# OCR - Oxford Cambridge and RSATeacher Delivery Guide Pure Mathematics: Exponentials and Logarithms

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: EXPONENTIALS AND LOGARITHMS (1)** |
| Exponentials and Logarithms | ME1 | Know and use the function and its graph. | For .  |  |  |
| E2 | Be able to convert from an index to a logarithmic form and vice versa. |  for  and . |  |  |
| E3 | Understand a logarithm as the inverse of the appropriate exponential function and be able to sketch the graphs of exponential and logarithmic functions. |  for  and  . Includes finding and interpreting asymptotes. |  |  |
| E4 | Understand the laws of logarithms and be able to apply them, including to taking logarithms of both sides of an equation. | Including, for example  and  |  | Change of base of logarithms. |
| E5 | Know and use the values of  and . | ,  |  |  |
| E6 | Be able to solve an equation of the form . | Includes solving related inequalities. |  |  |
| E7 | Know how to reduce the equations and  to linear form and, using experimental data, to use a graph to estimate values of the parameters.  | By taking logarithms of both sides and comparing with the equation.Learners may be given graphs and asked to select an appropriate model. |  |  |

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: EXPONENTIALS AND LOGARITHMS (1)** |
| Exponentials and natural logarithms | ME8 | Know and be able to use the function and its graph. |  |  |  |
| E9 | Know that the gradient of  is  and hence understand why the exponential model is suitable in many applications. |  |  |  |
| E10 | Know and be able to use the function and its graph. Know the relationship between  and . |  is the inverse function of . |  |  |
| Exponential growth and decay | E11 | Be able to solve problems involving exponential growth and decay; be able to consider limitations and refinements of exponential growth and decay models. | Understand and use exponential growth and decay: use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models. Finding long term values. |  |  |
| **Graphs with gradient proportional to one of the coordinates** |
|  results in a quadratic graph.  results in an exponential graph. Image of a quadratic graph Image of an exponential graph |

# Thinking Conceptually

### General approaches

The initial approach to this should be a recap of the work on indices. This forms the foundation of understanding of the work on logarithms.

The work on the introduction of the exponential function should be done graphically so that it is clear how this function works. This graphical process can also be used to show the inverse function through its reflection in the line . This helps justify the impossibility of a negative value within the logarithm.

With the work on indices reviewed, proving and demonstrating the laws of logarithms becomes a more natural process.

There are a multitude of examples of modelling with exponential functions within many other subject areas, including physics, chemistry, biology, economics, geography, psychology. This is not an area that will ever run dry and provides a depth of applications to stimulate interest.

### Common misconceptions or difficulties learners may have

The failure to link this work to indices often means that there is a failure to see the whole number values so that  or just become calculated values. This then helps to justify the values of , why logarithms of numbers smaller than  are negative and why it is not possible to find the logarithm of a negative number.

Familiarity with the exponential graph helps to avoid students believing that  can be have a negative value.

If reduction to linear form is not clearly related to the equation of a straight line then the connection is not properly made and this becomes an incomprehensible process.

The last section is not very different to growth and decay taught at GCSE and if this link is again made then it is not a difficult topic; it is not one that should ever stand alone as an add-on to the main course but as a culmination of all the previous skills.

### Conceptual links to other areas of the specification

Indices – the work on indices lays the foundation to the work on logarithms and if this first topic is not clearly understood then understanding of logarithms will not hold together particularly well.

Straight lines – this forms the foundation of reduction to linear form wherein is made the use of logarithms to reduce functions to straight line form.

Gradients – the whole understanding of the nature of rate of change and gradient is essential to being able to apply this to the curve of .

Calculus – Subsequent work on calculus will make use of natural logarithms so this section forms an important foundation for future study.

# Thinking Contextually

There are a considerable number of examples of applications using the exponential function at various levels. These range from financial implications like compound interest to calculating mortgage payments, through to the scientific applications like radioactive decay curves, the spread of disease and drug concentration decay, alongside Newton's cooling curve, through to geography and population growth or decline.

At a slightly higher level this can also include applications to music with sound decay curves or again to geography with methods to protect buildings from earthquakes by reducing the effect of the shock waves and also to engineering with a similar effect used in shock absorbers and car suspensions.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
| --- | --- | --- | --- |
| [Exponentials and logarithms (AS)](http://mei.org.uk/files/sow/12-exponentials-and-logarithms.pdf) | MEI | A starting point to consider the content, links to example resources, ideas for the effective use of technology, links across the curriculum, and common errors. | E1-E11 |
| [Exponential Function y=a^x](https://www.geogebra.org/m/SfDjuf54) | Geogebra | Use slider to compare graph with  ,  and  . | E1 |
| [When does this exponential function equal this linear one?](https://undergroundmathematics.org/exp-and-log/r8013) | Underground Mathematics | Introductory question on plotting exponential function. | E1 |
| [Reach for the stars](https://undergroundmathematics.org/exp-and-log/reach-for-the-stars/solution) | Underground Mathematics | Introductory puzzle to encourage learners to think about scales. | E1 |
| [Exponential Functions graphs](https://www.geogebra.org/m/YK6pcUsB) | Geogebra | An applet that allows the variables to be changed to see how the shape of the exponential graph will alter for different values. | E1 |
| [Graphs of exponential and logarithmic functions](https://www.intmath.com/exponential-logarithmic-functions/2-graphs-exp-log-fns.php) | Interactive Mathematics | Graphs of the exponential and logarithmic functions. | E1 and E8 |
| [Definitions of logarithmic and exponential functions](https://www.intmath.com/exponential-logarithmic-functions/1-definitions-exp-log-fns.php) | Interactive Mathematics | Basic definitions of both exponential and logarithmic functions. | E2 |
| [The Mathematics of chemistry: pH](https://integralmaths.org/course/view.php?id=166&section=20) | MEI | Use of logarithmic scales for pH in chemistry. | E2 |
| [Logarithmic functions graphs](https://www.geogebra.org/m/EUZVy9Br) | Geogebra | An applet that allows the variables to be changed to see how the shape of the logarithmic graph will alter for different values. | E3 |
| [Logarithmic Action!](https://www.geogebra.org/m/puEbQtBY) | Geogebra | Investigate transformational link between  and  . | E3 |
| [Laws of Logarithms](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-logarithms-2009-1.pdf) | Math Centre | A pdf worksheet that covers the basic laws of logs and with a worksheet to follow that includes the answers.  | E4 |
| [Laws of Logarithms](https://www.intmath.com/exponential-logarithmic-functions/3-logarithm-laws.php) | Interactive Mathematics | All the basic laws. | E4 |
| [Logarithms base 10](https://www.intmath.com/exponential-logarithmic-functions/4-logs-base-10.php) | Interactive Mathematics | Specific notes on logarithms to the base 10. | E4 |
| [Logarithms base e](https://www.intmath.com/exponential-logarithmic-functions/5-logs-base-e-ln.php) | Interactive Mathematics | Specific Notes on natural logarithms. | E4 |
| [Introduction to Logarithms](https://www.mathsisfun.com/algebra/logarithms.html) | Math is fun | A set of basic notes on this topic with 10 multiple choice questions at the end. | E4 |
| [Logarithms](http://www.themathpage.com/aprecalc/logarithms.htm) | The Math Page | A guide to the basic laws of logarithms. | E4 |
| [Risp 31: Building Log Equations](http://www.s253053503.websitehome.co.uk/risps/risp31.html) | Risps | Use the cards to build equations involving logs – and check which of them are true? | E4 and E5 |
| [Exponential and Logarithmic Equations](https://www.intmath.com/exponential-logarithmic-functions/6-logarithm-exponential-eqns.php) | Interactive Mathematics | Use of logarithms to solve equations involving exponentials. | E6 |
| [To log or not to log?](https://undergroundmathematics.org/exp-and-log/to-log-or-not-to-log) | Underground Mathematics | Encourage learners to consider when it is appropriate to solve equations using logarithms. | E6 |
| [Solving Logarithmic Equations](http://www.purplemath.com/modules/solvelog.htm) | Purple Math | A nice step by step method spread over three pages that details solving both logarithmic and exponential equations. | E6 |
| [Applications of Exponential Functions](http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_ExponentsApps.xml) | Algebra Lab | A guide to a variety of applications with questions and answers included at the end.  | E6 |
| [Graphs on logarithmic and semilogarithmic axes.](https://www.intmath.com/exponential-logarithmic-functions/7-graphs-log-semilog.php) | Interactive Mathematics | A nice introduction to the benefits of reduction to linear form. | E7 |
| [Reduction to linear form](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwi3oZ_V0Y3WAhVCIMAKHdAfCSIQFggtMAA&url=http%3A%2F%2Fpeople.exeter.ac.uk%2Fmsh209%2FFoundation%2FScans%2FReduce_to_straight_lines.pdf&usg=AFQjCNF_Y-P8chddgG8PPjyBr1-nRM_Fsw) | University of Exeter | Four worked examples on reducing data to linear form. | E7 |
| [Expressions reducible to linear form](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwjdyaCs0o3WAhWoL8AKHTLVDuYQFggtMAA&url=http%3A%2F%2Fwww3.ul.ie%2F~mlc%2Fsupport%2FTechMaths1%2Ffiles%2FReduceLinearForm.pdf&usg=AFQjCNH_cdeV63YpboLrTIsp5yp6eiUCMA) | University of Limerick | A simple pdf file that outlines the basic process. | E7 |
| [Calculating the value of e](https://www.intmath.com/exponential-logarithmic-functions/calculating-e.php) | Interactive Mathematics | A proof exercise rather than a purely graphical approach. | E8 |
| [When does f'(x)=f(x)](https://www.geogebra.org/m/Nd9qTNAf) | Geogebra | Another applet based exercise that focuses on the gradients, particularly in finding the value for which f'(x)=f(x). | E9 |
| [Graphs e^x and ln x](https://www.geogebra.org/m/azn5dkBe) | Geogebra | Graph y = e^x and inverse y = ln x | E10 |
| [Exponential Growth and Decay (illustrated Meaning)](https://www.geogebra.org/m/UNnfwuhQ) | Geogebra | Interactive demonstration with linked questions. | E11 |
| [Dow Jones Index](https://www.intmath.com/exponential-logarithmic-functions/dow-jones-industrial-ave-graph.php) | Interactive Mathematics | A simple application of modelling graphically. | E11 |
| [World population live](https://www.intmath.com/exponential-logarithmic-functions/world-population-live.php) | Interactive Mathematics | A simple modelling activity using world population data. | E11 |
| [Picture the process](https://undergroundmathematics.org/combining-functions/picture-the-process-ii) | Underground Mathematics | Match the description with the graph. | E11 |
| [Radioactive Decay](https://plus.maths.org/content/radioactive-decay-and-exponential-laws) | +plus magazine | This does go beyond the level of this course but the explanations provide a good background to what is going on and put the whole process in context. | E11 |
| [Exponential outbreaks, the Mathematics of Epidemics.](https://learning.blogs.nytimes.com/2014/11/05/exponential-outbreaks-the-mathematics-of-epidemics/?_r=0) | The learning network | Uses modelling to investigate the spread of Ebola in 2014 | E11 |
| [Exponential and Logarithmic Models](https://people.richland.edu/james/lecture/m116/logs/models.html) | Richland Community College | This looks at the different types of logarithmic and exponential models and the shapes of their graphs. | E11 |
| [Applications of Exponential Functions](https://www.ck12.org/algebra/applications-of-exponential-functions/) | cK-12 | A front page with links to separate applications most of which contain videos for use in class.  | E11 |
| [Interest Rate Word Problems](http://www.sosmath.com/algebra/logs/log5/log51/log51.html) | SOS Maths | Application of exponential and logarithm functions in modelling interest rates. | E11 |
| [Amortization Word Problems](http://www.sosmath.com/algebra/logs/log5/log52/log52.html) | SOS Maths | Application of exponential and logarithm functions in modelling monthly repayments of mortgages. | E11 |
| [Population Word Problems](http://www.sosmath.com/algebra/logs/log5/log53/log53.html) | SOS Maths | Application of exponential and logarithm functions in modelling population. | E11 |
| [Decay Word Problems](http://www.sosmath.com/algebra/logs/log5/log54/log54.html) | SOS Maths | Application of exponential and logarithm functions in modelling scientific decay. | E11 |
| [Earthquake Problems](http://www.sosmath.com/algebra/logs/log5/log56/log56.html) | SOS Maths | Application of exponential and logarithm functions in modelling eartquakes. | E11 |
| [Exponents in the Real World](http://passyworldofmathematics.com/exponents-in-the-real-world/) | Passy's World of Mathematics | Some background reading of the applications of exponential growth and decay. | E11 |
| [Half-Life Action!!!](https://www.geogebra.org/m/Ym5NQWJp) | Geogebra | Quick demonstration of the meaning of “Half-Life” | E11 |
| [The Mathematics of biology: exponential growth](https://integralmaths.org/course/view.php?id=166&section=5) | MEI | Resources about exponential growth of bacterial populations | E11 |
| [The Mathematics of business and finance: compound interest](https://integralmaths.org/course/view.php?id=166&section=8) | MEI | Exponential growth in the context of compound interest | E11 |

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