# Section Check In – 4.04 Further Vectors

## Questions

1. Find a vector which is perpendicular to the vectors and .

2. Find the Cartesian equation of the line passing through the points  and .

3.\* Find the equation of the plane through the points , and .

4.\* Find the point of intersection between the plane  and the line

.

5.\* The line *L1* passes through the points and . The line *L2*  passes through the point and is parallel to the vector .  
  
(i) Find an equation for *L*1 in the form.

(ii) Prove that *L*1 and *L*2 are skew.

6. Two lines have equations

 and ,

where  is a constant.   
 Given that the two lines intersect,

(i) find and the point of intersection,

(ii) find the acute angle between the lines.   
  
  
  
  
  
  
  
  
7. Determine whether the following vectors form a set of three perpendicular vectors:

 .

8.\* A plane is given by the equation .   
Determine an equation for the plane in each of the following forms:

(i) 

(ii) 

(iii) .

9. It is given that  where

.

Find the values of the constants and 

10.\* Four points have coordinates and  where is a constant.

(i) Find the perpendicular distance from C to the line AB.

(ii) Find in terms of, and show that the shortest distance between the lines AB and CD is given by

.

**Extension**

Web Activity – Advanced vector activities from NRICH <http://nrich.maths.org/10835>

## Worked solutions

1. Using the vector product, a vector perpendicular to the two given vectors is



2. The vector equation of a line is written in the form 

Choose  .   
 To find we have to find the direction vector between the 2 points; .

Hence the vector equation of the line is  whereis a real parameter.

The cartesian equation is found by saying that and rearranging each component of the vector equation for : . (Other equivalent forms are possible!)

3. We first find  and 

The cross product is then taken to find the normal to the plane:

 

The equation of the plane is  which in this case is .

To findwe use one of the points, say  , 

so that the vector equation is 

and hence the cartesian equation of the plane is .

4. First we write expressions forand using the equation of the line:



Next we substitute these expressions into the equation of the plane: 

Solving for :



Substituting this back into the equation of the line gives 

hence the point of intersection is.

5. (i) We need the direction vector and a point on the line. To find the direction vector we have   
 to find the difference between the position vectors of two points on the line, therefore:

****

A position vector on the line is  so an equation of the line is:

**,** whereis scalar parameter than can take any value.

(ii) First I’ll find the equation of the line. The direction of the line is parallel toso the direction vector of the line is:



A position vector on the line is. Therefore an equation of the line is:

**.**

If the 2 lines intersect then the position vectors of points on the lines will be equal for

some values of the parameters, so:

****

I can form 3 equations, but only two are needed to find values for *s* and *t*, for example

 and .

Solving these simultaneously gives  and .

Now, the lines only intersect if these values also work for the third equation;

LHS 

RHS 

These are not equal so the lines are skew.

6. (i) The lines are not parallel as the directions of each line are not scalar multiples of each other. They must either intersect or are skew. Therefore:

** = **

Forming 3 equations:



Re-arranging:



We choose any 2 of the equations, solve for *s* and *t* and then substitute into the one we haven’t used to see if it’s consistent. If we get ‘nonsense’ the lines are skew. We’ll use the 2 equations not involving*:*



Multiplying equation two by 4 to make one of the unknowns equal in size:



Subtracting: , 

Therefore: 

Substituting into the equation involving:

**

**

If the lines intersect then this equation must make sense. Therefore . The point of intersection can then be found by substitutingor into one of the equations, I’ll choose the equation not involving :

** **

So the point of intersection isand the value must take is 1.

(ii) To find the angle between 2 lines, we have to find the angle between the position   
 vectors of each line and use the formula: .



The magnitudes are:





The scalar product is: 

Therefore:  , to 1 dp

7.



Hence they are all perpendicular (note that it is not enough to show that , though it would be if you also showed that .)

8. We need to choose any point on the line, for example .

We will also need the normal to the plane .

Finally, we will need to choose any two (not collinear) vectors which are perpendicular to , for example and .

Then:

(i) 

(ii) 

(iii) 

9.



and hence we have to solve



Rearranging the first and second equations gives and .

Substituting into the third equation gives  and hence .

Substituting back gives  and .

10. (i) Using the formula for the perpendicular distance

.

The cross product is calculated as 

Therefore, the distance is 

(ii) The cross-product can be calculated as 



The dot product is 

which can be simplified to 

The modulus 



and hence the distance is

.

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