

# GCE

# **Mathematics**

Unit 4736: Decision Mathematics 1

Advanced Subsidiary GCE

## Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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### 1. Annotations and abbreviations

Annotation in scoris	Meaning
√and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
Λ	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
M1 dep*	Method mark dependent on a previous mark, indicated by *
сао	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
nfww	Not from wrong working

#### 2. Subject specific instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

### Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

#### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of

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answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain

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unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

### Mark Scheme

Question	Answer/Indicative content	Mark	Guidance
1	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1	<ul><li>Working for the first two marks must be shown on table.</li><li>Do <u>not</u> accept rows and columns interchanged.</li><li>Choosing <i>AB</i> from first column and both <i>BC</i> and <i>BE</i> from second column.</li></ul>
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A1	All correct (i.e. choosing <i>CF</i> from third column and <i>ED</i> from fifth column as well).
	AB, BC, BE, ED, CF	B1	These arcs listed in this order. May reverse the order of the letters (e.g. $BA$ rather than $AB$ ), but must be a list of arcs (rather than vertices or numbers on table or tree) and arcs must be in this order.
	$A \qquad B \qquad C \\ \bullet \qquad D \qquad E \qquad F$	M1 ft	Tree drawn (correct <u>or</u> follow through from ringed entries or arcs listed) Must have 6 vertices, 5 arcs and at least one vertex of order $\ge 3$ (i.e. a branching tree not a 'string')
	Weight = 27	A1 ft [ <b>5</b> ]	27 from correct tree or weight of <u>their</u> tree stated If their tree is not the correct tree, total their weights written (on diagram or in working) or ringed in table, but do not hunt for weights if no calculation shown

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Question	Answer/Indicative content	Mark	Guidance
2 (i)	The largest value, 53, is not at the end of the list or 26 is at the end of the list and this is not the largest value	B1 [1]	Recognising that after one pass of bubble sort (increasing) the largest value would be at the right-hand end of the list Allow 'bottom' or 'top' for 'end' Need not specifically identify values 53 or 26
(ii)	32       41       22       37       53       43       29       15       26         41       32       22       37       53       43       29       15       26	[1]	Need <u>both</u> of these <u>written out in full</u> for the mark A genuine misread can be identified here and credited in subsequent parts, but this mark will <u>not</u> be available for a MR
(iii)	22 32 41 [37 53 43 29 15 26]	[1]	Allow 22 32 41 or any list that starts with this (ignore errors later in list) If multiple lists are shown, mark the last one
(iv)	8	B1 [1]	cao (no follow through for lost values, 'nine bags' given in question)
(v)	[ 53 43 41 37 32 29 26 22 15 ] BIN 1: 53 43 BIN 2: 41 37 22 BIN 3: 32 29 26 BIN 4: 15	B1 M1 A1 [3]	<ul> <li>Condone (at most) one miscopy/misread/omission/extra for B and M, provided the intention for the mark is satisfied</li> <li>A packing of the 9 bags into at most four bins none of which has a total over 100, even if not packed in correct order. Try to interpret candidate's intention.</li> <li>53, 43, 41, 37, 32 (first five bags) correctly placed</li> <li>All correct, in four bins and packed in the correct order (cao)</li> <li>Final A mark will not be available if entries have been miscopied/misread or entries are omitted from list or extras added to list</li> </ul>
(vi)	BIN 1: 41 37 22 BIN 2: 53 32 15 BIN 3: 43 29 26	M1 A1 [2]	A packing of the 9 bags into exactly 3 bins No bin has a total over 100 Order of bins and order of bags in bins does not matter Miscopy/misread/omitted entries or extras get A0

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Qı	uestion	Answer/Indicative content	Mark	Guidance
3	(i)	P = 10x - 2y - 3z	B1 [1]	10x - 2y - 3z,  may imply ' P = 'Not P - 10x + 2y + 3z = 0  and not multiples of  10x - 2y - 3z
	(ii)	$5x - 5z \le 60 \text{ (or } x - z \le 12)$ $4x + 3y \le 100$ $x \ge 0, y \ge 0, z \ge 0$	M1 A1 B1 [3]	$5x - 5z (\leq, <, \geq, > \text{ or } =) 60 \text{ or } 4x + 3y ((\leq, <, \geq, > \text{ or } =) 100$ (or scaled versions of these) or with slack variables or similar, including slack variables and inequalities Both correct, with inequality signs and no slack variables No extra (incorrect) constraints isw subsequent inclusion of slack variables to make equalities Non-negativity for <i>x</i> , <i>y</i> and <i>z</i> (as inequalities, not in words)
	(iii)	i) Rows and columns may be reordered, try to follow candidate's intention Check tableau first, correct tableau $\Rightarrow 4$ marks		Candidates may use an intermediate tableau with the new pivot row but the other rows as original and then follow this with the augmented tableau
		Pivot on x column $60 \div 5 = 12, 100 \div 4 = 25, 12 < 25$ so pivot on 5 in column x (row 2 in column x)	B1	Correct pivot choice (may be implied from augmented tableau)
		Pivot row (new row 2) = row $2 \div 5$ New row 1 = row $1 + 10 \times \text{pivot row}$	M1	Dividing through their pivot row correctly, even if pivot choice was wrong (may be implied from working <u>or</u> augmented tableau)
		New row 3 = row 3 - 4 × pivot row $P x y z s t RHS$	M1	Augmented table has correct structure, in terms of existence of basis/non-basis columns, entries in RHS column are non-negative and $P$ value in RHS column (their 120) has increased (from 0)
		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A1 [4]	Correct tableau, using fractions and/or decimals Mark printed table in answer book unless crossed out and replaced (or candidate has made their intention clear)
	(iv)	No negative values in objective row	[1]	Recognising that all entries in top/first row are non-negative Or entries in top row are positive or zero (but not 'positive' without 'or zero') or equivalent using inequality signs
	(v)	x = 25, y = 0, z = 13 P = 211	B1 [2]	All three, including $y = 0$ . Need values and have them identified with <i>x</i> , <i>y</i> and <i>z</i> appropriately. Not just (25, 0, 13). 211, cao but may imply ' <i>P</i> ='

Question		on	Answer/Indicative content		Guidance		
4	(i)		All graphs have an even number of odd vertices <u>or</u> Cannot have an odd number of odd vertices <u>or</u> Sum of vertex orders is 15 so 7.5 arcs	B1 [1]	Recognising that a graph cannot have (exactly) three odd vertices (or nodes) <u>or</u> that the sum of the vertex (node) orders (or degrees or valencies) cannot be odd Sum (of orders) is 15/odd <u>but</u> it must be even/come in pairs/ Do not accept answers that say things like 'it can't be drawn' or 'cannot have a vertex of order 5'		
	(ii)	(a)	Vertex of order 6 must either join to itself or connect to another vertex twice (since there are only five other vertices) this means that the graph is not simple <u>or</u> On a simple graph with 6 vertices the maximum vertex order is 5 but this graph has a vertex of order 6	[2]	Identifying the vertex of order 6 (only) as causing the problem Explaining why the graph is not simple, must include both possibilities (repeated arc and loop) Giving only one possibility ('repeated arc' or 'loop') is not enough Candidates must identify that the problem is that the graph cannot be simple, and why, although they need not explicitly say that it cannot be simple		
		(b)	Eulerian No odd vertices	M1 A1 [2]	Correct choice, with an attempt at a reason, allowing misspelling Correct reason, all nodes are even (but not just using a diagram of a specific case)		
	(iii)	(a)	10	B1 [1]	Minimum is 10, sum $\geq$ 10		
		(b)	2, 4	B1 [1]	Both 2 and 4 stated (and no other values), but not just a specific example. Accept 'even <u>and</u> positive <u>and</u> less than 6'		
	(iv)	(a)	They can only be 2 or 4 <u>and</u> must sum to 20 2, 2, 4, 4, 4, 4 (in any order)	M1 A1 [2]	Six vertex orders listed, each is either 2 or a 4 or implied from correct list of vertex orders Two 2's and four 4's (in any order)		
		(b)	e.g.	M1 A1 [2]	Arcs may cross, provided intention for vertices is obvious A connected graph with exactly six vertices Graph is <u>simple</u> (no repeated arcs or loops) has exactly ten arcs and vertex orders 2, 2, 4, 4, 4, 4		

Question	Answer/Indicative content	Mark	Guidance
5 (i)	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	B1 M1 A1	<ul> <li>Permanent labels (shown in upper right boxes) and order of labelling (shown in upper left boxes) all correct</li> <li>Temporary labels 6 at <i>I</i>, 10 at <i>L</i> and 15 at <i>N</i></li> <li>No extra temporary labels (allow missing temporary values that becomes the permanent label)</li> <li>If values are crossed out, crossing out must be consistent (not a mixture of values we want and extras) and any values we want must be legible</li> </ul>
	Shortest distance = $12 \text{ km}$ Routes: $G J I L N$	B1 B1 [ <b>5</b> ]	12 Allow route written in reverse
(ii)	(ii) Shortest <i>G</i> to $K = 7$ , $KL = 2$ , shortest <i>L</i> to $N = 4 \Rightarrow 13$ km (shortest <i>G</i> to $L = 8$ , $LK = 2$ , shortest <i>K</i> to $N = 6 \Rightarrow 16$ km) Route: <i>G H K L N</i>		Showing using part (i) to calculate $G \dots KL \dots N = 13$ or $G \dots LK \dots N = 16$ (ft their distances) i.e. 'their 7' + 2+4 or 'their 7' + 6 or 'their 8' + 2+6 or 'their 8'+ 8 NOT 8 + 2 + 8 (and 13 on its own is not enough, even with correct route, we need evidence of how the answer to (i) was used) cao, this route (or in reverse)
(iii)	(iii) Odd vertices: $I, K, M, N$ IK = 5  IM = 6  IN = 7 $MN = \frac{5}{10}  KN = \frac{6}{12}  KM = \frac{5}{12}$ Minimum distance = 70 km		Odd vertices correct, or implied from subsequent working These three pairings seen (i.e. <i>IK</i> + <i>MN</i> , <i>IM</i> + <i>KN</i> , <i>IN</i> + <i>KM</i> , each with correct weights or totals) 70 (from correct method)
Repeat <i>IL</i> , <i>LK</i> and <i>MN</i> (written) Percy will pass through <i>L</i> four times		B1 B1 [ <b>5</b> ]	These <u>three arcs</u> (allow $IL+LK$ but not $IK$ or $ILK$ or route $I-L-K$ ) 4 times (allow 4 if it appears to be the answer to the final request)

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Q	uestion	Answer/Indicative content	Mark	Guidance		
6	(i)	Time constraint $20x + 15y \le 100 \Rightarrow 4x + 3y \le 20$ (as given)	B1	Need to see $20x + 15y$ , $100 \text{ and 'time' or 'hours'}$		
		Cost constraint: $(8 \times 8 + 35)x + (8 \times 3 + 80)y \le 600 \implies 99x + 104y \le 600$	M1	Using $8(8x + 3y)$ to cost the material (seen, or implied from $99x + 104y$ )		
		$(0 \times 0 + 35)x + (0 \times 5 + 00)y \le 000 \Rightarrow 77x + 10+y \le 000$	A1	$99x + 104y \le 600$ or any positive multiple of this constraint		
		Lower bound constraints: $x \ge 1$ and $y \ge 1$	B1	Both $x \ge 1$ and $y \ge 1$ and no extras (apart from non-negativity, which can be ignored)		
			[4]	(apare nom non nogan roy, when can ce ignored)		
	(ii)	Line $4x + 3y = 20$ , passing through (5, 0) and $(0, 6\frac{2}{3})$	M1	Line plotted so that it cuts axes at (4.75 - 5.25, 0) and (0, 6.5 - 6.75) (when extrapolated, if necessary)		
		Line $99x + 104y = 600$ passing close to (6, 0) and (0, 6)	M1	Line plotted so that it cuts axes at (6 - 6.25, 0) and (0, 5.75 - 6) (when extrapolated, if necessary)		
		Lines $x = 1$ and $y = 1$	M1	x = 1 and $y = 1$ (e.g. as boundaries of feasible region)		
			A1 [4]	Feasible region correct (indicated by, for example, shading in, shading out or labelling as <i>FR</i> , <i>R</i> or similar) The A mark is dependent on all three M marks.		
	(iii)	5		SC1 if (0, 6), (1, 5) or (2, 4) is written as a feasible point (or in $x = $ ,		
	()			y =  form) and total is shown as 6		
	(iv)	Total cost (£) = $15(20x+15y) + (99x+104y) = (399x+329y)$	[1] B1	$399x + 329y$ or $7(57x + 47y)$ , not follow through, need not give £, ignore $\leq 600$ if written		
		Need $x+y = 5$ with y as large as possible $\Rightarrow x = 1, y = 4$	M1	One of the points (1, 4), (2, 3), (3, 2), (4, 1) soi as solution (written or seen used in working)		
		1 bridesmaid and 4 page boys (costing £1715)	A1 [ <b>3</b> ]	This solution (cao) in context (in words), with or without cost		

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Q	uestic	on	Answer/Indicative content	Mark	Guidance
7	(i)			M1	Route starts Y Z R S W
			YZRSWXTQY		(allow correct list of arcs: YZ, ZR, RS, SW)
				A1	Correct cycle, including returning to <i>Y</i> (allow correct list of arcs)
			135 minutes	B1	135 cao, units not necessary
				[3]	
	(ii)		RZ=8 WX=8 SW=10 RS=12 TW=12 YZ=13 WY=14	B1	Using Kruskal's algorithm <u>on list</u> (indicating either which arcs are
			XZ = 22 QT = 25		used or which are not used, at least as far as $WY$ – i.e. dealing with
			$\dots \dots $		the whole of the top row correctly)
			Q $S$ $W$ $Y$	M1	Drawing a spanning tree for the eight vertices (i.e. any tree that has
				1411	7 arcs, whether it branches or not)
				A1	Correct tree
			$R \qquad T \qquad X \qquad Z$		
			88 (minutes)	B1	88 cao, units not necessary
				[4]	so eao, units not necessary
	(iii)	(a)	88 - 25 = 63	B1 ft	'their 88' – their arc weight(s) from Q, strict ft from (ii)
	. ,				Units not necessary
					If no numerical response to (ii) then accept 63
				[1]	
	(iii)	(b)	QT = 25, QR = 30, 63+25+30 = 118 minutes	B1 ft	'their 63' + 55 strict ft from (iii)(a), units not necessary
					If no numerical response to (iii)(a) then accept 118 only
				[1]	
	(iv)		Q T W X Y Z R S Q	B1	This closed route written, in this order starting and ending at Q
					(allow correct list of arcs: <i>QT</i> , <i>TW</i> , <i>WX</i> ,, <i>SQ</i> )
			= 130 mins, time for stops = 170 mins $\Rightarrow$ 300 minutes	B1	130, 170 and 300 seen or 300 from an appropriate calculation,
				D1	such as 130 (cao) and 5+5+10+30+30+30+30+30
			Route: $Y Z R S Q T W X Y$	B1	This route written (or in reverse) starting and ending at $Y$
				[2]	(allow correct list of arcs: YZ, ZR, RS,, XY)
				[3]	

reference

e	Q	R	S	Т	W	X	Y	Ζ
Q	-	30	35	25	37	40	43	32
R	30	-	12	15	15	20	20	8
S	35	12	-	20	10	18	25	16
Т	25	15	20	-	12	16	18	18
W	37	15	10	12	-	8	14	20
X	40	20	18	16	8	-	17	22
Y	43	20	25	18	14	17	-	13
Ζ	32	8	16	18	20	22	13	-

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