# OCR - Oxford Cambridge and RSATeacher Delivery Guide Pure Mathematics: Sequences and Series

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: SEQUENCES AND SERIES (1)** |
| Binomial expansions | Ms1 | Understand and use the binomial expansion of  where *n* is a positive integer.  |  |  |  |
| s2 | Know the notations *n*! and  and that is the number of ways of selecting *r* distinct objects from *n.* | The meaning of the term factorial.*n* a positive integer.Link to binomial probabilities. |  | will only be used in the context of binomial expansions and binomial probabilities. |
| **PURE MATHEMATICS: SEQUENCES AND SERIES (2)** |
| Binomial expansions | s3 | Use the binomial expansion of  where *n* is any rational number. | For  when  is not a positive integer. |  | General term. |
| s4 | Be able to write  in the form *an*  and hence expand . |  when *n* is not a positive integer. |  | Proof of convergence. |
| s5 | Be able to use binomial expansions with  rational to find polynomials which approximate . | Includes finding approximations to rational powers of numbers. |  |  |

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| **PURE MATHEMATICS: SEQUENCES AND SERIES (2)** |
| Sequences | Ms6 | Know what a sequence of numbers is and the meaning of finite and infinite with reference to sequences. |  |  |  |
| s7 | Be able to generate a sequence using a formula for the *k*th term, or a recurrence relation of the form . | e.g. ;  with . | *k*th term:  |  |
| s8 | Know that a series is the sum of consecutive terms of a sequence. | Starting from the first term. |  |  |
| s9 | Understand and use sigmanotation. |  |  |  |
|  | s10 | Be able to recognise increasing, decreasing and periodic sequences. |  |  |  |
|  | s11 | Know the difference between convergent and divergent sequences. | Including when using a sequence as a model or when using numerical methods. | Limit to denote the value to which a sequence converges. | Formal tests for convergence. |
| Arithmetic series | s12 | Understand and use arithmetic sequences and series. | The term arithmetic progression (AP) may also be used for an arithmetic sequence. | First term, *a*Last term, *l*Common difference, *d*. |  |
| s13 | Be able to use the standard formulae associated with arithmetic sequences and series. | The *n*th term, the sum to *n* terms.Including the sum of the first *n* natural numbers. |  *Sn* |  |
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| **Specification** | **Ref.** | **Learning outcomes**  | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: SEQUENCES AND SERIES (2)** |
| Geometric series | s14 | Understand and use geometric sequences and series. | The term geometric progression (GP) may also be used for a geometric sequence. | First term, *a*Common ratio, *r.* |  |
| s15 | Be able to use the standard formulae associated with geometric sequences and series. | The *n*th term, the sum to *n* terms. | *Sn* |  |
| s16 | Know the condition for a geometric series to be convergent and be able to find its sum to infinity. |  |  |  |
| Modelling | s17 | Be able to use sequences and series in modelling. |  |  |  |

# Thinking Conceptually

### General approaches

Prior to working with the subject content of this section of the specification, it is essential that learners have gained a thorough understanding of a number of topics such as the four rules of number including the priority of operations, signed numbers, rounding, algebra including substitution, bracket expansion and simplification of terms, products, factors, index notation and percentages. It would also be beneficial if learners have a good understanding of probability so that they can understand and appreciate the link to binomial probabilities. It is also essential that learners have the ability to recognize patterns.

Learners’ understanding should be deepened by a hands-on approach to this subject as they tend to struggle with the algebra involved.

### Common misconceptions or difficulties learners may have

The expansions of binomial expressions, sequences and series require learners to have a high level of skills in algebra. As the foundation of algebra is basic arithmetic, many misconceptions in algebra are found to be rooted in misconceptions in arithmetic.

There are many misconceptions concerning negative numbers as learners wrongly think that two negatives always make a positive when adding / subtracting negative numbers.

A common mistake is expanding  as if  had the value 1.

Learners often make mistakes with the signs when expanding, for example, 

Other common mistakes with expanding binomial expressions are raising only part of the term to the appropriate power and bracketing errors.

Also learners often waste time writing out the full expansion of the binomial expansion instead of just finding the coefficient of the required term.

Learners often find it difficult to remember the difference between series and sequences and there is a misconception that positive whole numbers are often the only numbers considered when generating sequences.

Learners often find it difficult to recognise when a sequence or series is arithmetic or geometric.

Learners struggle to correctly work out terms in a sequence by using the order of operations.

Also learners find it difficult to correctly find the value of, the common ratio, in geometric sequences.

In addition, learners find it difficult to use and apply the correct formulae for arithmetic and geometric series.

### Conceptual links to other areas of the specification

Indices – learners need a good understanding of the laws of indices when expanding binomial expressions.

Exponentials – learners need to have a good understanding of exponentials. The behaviour of a geometric sequence depends on the value of the common ratio. If the common ratio is greater than 1, there will be exponential growth towards positive or negative infinity (depending on the sign of the initial term) and if it is between −1 and 1 but not zero, there will be exponential decay towards zero. If the common ration is less than −1 then the magnitude of terms grows exponentially but their signs oscillate.

Inequalities – learners need to be able to use sequences and series in modelling and this sometimes involves solving inequalities involving exponentials and logarithms.

Logarithms – learners need to be able to use sequences and series in modelling and this involves solving inequalities involving exponentials and logarithms.

Polynomials – learners need to have a good understanding of polynomials to appreciate that a binomial is a polynomial that is the sum of two terms.

The modulus function – learners need to understand and be able to work with the sum to infinity of a convergent geometric series and this includes the use of modulus notation in the condition for convergence.

Probability – the expansion of binomial expressions has a similar structure to the calculation of binomial probabilities.

Graphs – learners need to a good understanding of graphs as often sequences and series can be analysed using graphs to determine whether they are converging or diverging.

# Thinking Contextually

Links to a range of resources that can be used to enhance and support the delivery of the ‘Sequences and Series’ topic are provided below.

Video tutorials, presentation slides, worked examples, exercises and interactive diagnostic tests are all used to reinforce the understanding of the different content areas.

Many learners fail to make connections between what they are learning and how that knowledge will be used. They struggle to understand the concepts in mathematics unless they can see the relevance to their everyday lives.

Learners will be more successful if they investigate mathematics through real life scenarios as they can see how these concepts are actually used outside of the classroom. They will then be able to discover the meaningful relationship between abstract ideas and practical applications in the real world. This in turn, will lead to greater motivation, enjoyment through discovery, improved confidence, independent thinking and better retention of skills.

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# Resources

| Title | Organisation | Description | Ref |
| --- | --- | --- | --- |
| [Pascal’s triangle and the Binomial Theorem](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-pascal-2009-1.pdf) | maths centre | Set of notes, examples and practice exercises. | s1 |
| [The mathematical secrets of Pascal’s triangle](https://www.youtube.com/watch?v=XMriWTvPXHI) | TED-Ed | This interesting video resource is aimed at AS level and demonstrates how Pascal’s triangle is used to expand a Binomial expression and looks at patterns within Pascal’s triangle. It also demonstrates how the Binomial expansion is used in probability.  | s1 |
| [Binomial Expansion: tutorial 1](https://www.youtube.com/watch?v=h3wdStMSfXk) | Exam Solutions | This youtube video is aimed at AS level and introduces learners to the binomial expansion for positive integer powers. | s1 |
| [Binomial Theorem](http://www.mathsisfun.com/algebra/binomial-theorem.html) | Math is Fun | This resource is aimed at AS level and demonstrates how to use the binomial expansion of positive integer powers. It includes ten interactive practice questions with worked answers for learners to reveal once they have attempted each question.  | s1 |
| [Binomial Theorem](http://ibmathstuff.wikidot.com/binomial-theorem) | IB Math Stuff | This excellent resource is aimed at AS level and demonstrates how to use the binomial expansion for positive integer powers and looks at the relationship of the binomial coefficients to Pascal’s triangle. This resource offers the opportunity to practice the important skills involved in binomial expansion to help address some common misconceptions and difficulties | s1 |
| [The Binomial Theorem](https://maths.mq.edu.au/numeracy/web_mums/module4/Worksheet412/module4.pdf) | Macquarie University | This excellent resource is aimed at AS level and demonstrates how to use the binomial expansion for positive integer powers and looks at the relationship of the binomial coefficients to Pascal’s triangle. This resource offers the opportunity to practice the important skills involved in binomial expansion to help address some common misconceptions and difficulties.  | s1 |
| [Genetics: Binomial Expansion](https://www.youtube.com/watch?v=mP3pBjHx7oo) | Christian D | This video resource is aimed at AS level and demonstrates how the binomial theorem is used in genetics. | s1 |
| [Can we use a binomial expansion to evaluate](https://undergroundmathematics.org/counting-and-binomials/r7978)   | Underground Mathematics | UCLES A level Mathematics 1 QP 186, 1951, Q3 requiring learners to determine the number of terms required in an expansion in order to obtain the requested accuracy. | s1 |
| [Binomial Expansion nCr method](https://www.youtube.com/watch?v=_SMFX9c8BLI) | Exam Solutions | This excellent video resource is aimed at AS level and demonstrates how to expand a Binomial expression for positive powers using the nCr method. | s1 and s2 |
| [The Binomial Expansion (AS)](http://mei.org.uk/files/sow/08-binomial-expansion.pdf) | MEI | A commentary of the underlying mathematics, a sample resource, a use of technology, links with other topics, common errors, opportunities for proof and questions to promote mathematical thinking. | s1 and s2 |
| [More Binomial Expansions](http://www.chartwellyorke.com/teachalevel/vol2sample2.ppt) | Mathematics Software for Education | This excellent resource is a Powerpoint presentation aimed at A level. The slides demonstrate how to use the binomial expansion for positive, fractional and negative powers.  | s1, s2, s3, s4 and s5 |
| [nCr, What is it?](https://www.youtube.com/watch?v=MqOhV9110y0) | Exam Solutions | This excellent short video resource is aimed at AS level and demonstrates how to calculate nCr with and without a calculator. | s2 |
| [Probability: Binomial Theorem](https://www.youtube.com/watch?v=W1_cSGUZbdY) | Rahul Patwari | This excellent video resource is aimed at AS level and demonstrates how the binomial theorem is used in probability. | s2 |
| [The Binomial Theorem](http://www.a-levelmathstutor.com/binomial.php) | A-Level Maths Tutor | This resource is aimed at A level as it demonstrates how to use the binomial expansion for positive integer powers and looks at the relationship of the binomial coefficients to Pascal’s triangle. It then looks at the binomial expansion of a fractional power. | s3 |
| [Binomial expansion – negative power](https://www.youtube.com/watch?v=DFRsdGigHis) | A Level Maths Help | This video resource is aimed at A level and demonstrates how to use the binomial expansion for negative powers.  | s3 and s4 |
| [The Binomial Theorem](https://www.intmath.com/series-binomial-theorem/4-binomial-theorem.php) | Interactive Mathematics | This excellent interactive resource is aimed at A level and demonstrates how to use the binomial expansion for positive integer powers and fractional powers. This resource offers the opportunity to practice the important skills involved in binomial expansion to help address some common misconceptions and difficulties.  | s3, s4 and s5 |
| [Binomial expansion – fractional power](https://www.youtube.com/watch?v=1__zL-aU5p4) | A Level Maths Help | This video resource is aimed at A level and demonstrates how to use the binomial expansion for fractional powers | s5 |
| [A2 Binomial Expansion: Negative Fractional Powers](http://www.alevelmathsnotes.com/2014/01/a2-binomial-expansion-negative.html) | A Level Maths Notes | This short resource is aimed at A level and demonstrates how to use the binomial expansion for negative fractional powers by giving four worked examples. | s5 |
| [Sequences and Series](http://mei.org.uk/files/sow/23-sequences-and-series.pdf) | MEI | A commentary of the underlying mathematics, a sample resource, a use of technology, links with other topics, common errors, opportunities for proof and questions to promote mathematical thinking. | s6 – s17 |
| [Sequences](https://www.stem.org.uk/resources/elibrary/resource/32141/sequences) | STEM Learning | These resources are aimed at A level and allow learners to become more familiar with sequences including Fibonacci and quadratic sequences. They include games, investigations, worksheets and practical activities. The resources are free but a login is required. | s7 |
| [Sequences](http://www.cimt.org.uk/primaryske/B1/Text.pdf) | CIMT | This resource is aimed at A level and provides the opportunity for learners to practice the important skills involved in sequences to help address some common misconceptions and difficulties. There are numerous exercises available for learners to attempt. | s7 |
| [Fibonacci Surprises](http://nrich.maths.org/11164) | NRICH | This interesting resource invites learners to investigate the Fibonacci sequence. | s7 |
| [Sigma Notation](http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/sigma.pdf) | maths centre | Set of notes, examples and exercises. | s9 |
| [Series](https://revisionmaths.com/advanced-level-maths-revision/pure-maths/algebra/series) | Revision Maths | This excellent resource is aimed at A level and introduces learners to the sigma notation and explains the difference between a series and a sequence. It provides examples of arithmetic sequences (progressions) including the formulae for the nth term and the sum to n terms. It also provides examples of geometric sequences (progressions) including formulae for the nth term and the sum to n terms.  | s9, s12 and s14 |
| [What is means for a Sequence to Converge, Diverge, be Periodic or Oscillate](https://www.youtube.com/watch?v=thzoShIFMDQ) | Jack Brown | This short video resource is aimed at A level and explains the differences between converging, diverging and periodic sequences.  | s11 |
| [Solving Sequences, Converging or Diverging? Calculus Tips](https://www.youtube.com/watch?v=0_D66IgOWoI) | Straighter Line | This short youtube video is aimed at A level as it introduces learners to the difference between convergent and divergent geometric sequences and series.  | s11 |
| [Staircase sequences](https://undergroundmathematics.org/thinking-about-numbers/staircase-sequences) | Underground Mathematics | Open ended sequence problem that leads into ideas of convergence.  | s11 |
| [Arithmetic Sequences Harder Questions AS A2 Maths](https://www.youtube.com/watch?v=uYq4Uu0ZaG8) | UK Maths Teacher | This challenging resource is aimed at A level and demonstrates how to find the nth term of an arithmetic sequence using sequence formulae and simultaneous equations. This resource is aimed at learners studying at A level. | s12 |
| [Arithmetic Sequences and Series](https://www.slideshare.net/itutor/arithmetic-sequence-and-series) | Slide Share | This resource is aimed at at A level and includes fourteen professional looking slides on arithmetic sequences. The slides include how to find the nth term and the sum to n terms by showing worked examples. | s12 and s13 |
| [Sequences and Series (2) – Introduction to Arithmetic Sequences](https://www.youtube.com/watch?v=DoZCRAW_vRc) | UK Maths Teacher | This excellent video resource is aimed at A level and introduces the learner to arithmetic sequences. Learners are asked to identify arithmetic sequences, generate terms and find nth terms of increasing and decreasing sequences. | s12 and s13 |
| [Arithmetic Sequence Calculator](https://www.geogebra.org/m/eK2b6XtA) | Geogebra | Two demonstrations using geogebra software.Question 1 allows learner to enter First term and common difference and nth term rule, in the form , is generated.Question 2 allows learner to specify two specific terms and the nth term rule, in the form , is generated. | s12 and s13 |
| [The nth term for Fractional Sequences](https://corbettmaths.com/2012/08/20/the-nth-term-for-fractional-sequences/) | Corbettmaths | This video resource is aimed at A level and demonstrates how to calculate the nth term for fractional arithmetic sequences. | s12 and s13 |
| [Arithmetic and Geometric Progressions](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-apgp-2009-1.pdf) | maths centre | This resource is aimed at A level and offers the opportunity to practice the important skills involved in arithmetic and geometric sequences to help address some common misconceptions and difficulties. There are exercises to complete along with answers. | s12, s13, s14, s15 and s16 |
| [What’s the next number to occur in both sequences](https://undergroundmathematics.org/divisibility-and-induction/r9916) | Underground Mathematics | UKMT SMC 2005 Q7 problem looking at common differences of arithmetic sequences | s13 |
| [Arithmetic sequence real life car rental](https://www.youtube.com/watch?v=zxQPCLa9UhI) | Heritagealgebra | This video resource is aimed at A level and demonstrates real – life practical applications of using arithmetic sequences by investigating car rental. | s13 and s17 |
| [Geometric sequence calculator](https://www.geogebra.org/m/TVdvyvgy) | Geogebra | Two demonstrations using geogebra software. Question 1 allows learner to enter first term and common ratio and the nth term rule, in the form  , is generated.Question 2 allows learner to specify two specific terms and the nth term rule, in the form  , is generated. | s14 and s15 |
| [Compound interest: establishing a formula](https://www.youtube.com/watch?v=luQ-4tJzC30) | Metal Maths Project | This excellent video resource shows an example of where sequences can be used in the real world. The video gives the example of a student who is saving for a round-the-world trip and shows how to calculate how long this will take. | s14 and s15 |
| [Geometric Sequences and Series (1) – Sequences Brief Introduction](https://www.youtube.com/watch?v=ryWO3wYJ8Iw) | UK Maths Teacher | This excellent video resource is aimed at A level and introduces the learner to geometric sequences and series and explains the difference between the two. It demonstrates how to find the nth term of increasing and decreasing geometric sequences and also looks at converging and diverging sequences. | s14, s15 and s16 |
| [Geometric Sequence](https://www.slideshare.net/chelseaanneberganio/geometric-sequence-51673284?next_slideshow=1) | Slide Share | This resource is aimed at A level and includes forty one professional looking slides on finite and infinite geometric sequences. The slides include how to find the nth term, the sum of a finite series and the sum to infinity of convergent geometric series. It includes questions for learners to attempt along with worked answers. | s14, s15 and s16 |
| [The Mathematics of Business and finance: multipliers](https://integralmaths.org/course/view.php?id=166&section=6) | MEI | Use of multipliers in ecomomic modelling. Teacher’s notes, powerpoints, spreadsheet and worksheet | s14, s16 |
| [Compound Interest](file:///%5C%5Cfilestorage%5COCR%5CPD%5CProdSup%5CProjMan%5CProjects%5CGCE%5CMaths%5CMaths_T3%5CMathsB_MEIonlyresources%5CDG_06_Sequences%5CPub%5Cjustmaths.co.uk%5CWorksheets%5CNumber%5CCompound%20Interest%20-%20EXAM%20QUESTIONS.pdf) | Just Maths | This resource invites learners to answer questions on compound interest and reviews GCSE content. | s15 |
| [Double a Penny Everyday for 31 Days -Learn The Power of Compound Interest](https://www.youtube.com/watch?v=XUCYdGAWcbo) | Your Plan B Income | This short video resource looks at the “power” of compound interest. | s15 |
| [Finite geometric series word problem: mortgage](https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-finite-geometric-series-word-problems/v/geometric-series-sum-to-figure-out-mortgage-payments) | Khan Academy | This video resource is aimed at A level and looks at how finite geometric series are used in the real world by looking at mortgage calculations. | s15 |
| [Geometric Series](http://www.projectmaths.ie/documents/T%26L/GeometricSeries.pdf) | Project Maths | This challenging resource (pages 20 - 24) is aimed at A level and offers further practice in solving real life problems using geometric sequences. | s15 and s17 |
| [Sequences word problems](https://www.khanacademy.org/math/algebra/sequences/modeling-with-sequences/v/modeling-situations-with-arithmetic-and-geometric-sequences) | Khan Academy | This video resource is aimed at A level and demonstrates how to solve problems with modelling real life situations using arithmetic and geometric sequences. | s17 |
| [The Geometric Series in Finance](http://www.konfluence.org/geomser/geomser.html) | Konfluence Research Institute | This challenging resource is aimed at A level and demonstrates how geometric series are used in finance by looking at mortgages and investments. | s17 |
| [Finance and Growth](http://www.metalproject.co.uk/METAL/Resources/Teaching_learning/55848.pdf) | Metal Project | This challenging resource looks at how arithmetic and geometric series can be used to calculate simple and compound interest, the time value of money, investment appraisal and the NPV rule and the IRR rate of return rule. There are worked examples and exercises for learners to complete along with links to video resources. The content is at A level standard. | s17 |
| [Applications of Sequences and Series](http://www.algebralab.org/lessons/lesson.aspx?file=algebra_seqseriesapps.xml)  | Algebra LAB | This interactive resource is aimed at A level and offers further practice in solving real life problems using arithmetic and geometric sequences. | s17 |
| [Geometric sequence real life retirement](https://www.youtube.com/watch?v=-Wk2pJj3FH4) | Heritagealgebra | This video resource demonstrates real – life practical applications of using geometric sequences by investigating retirement figures. It is aimed at A level. | s17 |
| [Geometric Sequences and Series](http://www.hec.ca/en/cam/help/topics/geometric_sequences_and_series.pdf) | HEC Montréal | This challenging resource is aimed at A level and looks at geometric sequences and series applications in financial mathematics. | s17 |
| [Population Growth and Sequences](https://www.math.ku.edu/~mandal/math105/growth105C11.html) | University of Kanas | This resource is aimed at A level and looks at population growth and its link to sequences. It offers further practice in solving problems using sequences. | s17 |
| [Multiplying bacteria - a geometric sequence problem](http://www.bbc.co.uk/education/clips/zjyykqt) | BBC | This short video resource poses a real – life problem using sequences. The video can be paused so that learners can discuss and attempt to solve the problem. | s17 |
| [Sequences and Series](http://tentotwelvemath.com/classroom-resources/sequences-and-series-2/) | 10 to 12 | Set of notes, demonstrations and activities on AP and GP  | s17 |

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