

# OCR

Oxford Cambridge and RSA

## Monday 27 June 2016 – Morning

### A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4726/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

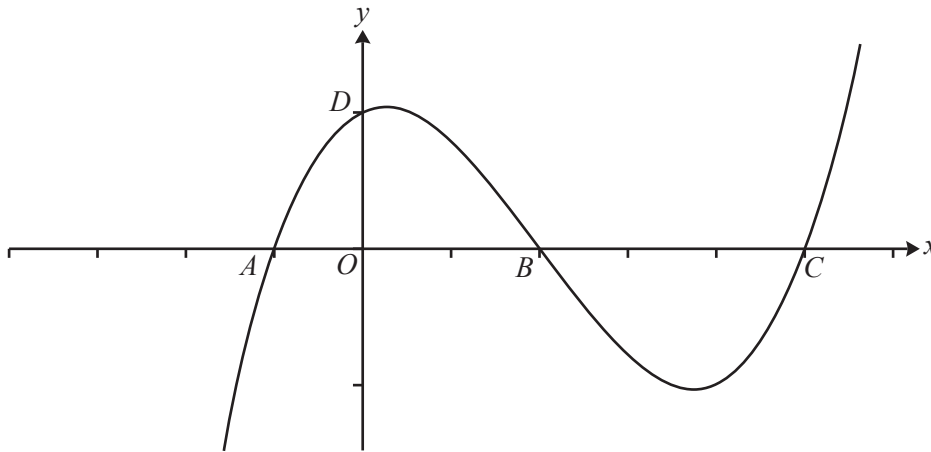
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Answer **all** the questions.

- 1 (i) By first expanding  $(e^x + e^{-x})^3$ , or otherwise, show that  $\cosh 3x \equiv 4 \cosh^3 x - 3 \cosh x$ . [4]
- (ii) Solve the equation  $\cosh 3x = 6 \cosh x$ , giving your answers in exact logarithmic form. [5]

- 2 It is given that  $f(x) = \frac{x(x-1)}{(x+1)(x^2+1)}$ . Express  $f(x)$  in partial fractions and hence find the exact value of  $\int_0^1 f(x) dx$ . [6]

- 3 The diagram shows the curve  $y = f(x)$ . Points  $A$ ,  $B$ ,  $C$  and  $D$  on the curve have coordinates  $(-1, 0)$ ,  $(2, 0)$ ,  $(5, 0)$  and  $(0, 2)$  respectively.



On the copy of this diagram in the Printed Answer Book, sketch the curve  $y^2 = f(x)$ , giving the coordinates of the points where the curve crosses the axes. [5]

4 You are given the equation  $(2x-1)^2 - e^x = 0$ .

(i) Verify that 0 is a root of the equation. [1]

There are also two other roots,  $\alpha$  and  $\beta$ , where  $0 < \alpha < \beta$ .

(ii) The iterative formula  $x_{r+1} = \ln(2x_r - 1)^2$  is to be used to find a root of the equation.

(a) Sketch the line  $y = x$  and the curve  $y = \ln(2x-1)^2$  on the same axes, showing the roots 0,  $\alpha$  and  $\beta$ . [3]

(b) By drawing a 'staircase' diagram on your sketch, starting with a value of  $x$  that is between  $\alpha$  and  $\beta$ , show that this iteration does not converge to  $\alpha$ . [1]

(c) Using this iterative formula with  $x_1 = 3.75$ , find the value of  $\beta$  correct to 3 decimal places. [3]

(iii) Using the Newton-Raphson method with  $x_1 = 1.6$ , find the root  $\alpha$  of the equation  $(2x-1)^2 - e^x = 0$  correct to 5 significant figures. Show the result of each iteration. [4]

5 It is given that  $y = \tan^{-1}2x$ .

(i) Find  $\frac{dy}{dx}$  and show that  $\frac{d^2y}{dx^2} + 4x\left(\frac{dy}{dx}\right)^2 = 0$ . [3]

(ii) Find the Maclaurin series for  $y$  up to and including the term in  $x^3$ . Show all your working. [4]

(iii) The result in part (ii), together with the value  $x = \frac{1}{2}$ , is used to find an estimate for  $\pi$ . Show that this estimate is only correct to 1 significant figure. [2]

- 6 The equation of a curve in polar coordinates is  $r = \sin 5\theta$  for  $0 \leq \theta \leq \frac{1}{5}\pi$ .
- (i) Sketch the curve and write down the equations of the tangents at the pole. [4]
- (ii) The line of symmetry meets the curve at the pole and at one other point  $A$ . Find the equation of the line of symmetry and the cartesian coordinates of  $A$ . [2]
- (iii) Find the area of the region enclosed by this curve. [4]
- 7 (i) By using a set of rectangles of unit width to approximate an area under the curve  $y = \frac{1}{x}$ , show that  $\sum_{x=1}^{\infty} \frac{1}{x}$  is infinite. [4]
- (ii) By using a set of rectangles of unit width to approximate an area under the curve  $y = \frac{1}{x^2}$ , find an upper limit for the series  $\sum_{x=1}^{\infty} \frac{1}{x^2}$ . [5]
- 8 It is given that  $I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \, dx$  where  $n$  is a positive integer.
- (i) By writing  $\sec^n x = \sec^{n-2} x \sec^2 x$ , or otherwise, show that
- $$(n-1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2} \text{ for } n > 1. \quad [5]$$
- (ii) Show that  $I_8 = \frac{96}{35}$ . [3]
- (iii) Prove by induction that  $I_{2n}$  is rational for all values of  $n > 1$ . [4]

**END OF QUESTION PAPER**

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