Level 3 Cambridge Technical in Engineering
05822/05823/05824/05825 /05873

Unit 23: Applied mathematics for engineering

Wednesday 18 January 2017 – Afternoon
Time allowed: 2 hours

You must have:
- the formula booklet for Level 3 Cambridge Technical in Engineering (inserted)
- a ruler (cm/mm)
- a scientific calculator

First Name		Last Name	
Centre Number		Candidate Number	
Date of Birth			

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number, candidate number and date of birth.
- Answer **all** the questions.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number centre number and question number(s).
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- An answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- This document consists of **16** pages.

FOR EXAMINER USE ONLY	
Question No	Mark
1	/11
2	/11
3	/14
4	/12
5	/11
6	/12
7	/9
Total	**/80**

© OCR 2017 [601/4594/8, 601/4600/X, 601/4599/7, 603/0322/0]

Answer **all** questions.

1 For this question you may refer to the constant acceleration formulae in section 6.4 of the formula booklet provided.

A new car is being tested for braking distance on a test track. The car is travelling at a speed of $v\,\text{m}\,\text{s}^{-1}$ when the brakes are applied and the car comes to rest after 3 s. The distance, s m, that the car travels in t s after the brakes are applied is recorded. Recordings are made at 0.5 s intervals and the results are shown in Table 1.

Time *t* (s)	**Distance travelled *s* (m)**
0	0
0.5	13.75
1	25
1.5	33.75
2	40
2.5	43.75
3	45

Table 1

(a) On the grid below, draw a graph of s against t for $0 \le t \le 3$.

[3]

© OCR 2017

(b) The car decelerates uniformly. The distance the car travels may be modelled by the following equation.

$$s = At^2 + Bt \qquad \text{for } 0 \le t \le 3$$

(i) Use information from Table 1 to calculate the values of A and B.

....

....

....

....

....

....

.... **[3]**

(ii) Calculate the time taken for the car to travel the first 30 m after the brakes are applied.

....

....

....

....

....

....

.... **[3]**

(iii) Calculate the speed of the car when $t = 0$.

....

.... **[1]**

(iv) Calculate the deceleration of the car.

....

.... **[1]**

© OCR 2017

2 The isosceles triangle shown in Fig. 1 has height D and area A.

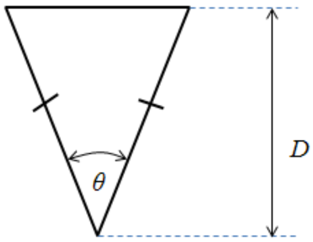


Fig. 1

(a) Show that $A = D^2 \tan\left(\frac{\theta}{2}\right)$.

....

....

....

....

....

.... **[3]**

(b) The cross-section of the hull of a boat is modelled by an isosceles triangle as shown in Fig.1 above. The hull has a length of 5 m and a total mass of 4000 kg. When the hull is resting in water, as shown in Fig. 2, the base of the hull is at a depth d m below the surface of the water. In this position the mass of water displaced by the hull is equal to the total mass of the boat.

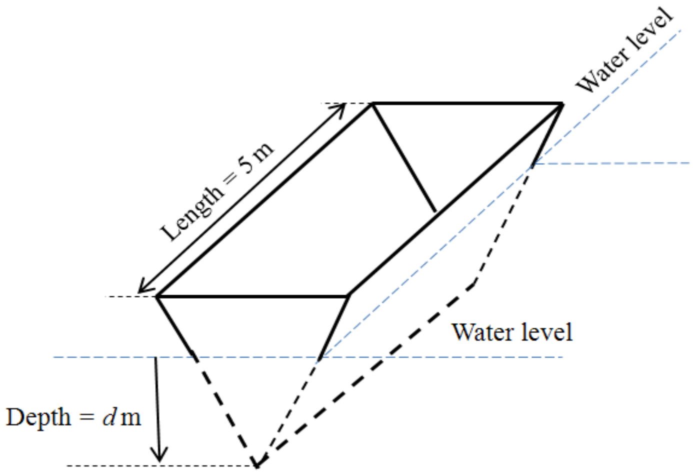


Fig. 2

The density of water is 1000 kg m^{-3}.

© OCR 2017

(i) Show that

$$d = 2\sqrt{\frac{1}{5\tan\left(\frac{\theta}{2}\right)}}\ .$$

....

....

....

....

....

....

....

....

....

....

.... **[4]**

(ii) Given that $d = 1.7$, calculate angle θ.

....

....

....

....

....

....

....

....

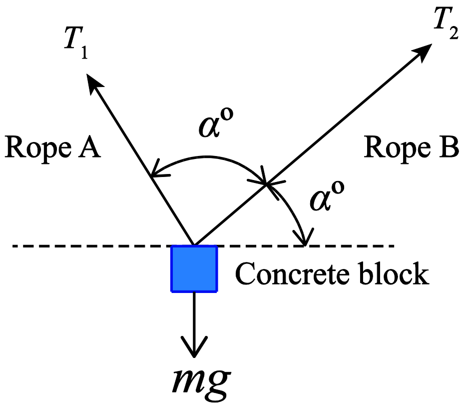
....

....

.... **[4]**

© OCR 2017

3 Fig. 3 shows a concrete block of mass m kg supported at rest in equilibrium by two ropes, A and B. Rope A has tension T_1 N and Rope B has tension T_2 N. The angle between the two ropes is $\alpha°$ and the angle between Rope B and the horizontal is $\alpha°$ where $0<\alpha<90$.



Not to scale

Fig. 3

(a) Express the tensions T_1 and T_2 as vectors $\overline{T_1}$ and $\overline{T_2}$ in component form, expressing each component in terms of α and T_1 or T_2 as appropriate.

....

....

....

.... **[4]**

(b) Sketch a vector diagram showing the relationship between the resultant vector $\overline{R} = \overline{T_1} + \overline{T_2}$ and the vectors $\overline{T_1}$ and $\overline{T_2}$.

[2]

© OCR 2017

(c) Write the resultant vector, $\overline{\mathrm{R}}$, in component form, expressing both horizontal and vertical components in terms of α, T_1 and T_2 .

....

....

.... **[2]**

(d) The concrete block is at rest in equilibrium, so the horizontal component of $\overline{\mathrm{R}}$ is zero.

(i) Find an expression for $\cos\alpha$ in terms of T_1 and T_2 .

....

....

....

....

....

....

....

....

....

....

....

....

.... **[4]**

(ii) Given that $T_1 = T_2$, find angle α .

....

....

....

....

.... **[2]**

© OCR 2017

4 The operating efficiency, η, of a particular car engine is modelled by the equation

$$\eta = -0.01\left(\frac{N}{10}\right)^2 + 0.06\left(\frac{N}{10}\right) + 0.2,$$

where N is the speed of the engine in revolutions per second (rps).

(a) Using calculus, calculate the speed of the engine that will result in maximum operating efficiency.

....

....

....

....

....

....

....

....

....

.... **[3]**

(b) The power, P W, of the engine is modelled by the equation

$$P = 10000\eta N.$$

Calculate the power of the engine at the speed calculated in part **(a)**.

....

....

....

....

....

....

....

.... **[2]**

© OCR 2017

(c) By combining the equation for efficiency and the equation for power calculate the speed of the engine that will result in maximum power.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

.... **[5]**

(d) A student claims that, using these models, maximum power is achieved when the engine is operating at maximum efficiency. State whether or not this claim is true and justify your answer by referring to your calculations above.

....

....

....

....

....

....

....

.... **[2]**

© OCR 2017

THIS PAGE HAS BEEN LEFT INTENTIONALLY BLANK

© OCR 2017

5 A domestic storage heater produces heat over a 10-hour period. During this time the instantaneous power, P kW, produced is modelled by the equation

$$P = 4he^{-\frac{h}{2}},$$

where h is the time, measured in hours, from when the heating system is turned on.

(a) Calculate the time at which the instantaneous power produced is a maximum.

....

....

....

....

....

....

....

.... **[4]**

(b) Calculate $\frac{1}{10}\int_0^{10} 4he^{-\frac{h}{2}}dh$ to find the average power produced during the 10-hour period.

....

....

....

....

....

....

....

....

....

....

....

....

.... **[7]**

© OCR 2017

6 The passive impedances, Z Ω, of a resistor, an inductor and a capacitor when subjected to an input voltage of the form $v = V\sin(\omega t)$ are given below.

Resistor: Z = R, where R Ω is resistance

Inductor: Z = $\omega L\mathrm{j}$, where L H is inductance

Capacitor: Z = $\dfrac{1}{\omega C\mathrm{j}}$, where C F is capacitance

Note $\mathrm{j} = \sqrt{-1}$

When the components are connected in parallel the total passive impedance, Z_t, satisfies the following formula.

$$\frac{1}{Z_t} = \sum_{i=1}^{n} \frac{1}{Z_i}$$

Fig. 4 shows a circuit containing a resistor, an inductor and a capacitor connected in parallel.

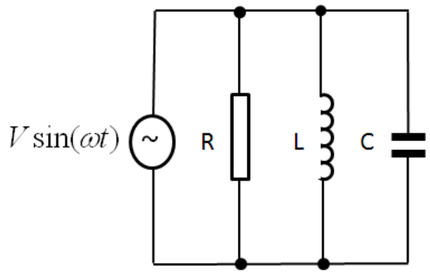


Fig. 4

(a) Show that the total impedance for this circuit, Z_t, is given by the following formula.

$$Z_t = \frac{\omega RL}{\omega L + R(\omega^2 LC - 1)\mathrm{j}}$$

....

....

....

....

....

....

....

....

.... **[5]**

© OCR 2017

(b) You are given following values.

$\omega = 10^4$ $R = 1$ $L = 10^{-2}$ $C = 10^{-5}$

(i) Express Z_t in the form $a + \mathrm{j}b$ where a and b are real values.

....

....

....

....

....

....

....

....

....

.... **[3]**

(ii) Express Z_t in the form $r(\cos\theta + \mathrm{j}\sin\theta)$ where r is a real value and θ is an angle in degrees.

....

....

....

....

....

....

....

....

....

....

....

.... **[4]**

© OCR 2017

7 **(a)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x}{1+x}\right)=\dfrac{1}{(1+x)^2}$.

....

....

....

....

....

....

.... **[2]**

The rotational speed, ω, of an unpowered flywheel as it is slowing down is modelled by the differential equation

$$(1+t)\frac{\mathrm{d}\omega}{\mathrm{d}t}=\omega_0\left(\frac{t}{1+t}-1\right),$$

where t is time and ω_0 is the speed of the flywheel when $t = 0$.

(b) Solve the differential equation to show that

$$\omega=\omega_0\left(1-\frac{\mathrm{t}}{1+\mathrm{t}}\right).$$

[5]

....

....

....

....

....

....

....

....

....

....

....

....

© OCR 2017

....

....

....

....

....

(c) Comment on whether or not the model appears to provide a realistic description of the speed of the flywheel as it is slowing down.

....

....

....

....

....

....

....

.... **[2]**

END OF QUESTION PAPER

© OCR 2017

PLEASE DO NOT WRITE ON THIS PAGE

Oxford Cambridge and RSA

Copyright Information:

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.
If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material OCR will be happy to correct its mistake at the earliest possible opportunity.
For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.
OCR is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2017