# Teacher Delivery Guide Core Pure: Proof

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **CORE PURE: PROOF (a)** |
| Proof | \* | Be able to prove mathematical results by deduction and exhaustion, and disprove false conjectures by counter example. | Includes proofs of results used in this specification, where appropriate. |  |  |
| Induction | Pp4 | Be able to construct and present a proof using mathematical induction for given results for a formula for the *n*th term of a sequence, the sum of a series or the *n*th power of a matrix.  | The result to be proved will be given. E.g. for the sequence given by ,  prove that . | ,  |  |
| **CORE PURE: PROOF (b)** |
| Proof | \* | Be able to prove mathematical results by contradiction. |  |  |  |
| Induction | Pp5 | Be able to construct and present a proof using mathematical induction. | E.g. proofs of divisibility, proof of De Moivre’s theorem. The result to be proved will always be given explicitly. |  |  |

# Thinking Conceptually

### General approaches

The key concept in the method of proof by induction is that truth of a value implies truth for the next, partnered with the truth of one particular instance, which is usually that of the first term. This idea shows that the truth of the 1st result provides the truth of the 2nd; and the 2nd implies the 3rd; the 3rd implies the 4th; and so to infinity. A wonderful idea and concept. Once grasped - easy to comprehend.

In the approach to teaching it is good to give a clear presentation of this idea before beginning any actual proofs. A common visual aid is a line of dominoes which begin to fall in turn. This visual demonstration can be found in the resources list. Once example proofs have been given to the students, it is probably a good idea to break the proof down into stages. Various approaches are given in the activities. Provide four clear stages for beginners:

* Proving the first instance  with conclusion;
* Assuming the  instance ;
* Using the assumption to prove the  case;
* Explaining why the result is proven in a conclusion which summarises both the method of induction and the reasons why the result is true for all integers.

Proof by induction is used in various areas of mathematics, such as: proof of summations; divisibility results; proof of matrix results and derivatives; proof of recurrence relations. Each type, in general, has its own typical helpful strategies. Summation, matrices, derivatives and recurrence relations tend to use a similar approach which follows the initial method outlined in the key concept part above. For divisibility, it is often easier to do using  notation and then assuming  and using a strategy which considers whether  is divisible. If using this approach, first show that  is divisible.

### Common difficulties learners may have

Proof by induction is more formal than most other parts of A Level maths and further maths, and so it can be a hurdle for many learners. It is conceptually challenging, especially for beginners. With time, the ideas develop and new concepts are formed. There is not one unique way to approach the proofs and so different strategies are gained with experience. In addition, each area in maths will have its own approaches and so it will probably take time for a student to feel at ease with the whole range of ideas and strategies involved. This work is abstract and that can also cause problems. In terms of the mathematics itself, it can at times be very hard to establish the truth of the case.

### Common misconceptions learners may have

Learners may underestimate the amount of detail needed in proof by induction. They may lack formality in their work. The work itself may omit essential parts of the reasoning. Students may not be aware of the difficulty involved in some of the proofs and may feel inadequate if they cannot complete them easily. They need to know that this is a misconception since many proofs are hard or elusive. Beginners may underestimate the level of mathematical reasoning required for mathematical induction.

### Conceptual links to other areas of the specification

This topic has conceptual links with all other areas of proof and with many algebraic or geometric results. There are links with matrices and general number. This also links with series defined by recurrence relations or otherwise, and their summations. In addition, proof by induction can be linked with indices, derivatives and inequalities.

The main conceptual link is with mathematical reasoning at a deeper level. We ask ourselves ‘is this argument convincing?’ and an induction argument if written clearly, should be a convincing proof.

Proof by induction demands extremely good algebraic skills. It is a useful part of maths to encourage students to feel more at ease with very formal notation and with formal reasoning. Induction is a topic which overlaps with university mathematics courses.

# Thinking Contextually

The contexts which are used for proof by induction are wide in range and many of them can be found in the suggested activities. Obvious areas are matrices, indices and series summation. Proof by induction can also be taught using the contexts of recurrence formulae and inequalities. Less obvious areas are graph theory, geometry and derivatives. Proof by induction also links with computer programming, number theory (with theorems concerning primes and factorials), and set theory.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
| --- | --- | --- | --- |
| [Mathematical Induction](http://legacy.earlham.edu/~peters/courses/logsys/math-ind.htm) | Earlham College | Background reading for students. May be a helpful resource for some learners. | p4 |
|   | Underground Mathematics | This problem can be accessed and explored by any student armed with a calculator; indeed this will give them a good feel for what’s going on. However, being able to generalise this requires some more sophisticated thinking. Students will find that if they factorise the general term then they can apply the difference of two squares in order to break it down into manageable parts that can be analysed. A basic understanding of evens, odds and consecutive numbers will then help students to come to a conclusion, although some support may be required to fully justify this. | p4 |
| [Can we prove these Fibonacci number results?](https://undergroundmathematics.org/divisibility-and-induction/r9868) | Underground Mathematics | UCLES STEP Mathematics II, 1996, Q3. Links to suggested approach and worked solution. | p4 |
| [One step, two step](https://undergroundmathematics.org/divisibility-and-induction/one-step-two-step) | Underground Mathematics | A resource where students can naturally find themselves using inductive reasoning without needing to be formal about it. It’s possible to tackle this one using knowledge of binomial coefficients, but there’s a more elegant approach that students can find by studying smaller numbers of stairs and building up. | p4 |
| [Mathematical Induction](file:///C%3A%5CUsers%5Cstella.dudzic%5CWork%20Folders%5CDocuments%5CStella%5CCurricDev%5CALevel%5CMEI%20A%20level%20Development%202014%5CResources%5COCRres%5CFMres%5Cweb.stanford.edu%5Cclass%5Carchive%5Ccs%5Ccs103%5Ccs103.1126%5Clectures%5C03%5CSlides03.pdf). | Stanford University | PowerPoint presentation on Induction which is student centred and fairly accessible. Good motivational material. Fairly sophisticated approach. | p4 and p5 |
| [Proof by mathematical induction explained](http://www.smh.com.au/video/video-news/video-education-promotion/proof-by-mathematical-induction-explained-20160614-4gh8v.html) | The Sidney Morning Herald | Very good and clear 5 minute video which introduces proof by induction. Good starter. | p4 and p5 |
| [Some Induction Examples](http://nrich.maths.org/10658) | Nrich | Ideal worked example for beginners to proof by induction. Could be used for flipped learning. | p4 and p5 |
| [A level Maths: Proof by Induction](https://www.tes.com/teaching-resource/a-level-maths-proof-by-induction-6146951) | TES | Good resource with templates for different types of proof by induction. Also some past exam questions with worked solutions. | p4 and p5 |
| [Lecture 2: Induction](https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-042j-mathematics-for-computer-science-fall-2010/video-lectures/lecture-2-induction/) | Massachusetts Institute of Technology | An introduction to proof techniques, covering proof by contradiction and induction, with an emphasis on the inductive techniques used in proof by induction. Induction starting from 20 minutes into the class. Quite a sophisticated approach & fairly challenging to students who are new to the topic. Could be used for flipped learning. | p4 and p5 |
| [Mathematical Induction](https://cseweb.ucsd.edu/classes/sp14/cse20-a/InductionNotes.pdf) | UC San Diego | Fairly basic good introduction to induction with plenty of examples. | p4 and p5 |
| [Proof by Mathematical Induction](http://www.math.umaine.edu/~farlow/sec16.pdf) | The University of Maine | Introduction to Induction with some historical aspects. Includes Peano’s fifth axiom from the nineteenth century. | p4 and p5 |
| [Dirisibly Yours](http://nrich.maths.org/1933) | Nrich | Divisibility proof to construct. Could be a plenary exercise. | p4 and p5 |
| [Elevens](http://nrich.maths.org/5510) | Nrich | Proof of an algebraic property to construct. | p4 and p5 |
| [Proof by induction (Creating connections between topics)](https://www.ncetm.org.uk/public/files/4597078/Mathematical%2BMoment%2BProof%2Bby%2Binduction.pdf) | NCETM | Class activity for work in pairs. Motivates linked starter activities. Focuses on recurrence relations and induction. | p4 and p5 |
| [Induction in Pascal’s Triangle](file:///C%3A%5CUsers%5Cstella.dudzic%5CWork%20Folders%5CDocuments%5CStella%5CCurricDev%5CALevel%5CMEI%20A%20level%20Development%202014%5CResources%5COCRres%5CFMres%5Ceuclid.ucc.ie%5CMATHENR%5CExercises%5CCombinatoricsInductionSums.pdf) | UCC | Interesting Induction PowerPoint linked with Pascal’s triangle. Could be used for a more challenging class activity with group work. | p4 and p5 |
| [Fermat Numbers](file:///C%3A%5CUsers%5Cstella.dudzic%5CWork%20Folders%5CDocuments%5CStella%5CCurricDev%5CALevel%5CMEI%20A%20level%20Development%202014%5CResources%5COCRres%5CFMres%5Cwstein.org%5Cedu%5C2010%5C414%5Cprojects%5Ctsang.pdf) | William A. Stein | Stretch activity material for high performing students. Document about Fermat Numbers, with some historical content. Lots of proof including proof by induction. Could be used for activity based time in class, or for project work. | p4 and p5 |
| [Mathematical induction](http://web.mat.bham.ac.uk/R.W.Kaye/seqser/induction.html) | University of Birmingham | Good general account of induction. Recommended as reading once some experience has been gained. | p4 and p5 |
| [Converging Product](http://nrich.maths.org/253) | Nrich | More challenging proof to construct and result to show. | p5 |
| [7.4 - Mathematical Induction](https://people.richland.edu/james/lecture/m116/sequences/induction.html) | Richland Community College | Clear summary of stages of the proof with worked examples. Could be used for flipped learning. | p5 |
| [OK! Now Prove It](http://nrich.maths.org/297) | Nrich | Activity to find the conjecture and prove it.  | p5 |
| [Proof by Induction](https://www.plymouth.ac.uk/uploads/production/document/path/3/3745/PlymouthUniversity_MathsandStats_proof_by_induction.pdf) | Plymouth University | Good induction PowerPoint with many examples. Includes conjecture. Also has exercises and a quiz with worked solutions. | p5 |
| [Mathematical Induction](http://www.people.vcu.edu/~rhammack/BookOfProof/Induction.pdf) | VCU | Good formal account of conjecture and induction proof. Includes many examples and also some in context. | p5 |
| [Triominoes](https://undergroundmathematics.org/divisibility-and-induction/triominoes) | Underground Mathematics | This resource gives students a situation where they can naturally see how understanding the problem for one board size can lead to understanding the next size up. There are two rather lovely standard arguments for this problem, both using inductive thinking but with slightly different inductive steps. | p5 |
| [Picture this!](https://undergroundmathematics.org/divisibility-and-induction/picture-this) | Underground Mathematics | This presents students with opportunities to investigate and to explore. The interactivity shows how the computer takes two numbers and produces a picture. | p5 |
| [DeMoivre's Theorem (Mathematical Induction)](https://www.youtube.com/watch?v=k4ZddA4e3a4) | DaveAcademy | A video demonstration looking at De Moivre's theorem and a qualitative explanation on how to prove something with mathematical induction. | p5 |

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