt{3}}$. Write down the corresponding values of y. **[2]**

(ii) Find the radius and centre of curvature when $t = \frac{1}{\sqrt{3}}$. **[11]**

The arc of the curve given by $0 \leqslant t \leqslant \frac{1}{\sqrt{3}}$ is denoted by C.

(iii) Find the length of C. **[5]**

(iv) Show that the area of the curved surface generated when C is rotated about the y-axis through 2π radians is $\frac{\pi}{3}$. **[6]**

Option 4: Groups

4 **(a)** The elements of the set $P = \{1, 3, 9, 11\}$ are combined under the binary operation, $*$, defined as multiplication modulo 16.

(i) Demonstrate associativity for the elements 3, 9, 11 in that order.

Assuming associativity holds in general, show that P forms a group under the binary operation $*$. **[6]**

(ii) Write down the order of each element. **[2]**

(iii) Write down all subgroups of P. **[1]**

(iv) Show that the group in part **(i)** is cyclic. **[1]**

(b) Now consider a group of order 4 containing the identity element e and the two distinct elements, a and b, where $a^2 = b^2 = e$. Construct the composition table. Show that the group is non-cyclic. **[4]**

(c) Now consider the four matrices $\mathbf{I}$, $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$ where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbf{Y} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The group G consists of the set $\{\mathbf{I}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}\}$ with binary operation matrix multiplication. Determine which of the groups in parts **(a)** and **(b)** is isomorphic to G, and specify the isomorphism. **[6]**

(d) The distinct elements $\{p, q, r, s\}$ are combined under the binary operation $\circ$. You are given that $p \circ q = r$ and $q \circ p = s$.

By reference to the group axioms, prove that $\{p, q, r, s\}$ is not a group under $\circ$. **[4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Each day that Adam is at work he carries out one of three tasks A, B or C. Each task takes a whole day. Adam chooses the task to carry out on each day according to the following set of three rules.

1. If, on any given day, he has worked on task A then the next day he will choose task A with probability 0.75, and tasks B and C with equal probability.
2. If, on any given day, he has worked on task B then the next day he will choose task B or task C with equal probability but will never choose task A.
3. If, on any given day, he has worked on task C then the next day he will choose task A with probability p and tasks B and C with equal probability.

(i) Write down the transition matrix. **[3]**

(ii) Over a long period Adam carries out the tasks A, B and C with equal frequency. Find the value of p. **[4]**

(iii) On day 1 Adam chooses task A. Find the probability that he also chooses task A on day 5. **[3]**

Adam decides to change rule 3 as follows.

If, on any given day, he has worked on task C then the next day he will choose tasks A, B, C with probabilities 0.4, 0.3, 0.3 respectively.

(iv) On day 1 Adam chooses task A. Find the probability that he chooses the same task on day 7 as he did on day 4. **[5]**

(v) On a particular day, Adam chooses task A. Find the expected number of consecutive further days on which he will choose A. **[3]**

Adam changes all three rules again as follows.

- If he works on A one day then on the next day he chooses C.
- If he works on B one day then on the next day he chooses A or C each with probability 0.5.
- If he works on C one day then on the next day he chooses A or B each with probability 0.5.

(vi) Find the long term probabilities for each task. **[6]**

END OF QUESTION PAPER

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