# Section Check In – 4.01 Proof

## Questions

1. Prove by induction that  is divisible by 2 for .

2.\* Prove by mathematical induction that is true for all .

3. Let the matrix . Prove by induction that  for .

4. A sequence is given by , . Prove by induction that .

5.\* Prove that the *n*th derivative of isfor .

6.\* Let f be the function given by  with . Using composition of functions and mathematical induction prove that  for all .

7.  is thought to be divisible by 3, 5 and 7 for all .

(i) Show that this is not true.

(ii) State which of 3, 5 and 7 do divide into  for all , and prove this by induction.

8.\* Prove by mathematical induction that  is true for all .

9. Prove that  for positive integer *n*.

10.\* The number of members of an online forum after *n* days, , is modelled by where  and *a* and *k* are constants.

Given that the forum has one member on day 1 and welcomes four new members per day, prove by induction that on the th day the number of members can be modelled by .

**Extension**

Let  be an integer. Show that .  
[You may use the result that  is an increasing function and .]

## Worked solutions

1. \*Consider 

 divisible by .

Therefore true for .

\*Assume true for 

Let 

Then  is assumed divisible by  and  for some .

\*Consider 



so  is divisible by .

So  for some 

hence  which is divisible by .

In conclusion

The statement is true for . If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

2. Consider 

LHS is 

RHS is 

LHS  RHS

Hence true for .

Assume true for , so  assumed true.

Consider 



Taking out a factor of 

.

The statement is true for . If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

3. If 

 which is true

Assume true for , so that 

Consider 

 Therefore true when .

The statement is true for . If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

4. Basis case

When  we have  and  is given, therefore true when .

Inductive step

Assume true when  therefore  assumed true.

Consider 

 so true for .

If assumed true for  the result has then been shown to be true for .

Therefore by mathematical induction the result is true for all .

5. Basis case

When 

 so true for 

Inductive step

Assume true for , so that 

Consider 





 so true for 

If assumed true for  the result has then been shown to be true for .

Therefore by mathematical induction the result is true for all .

6. Consider 

 and  hence true for 

Assume true for  so  assumed true.

Consider 



Therefore true for .

The statement is true for .

If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

7. (i) The values of  for the first few natural numbers are 102, 10 002, 1 000 002, …

These are not divisible by  because of the final digit .

102 is not divisible by 7, so we need to prove that  is divisible by .

(ii) Basis case  
Consider 

 divisible by .

Therefore true for .

Inductive step  
Assume true for 

Therefore assumed divisible by .

Consider 

Now let 

and consider the difference 



So  is divisible by .

Now , and both and  are divisible by .  
Hence  is divisible by .

If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

8. Consider 

LHS 

RHS 

LHS  RHS

Hence true for .

Assume true for , so  assumed true.

Consider 

















Therefore true for .

If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

9. When   and , so we have equality.

Hence true for .

Assume true for , so that 

Then for ,

so true for .

If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

10. The forum starts with one member, so .

The number of members increases by 4 every day so  and .

Using the formula given, when  we have , therefore true when .

Assume true when , so that  assumed true.

Consider 

 so true for .

If assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all .

**Extension**

**(NB some explanatory steps have been left out – make sure you can justify each step)**

Basis case

When  we have  which evaluates as  so it is true.

For the inductive step we show each half of the inequality separately for clarity.

1. First 

Assume true for so that 

Then for  we have 

Since we can assume that  is an increasing function and , we can see that  so  as required.

2. Now 

In a similar way, assume true for , i.e. that 

Then for  we have 

Since we can assume that  is an increasing function and , we can see that  so  as required.

If  assumed true for  the result has then been shown to be true for . Therefore by mathematical induction the result is true for all integers .

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