



Friday 20 May 2016 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:

Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer all the questions.

- Find $\sum_{r=1}^{n} (3r+1)(r-1)$, giving your answer in a fully factorised form. 1 [5]
- The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{1}{3}\pi$. Giving your answers in the form x+iy, 2 where x and y are exact real numbers, and showing clearly how you obtain them, find

$$[2]$$

(ii)
$$\frac{1}{(z^*-5i)^2}$$
. [5]

- The quadratic equation $kx^2 + x + k = 0$ has roots α and β . 3
 - (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(ii) Find the value of
$$\left(\alpha + \frac{1}{\alpha}\right) \left(\beta + \frac{1}{\beta}\right)$$
 in terms of k .

The matrices **A**, **B** and **C** are given by $\mathbf{A} = (a \ 2 \ 3)$, $\mathbf{B} = (b \ 0 \ 5)$ and $\mathbf{C} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}$. Find 4

(i)
$$5A - 3B$$
, [2]

The sequence u_1, u_2, u_3, \dots is defined by $u_1 = 5 \text{ and } u_{n+1} = 3u_n + 2 \text{ for } n \geqslant 1.$ 5

$$u_1 = 5$$
 and $u_{n+1} = 3u_n + 2$ for $n \ge 1$.

Prove by induction that $u_n = 2 \times 3^n - 1$. [4]

- In an Argand diagram the points A and B represent the complex numbers 5+4i and 1+2i respectively. 6
 - (i) Given that A and B are the ends of a diameter of a circle C, find the equation of C in complex number form. [4]

The perpendicular bisector of AB is denoted by l.

- (iii) Find the complex numbers represented by the points of intersection of C and l. [3]
- The matrix $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ represents a transformation P. 7
 - (i) Describe fully the transformation P. [2]

The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} -3 & -1 \\ -1 & 0 \end{pmatrix}$.

(ii) Given that M represents transformation Q followed by transformation P, find the matrix that represents the transformation Q and describe fully the transformation Q. [6]

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8 (i) Show that
$$\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$$
. [1]

(ii) Hence find
$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)}$$
, giving your answer as a single fraction. [6]

(iii) Find
$$\sum_{r=n}^{\infty} \frac{1}{(2r+1)(2r+3)}$$
, giving your answer as a single fraction. [3]

9 (i) The matrix **X** is given by
$$\mathbf{X} = \begin{pmatrix} a & 3 & -2 \\ 0 & a & 5 \\ 1 & 2 & 1 \end{pmatrix}$$
. Show that the determinant of **X** is $a^2 - 8a + 15$. [3]

(ii) Explain briefly why the equations

$$3x+3y-2z = 1$$
$$3y+5z = 5$$
$$x+2y+z=2$$

do not have a unique solution and determine whether these equations are consistent or inconsistent. [3]

- 10 (i) Use an algebraic method to find the square roots of the complex number 9+40i. [6]
 - (ii) Show that 9+40i is a root of the quadratic equation $z^2-18z+1681=0$. [1]
 - (iii) By using the substitution $z = \frac{1}{u^2}$, find the roots of the equation $1681u^4 18u^2 + 1 = 0$. Give your answers in the form x + iy, where x and y are real.

END OF QUESTION PAPER

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