# Teacher Delivery Guide Core Pure: Differential Equations (b)

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **CORE PURE: DIFFERENTIAL EQUATIONS (b)** | | | | | |
| Modelling with differential equations | P  p19 | Understand how to introduce and define variables to describe a given situation in mathematical terms. |  |  |  |
| p20 | Be able to relate 1st and 2nd order derivatives to verbal descriptions and so formulate differential equations. | The differential equations will not be restricted to those which candidates can solve analytically. |  |  |
| p21 | Know the language of kinematics, and the relationships between the various variables. | Including acceleration = . |  |  |
| \* | Know Newton's 2nd law of motion. | In the form . |  | Variable mass. |
| p22 | Use differential equations in modelling in kinematics and in other contexts. | Sufficient information will be given about contexts which may be unfamiliar. |  |  |

***DISCLAIMER***

This resource was designed using the most up to date information from the specification at the time it was published. Specifications are updated over time, which means there may be contradictions between the resource and the specification, therefore please use the information on the latest specification at all times.If you do notice a discrepancy please contact us on the following email address: [resources.feedback@ocr.org.uk](mailto:resources.feedback@ocr.org.uk)

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| **CORE PURE: DIFFERENTIAL EQUATIONS (b)** | | | | | |
| Solutions of differential equations | Pc7 | Know the difference between a general solution and a particular solution. Be able to find both general and particular solutions. |  |  |  |
| Integrating factor method | c8 | Recognise differential equations where the integrating factor method is appropriate. | Equations which can be rearranged into the form . |  |  |
|  | c9 | Be able to find an integrating factor and understand its significance in the solution of an equation. | Integrating factor, . |  |  |
|  | c10 | Be able to solve an equation using an integrating factor and find both general and particular solutions. | E.g. a particular solution through a given point. |  |  |
| Second order differential equations | c11 | Be able to solve differential equations of the form , using the auxiliary equation. | and  are constants. | Homogeneous.  Complementary function. |  |
| c12 | Understand and use the relationship between different cases of the solution and the nature of the roots of the auxiliary equation. | Discriminant > 0.  Discriminant = 0.  Discriminant < 0. |  |  |
| c13 | Be able to solve differential equations of the form , by solving the homogeneous case and adding a particular integral to the complimentary function. | and  are constants. |  |  |
| c14 | Be able to find particular integrals. Understand the relationship between different cases of the solution and the nature of the roots of the auxiliary equation. | Cases where  is a polynomial, trigonometric or exponential function.  Includes cases where the form of the complementary function affects the form required for the particular integral. |  |  |

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| **CORE PURE: DIFFERENTIAL EQUATIONS (b)** | | | | | | |
| Simple harmonic motion | Pc15 | Be able to solve the equation for simple harmonic motion, , and be able to relate the solution to the motion. | Learners may state that they recognise the differential equation is that for SHM, and quote the solution in an appropriate form (e.g.  or), unless specifically required to solve the equation, e.g. by using the techniques of Pc11. | amplitude.  period |  | |
| Damped oscillations | c16 | Be able to model damped oscillations using 2nd order differential equations. |  |  |  | |
| c17 | Be able to interpret the solutions of equations modelling damped oscillations in words and graphically. | The damping will be described as ‘over-‘, ‘critical’ or ‘under-‘ according to whether the roots of the auxiliary equation are real distinct, equal or complex. | Where applicable, the amplitude refers to the local maximum distance from the equilibrium position.  The amplitude decreases with time. |  | |
| Simultaneous differential equations | c18 | Analyse and interpret model situations with one independent variable and two dependent variables which lead to coupled 1st order simultaneous linear differential equations and find the solution. | Applications include predator-prey models and other population models. E.g. solve by eliminating one variable to produce a single, 2nd order equation. |  |  | |

# Thinking Conceptually

### General approaches

This topic can easily be divided into two clear sections, the work on first order differential equations and the work on second order differential equations. It would also be advisable to treat it like that as the techniques used on the two are very different.

It is worth considering covering the work on second order differential equations first as this is sometimes an easier topic because the solution techniques are linked to quadratics rather than specific integration skills. A brief revision of the different solutions to a quadratic is worth conducting first. This topic connects with applications in Mechanics and so a practical focus is in order.

The first order differential equations cover a much broader range of techniques and it is useful to ensure that the learners have a good grounding in integration before beginning. It will, however, provide a massive boost to their understanding of integration in the A Level Mathematics course. This is more of a pure topic although the modelling aspect does connect to both Mechanics and Statistics with population modelling.

We would also suggest not shying away from proofs here to demonstrate the various solution types, firstly because it leads to a better understanding and secondly these because these are Further Mathematicians.

A fairly practical approach to the later sections which are very modelling based will give the learners a clear idea of what they are trying to achieve and why it works.

### Common misconceptions or difficulties learners may have

The most common misconception here is that learners often see dy and dx as notation rather than operators as it can be taught that way at A Level. This does not allow them to easily understand the way in which second order differential equations can then be solved. An initial discussion on the role and use of operators will often facilitate that.

The greatest difficulty is usually an insufficient grounding in integration. It would not hurt to run over all their integration again so that they can refresh it in their own minds and also fill any gaps that they have before moving on. The principle of learning to walk before you can run is certainly valid here.

Teaching this topic as two separate units (1st ODE and 2nd ODE) also helps overcome any confusion between the two techniques which are contrasting in their methodology.

### Conceptual links to other areas of the specification

### A Level Mathematics

Quadratic Equations – the solution to these needs to be understood in order to be able to solve the second order differential equations and relate it to the different types of solutions that then arise.

Trigonometry – specifically the graphical work, however all the work on radians and a significant number of the trigonometric identities could be used here. It is worth emphasising that all work in this section is conducted in radians.

Exponentials and Logarithms – specifically the exponential function, however, as with the trigonometry all other aspects of logarithms could be put to use here. Understanding the differentiation of the exponential function helps with understanding how the integrating factor transforms a first order differential equation into the correct format.

Differentiation – an understanding of differentiation and its uses, particularly the differentiation of standard functions and the introduction of constructing differential equations. Although this in an integration topic there is still a significant quantity of differentiation techniques to be used here.

Integration – as the principal focus of this topic is integration then all the skills here may well be drawn upon to complete the solutions of the various differential equations. A clear understanding of this work will avoid any confusion during this stage of the course.

Non uniform acceleration – which leads to the work on simple harmonic motion and damped oscillations. The links between displacement, velocity and acceleration remain the same irrespective of the complexity of the Mathematics.

A Level Further Mathematics

Complex Numbers – particularly in reference to one set of solutions of the second order differential equations. It is possible to teach this topic without detailed reference to complex numbers but it helps them to understand much more if a direct link is made.

Hyperbolic Functions – as this is an A Level Further Mathematic topic it is possible that the hyperbolic functions may also be included in the first order differential equations.

Further Integration – particularly the techniques at Further Mathematics level which could be drawn upon, particularly in the First Order Differential equations work.

# Thinking Contextually

Applications of 1st Order differential equations which require an integrating factor include mixing problems.

The second order differential equations focus far more on the application to Harmonic Motion, in this case moving towards damped motion. This then links to suspension systems and the mechanics of the underworkings of a car. However suspension systems go far beyond that with its impact on large scale engineering, particularly in earthquake damping techniques.

Stepping away from the physical engineering, sound waves are also linked to damped harmonic motion as sound waves decay, but also in terms of harmonic and resonance frequencies.

Linear systems tend to be more biological in their applications as they are used to model predator prey populations as in the rate of growth of predators and the decline of prey or vice versa. These models are not always linear but at least get across that there is an application that can be referenced easily.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
| --- | --- | --- | --- |
| [Example STEP Differential Equation Questions](https://nrich.maths.org/11055) | Nrich | Several example questions taken from STEP past papers. Includes questions covering formulating a differential equation, as well as solving first order and second order problems (Hints available). | c7 – c18 |
| [Linear First Order Differential Equations Calculator](https://www.symbolab.com/solver/linear-first-order-differential-equation-calculator) | Symbolab | This is an online calculator that allows the entry of a 1st order differential equation and then provides the solution. | c8 |
| [First Order Linear Differential Equations](http://www.maths.surrey.ac.uk/explore/vithyaspages/firstorder.html) | University of Surrey | This set of pages deals with 1st ODE. It is broken into variables separable and Integrating factors.  Each section has a set of test questions at the end. | c8, c9 and c10 |
| [First Order Differential Equations](https://nrich.maths.org/11053) | Nrich | Article covering separation of variables and integrating factors. | c8, c9 and c10 |
| [First Order Ordinary Differential Equation](http://www.mathcentre.ac.uk/resources/uploaded/mccp-richard-1.pdf) | mathcentre | This is a pdf file that covers all aspects of 1st ODE. It includes a good number of examples, questions and solutions. | c8, c9 and c10.  p21 and p22 |
| [Second Order Differential Equations](http://www.maths.surrey.ac.uk/explore/vithyaspages/secondorder.html) | University of Surrey | This set of pages deals with 2nd ODE. It is broken into homogeneous and non-homogeneous equations.  Each section has a set of test questions at the end. | c11, c12, c13 and c14 |
| [Second Order Differential Equations](https://www.symbolab.com/solver/linear-first-order-differential-equation-calculator) | Symbolab | This is an online calculator that allows the entry of a 2nd order differential equation and then provides the solution. | c11, c12, c13 and c14 |
| [Second Order Differential Equations](https://nrich.maths.org/11054) | Nrich | Article covering homogeneous and inhomogeneous second order differential equations. | c11, c12, c13 and c14 |
| [Damped Harmonic Oscillator](http://hyperphysics.phy-astr.gsu.edu/hbase/oscda.html) | HyperPhysics | A physics based approach to Damped Harmonic Motion, but the principles are the same. | c16 and c17 |
| [Animation of a damped harmonic oscillator (physics, mechanics)](https://www.youtube.com/watch?v=HRcjtVa1LfM) | Ya physics animations | This youtube video shows a damped harmonic oscillator with graph. Very simple and very clear. | c16 and c17 |
| [The damped harmonic oscillator](https://www.geogebra.org/m/sAAwEXgy) | Geobebra | This simulator shows how changing the values affects the different types of damping and hence the solutions. | c16 and c17 |
| [Second Order Ordinary Differential Equations](http://www.mathcentre.ac.uk/resources/uploaded/mccp-richard-2.pdf) | mathcentre | This is a pdf file that covers all aspects of 2st ODE. It includes a good number of examples, questions and solutions. | c16 and c17 |
| [Differential Equations Second Order (Homogeneous)](http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/second-order-differential-equations-homog.pdf) | University of Salford | This is a power point style pdf file that is less of a tutorial than a quick revision with questions and full worked solutions. Ideal as a class exercise to find out if they can do the work and check their answers. | c16 and c17 |
| [Differential Equations Second Order (Inhomogeneous)](http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/second-order-differential-equations-inhomog.pdf) | University of Salford | This is a power point style pdf file that is less of a tutorial than a quick revision with questions and full worked solutions. Ideal as a class exercise to find out if they can do the work and check their answers. | c16 and c17 |
| [Simple Harmonic Motion](http://www.acoustics.salford.ac.uk/feschools/waves/shm.php) | University of Salford | This is a series of animations that covers all aspects of simple harmonic motion. An excellent visual demonstration. | c16 and c17 |
| [London Tour](https://www.maths.ox.ac.uk/study-here/undergraduate-study/outreach/maths-city/london-tour-0)  [Harmonic Motion](https://www.maths.ox.ac.uk/study-here/undergraduate-study/outreach/maths-city/london-tour-0) | University of Oxford | This is an article on Mathematics in London, but included is a section on the Millennium Bridge and resonance frequencies. | c16 and c17 |
| [Tacoma Narrows Bridge Collapse](https://www.youtube.com/watch?v=lXyG68_caV4) | Garrett Lenz | What happens when they get the Maths wrong. | c16 and c17 |
| [First order Linear Systems](https://ocw.mit.edu/courses/mathematics/18-03sc-differential-equations-fall-2011/unit-iv-first-order-systems/linear-systems-introduction/MIT18_03SCF11_s32_1text.pdf) | MIT Opencourseware | This is a simple introduction to linear systems with a loosely practical example. | c18 |
| [Systems of first-order linear differential equations](https://mjo.osborne.economics.utoronto.ca/index.php/tutorial/index/1/sim/t) | University of Toronto | An economics based approach to this topic. | c18 |
| [Differential Equations for Engineers](http://www.civil.uwaterloo.ca/xie/Graphics/XIE_Differential%20Equations%20for%20Engineers_excerpt.pdf) | University of Waterloo | This file is just devoted to the modelling application of all aspects of differential Equations. | p19, p20, p21 and p22 |

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