OCR
Oxford Cambridge and RSA

Friday 17 June 2016 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The differential equation

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+8\frac{\mathrm{d}x}{\mathrm{d}t}+3x=\mathrm{f}(t)$$

is to be solved.

Consider first the case where $\mathrm{f}(t)=4\mathrm{e}^{0.5t}$.

(i) Find the general solution for x. **[7]**

(ii) Given that when $t=0$, $x=6$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=-4$, find the particular solution for x. **[4]**

(iii) Given that x has a minimum value, find the value of t for which this minimum occurs. **[4]**

Now consider the case where $\mathrm{f}(t)=4\mathrm{e}^{-0.5t}$.

(iv) Given that when $t=0$, $x=6$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=-4$, find the particular solution for x. **[9]**

2 **(a)** The differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}-3y=x^5\cos x$$

is to be solved.

(i) Find the general solution for y in terms of x. **[8]**

(ii) Find the particular solution for which $y=0$ when $x=\frac{1}{2}\pi$. **[2]**

(b) Now consider the differential equation $\sec x\frac{\mathrm{d}y}{\mathrm{d}x}-3y^2=0$.

(i) Find the particular solution for which $y=1$ when $x=\frac{1}{2}\pi$. **[7]**

(ii) Show that the maximum value of y is 1. **[2]**

(c) Now consider the differential equation $\sec x\frac{\mathrm{d}y}{\mathrm{d}x}-3y^2=x$, where $y=0$ when $x=1$.

This is to be solved numerically using Euler's method. The algorithm is given by

$$x_{r+1}=x_r+h, \qquad y_{r+1}=y_r+hy'_r.$$

Use a step length of 0.01 to estimate y when $x=1.02$. **[5]**

3 A parachutist of mass 90 kg falls vertically from a stationary helicopter. When he is x m below the helicopter, his velocity is $v\,\mathrm{m\,s^{-1}}$. The forces acting on the parachutist are his weight and a resistive force of magnitude $0.36v^2$ N.

(i) Show that his motion can be modelled by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = 9.8 - 0.004v^2. \quad \textbf{[2]}$$

(ii) Solve this differential equation to show that

$$v^2 = 2450(1 - \mathrm{e}^{-0.008x}). \quad \textbf{[6]}$$

(iii) Sketch the graph of v against x. **[2]**

(iv) Find how far the parachutist has fallen when his speed has reached $48\,\mathrm{m\,s^{-1}}$. **[2]**

The parachutist opens his parachute when his speed is $48\,\mathrm{m\,s^{-1}}$. The forces acting on him now are his weight and a resistive force of magnitude $72v$ N.

(v) Find an expression for v in terms of t, where t is the time in seconds that has elapsed since the parachute was opened. **[8]**

(vi) Find the distance that the parachutist falls during the first 5 seconds after his parachute has opened. **[4]**

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x - y + 3\cos t$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 5x - y - 12\sin t$$

are to be solved for $t \geqslant 0$, where t denotes time.

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[12]**

(ii) Find the corresponding general solution for y. **[3]**

When $t = 0$, $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 5$.

(iii) Find the particular solutions for x and y. **[4]**

(iv) Find the time that elapses between the first two occasions on which $x = y$. **[5]**

END OF QUESTION PAPER