# Teacher Delivery Guide Statistics: Discrete Random Variables

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | | **Exclusions** |
| **Y422 STATISTICS MAJOR: DISCRETE RANDOM VARIABLES (a)**  **Y432 STATISTICS MINOR: DISCRETE RANDOM VARIABLES**  **Y412 STATISTICS a: DISCRETE RANDOM VARIABLES** | | | | | | |
| Probability distributions | SR1 | Be able to use probability functions, given algebraically or in tables.  Be able to calculate the numerical probabilities for a distribution.  Be able to draw and interpret graphs representing probability distributions. | Other than the Poisson and geometric distributions, the underlying random variable will only take a finite number of values.  An understanding that probabilities are non-negative and sum to 1 is expected. | |  |  |
| Expectation and variance | R2 | Be able to calculate the expectation (mean), , and understand its meaning. |  | |  |  |
|  | R3 | Be able to calculate the variance,, and understand its meaning. | Knowledge of .  Standard deviation = . | |  |  |
|  | R4 | Be able to use the result  and understand its meaning. |  | |  |  |
|  | R5 | Be able to use the result  and understand its meaning. |  | |  |  |

***DISCLAIMER***

This resource was designed using the most up to date information from the specification at the time it was published. Specifications are updated over time, which means there may be contradictions between the resource and the specification, therefore please use the information on the latest specification at all times.If you do notice a discrepancy please contact us on the following email address: [resources.feedback@ocr.org.uk](mailto:resources.feedback@ocr.org.uk)

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | | **Exclusions** |
| Expectation and variance  (cont) | SR6 | Be able to find the mean of any linear combination of random variables and the variance of any linear combination of independent random variables. |  | |  | Proofs. |
| The discrete uniform distribution | R7 | Recognise situations under which the discrete uniform distribution is likely to be an appropriate model. | E.g. *X* has a uniform distribution over the values {4, 5, 6, 7, 8, 9}.  E.g. a fair spinner with six equally-sized sections, labelled 4, 5, 6, 7, 8, 9. |  | |  |
| R8 | Be able to calculate probabilities using a discrete uniform distribution. |  |  | |  |
| R9 | Be able to calculate the mean and variance of any given discrete uniform distribution. | If *X* has a uniform distribution over the values {1, 2, … *n*} then  and . The formulae for this particular uniform distribution will be given but their derivations may be asked for. |  | |  |
| The binomial distribution | R10 | Recognise situations under which the binomial distribution is likely to be an appropriate model, and be able to calculate probabilities to use the model.  Know and be able to use the mean and variance of a binomial distribution,  and . Prove these results in particular cases. | E.g. prove results by considering a binomial random variable as the sum of  independent Bernoulli random variables: where each takes the value 1 with probability and 0 with probability  This proof assumes the relationship about variance in SR6. |  | |  |

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| The Poisson distribution | SR11 | Recognise situations under which the Poisson distribution is likely to be an appropriate model. | Modelling the number or events occurring in a fixed interval (of time or space) when the events occur randomly at a constant average rate, and independently of each other.  It is expected that these conditions can be applied to the particular context.  If the mean and variance of the data do not have a similar value then the Poisson model is unlikely to be suitable. |  |  |
| R12 | Recognise situations in which both the Poisson distribution and the binomial distribution might be appropriate models. | In a situation where the binomial model is appropriate, if  is large and  is small, then the conditions for a Poisson distribution to be appropriate are approximately satisfied. In the absence of guidance either model can be used. |  | Formal criteria. Using the Poisson distribution as a numerical approximation for calculating binomial probabilities. |
| R13 | Be able to calculate probabilities using a Poisson distribution. | Including use of a calculator to access Poisson probabilities and cumulative Poisson probabilities. |  |  |
| R14 | Know and be able to use the mean and variance of a Poisson distribution. |  |  | Proof. |
| R15 | Know that the sum of two or more independent Poisson distributions is also a Poisson distribution. | and  when and  are independent. |  | Proof. |

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| The geometric distribution | SR16 | Recognise situations under which the geometric distribution is likely to be an appropriate model. | Link with corresponding binomial distribution. | , wherenumber of Bernoulli trials up to and including the first success. | The alternative definition which counts the number of failures. |
| R17 | Be able to calculate the probabilities within a geometric distribution, including cumulative probabilities. | where  probability of success and .  .  An understanding of the calculation is expected. |  |  |
| R18 | Be able to use the mean and variance of a geometric distribution. | . |  | Proof. |

# Thinking Conceptually

### General approaches

The work in this section builds on previous work on Probability and Statistical Distributions in the AS and A Level specification. Students will need to recall the idea of a random variable and the subject matter of probability distributions. They will also need to use their knowledge of probability in general.

### Conceptual links to other areas of the specification

This section covers all of the further maths advanced level work on discrete random variables and includes various different probability distributions. At A Level, or for those AS students doing both Statistics a and Statistics b, it links with the subsequent work on continuous random variables and linear combinations of random variables. Indeed, the more able students might make the conceptual leap and conjecture that there should be *continuous* probability distributions too.

# Thinking Contextually

Use examples from the A Level Mathematics course to illustrate what a discrete random variable is. Something simple like a die rolled and the probability distribution expressed in a table can be used to look at the effect of linear coding. Once the students calculate the expectation and variance of , they can then find the values of the expectation and variance of  for various values of  and  directly and probably manage to conjecture the effect that linear coding has on these values. The use of technology, such as a spreadsheet, would be useful here.

The mathematics of discrete variables and their probabilities can be found in many different contexts. These range from card and coin games to medical research; sports; phone calls and the birth of babies. The resources list contains many examples and ideas of contexts which require discrete models and where we want to find probabilities.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
| --- | --- | --- | --- |
| [PowerPoint with clear description of a probability distribution](https://projects.ncsu.edu/crsc/events/ugw06/presentations/scheywar/finalundergrad.pdf) | North Carolina State University | Good general PowerPoint. Clear slides include random variable, probability distribution and probability mass function, with an example. | R1 |
| [Statistics 101: Discrete Random Variables Basics](https://www.youtube.com/watch?v=fGKd6ZtuTzM) | Brandon Foltz | An introduction to discrete random variables. A 13 minute video. May be useful within a starter or as part of homework activities. | R1 |
| [Examples of Discrete Probability Distributions](http://homepage.divms.uiowa.edu/~rdecook/stat2020/notes/ch3_pt1.pdf) | University of Iowa | Some clear example slides which could be used within an explanation in class or as a reference resource. | R1 |
| [Discrete Random Variable Simulator](https://www.geogebra.org/m/cqMQwFC5) | Geogebra | Create your own Discrete Random Variable by inputting probabilities into spreadsheet. And compare simulation with theoretical distribution. Could be used within a plenary. | R1 |
| [Discrete Probability Distributions Introductory Lecture](https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-041-probabilistic-systems-analysis-and-applied-probability-fall-2010/video-lectures/lecture-5-discrete-variables-probability-expectations/) | MIT | Accessible lecture which explores discrete random variables and discrete probability distributions. The lecturer is John Tsitsiklis. 50 minutes long. Could be used for flipped learning. | R1, R2 and R3 |
| [Demonstration of E(X) and Var(X)](http://www.jbstatistics.com/expected-value-and-variance-of-discrete-random-variables/) | JBstatistics | 8 minute video which demonstrates how to obtain E(X), Var(X) and the standard deviation. Good as a starter. | R2 and R3 |
| [DRVs from a Bag](http://www.s253053503.websitehome.co.uk/msv/msv-5.html) | Making Statistics Vital (MSV) | An exercise designed to ensure that students know how to find E(X) and Var(X) for a discrete random variable, and that the probabilities in a probability distribution will always add to one. | R2 and R3 |
| [The four sided Dice](http://www.s253053503.websitehome.co.uk/msv/msv-31.html) | MSV | Investigate whether a four-sided dice with positive integer faces including one or more odd number faces can ever have the expectation of its score equal to the variance. | R2 and R3 |
| [Double or Add](http://www.s253053503.websitehome.co.uk/msv/msv-19.html) | MSV | Investigation of how does 'rolling a dice and doubling' differ from 'rolling two dice and adding'? | R4, R5 and R6 |
| [Linear Combinations of Discrete Random Variables](https://www.examsolutions.net/tutorials/linear-combinations-discrete-random-variables/?level=A-Level&board=Edexcel&module=S1&topic=1836) | Exam Solutions | A collection of 3 videos showing how linear combinations work with the mean and variance. | R4, R5 and R6 |
| [Uniform Distribution Introduction](https://www.youtube.com/watch?v=pc92J_DIwZo) | University of Oklahoma  Dr Kash Barker | 2.5 minute video introduction. Good starter resource. | R7, R8 and R9 |
| [Uniform Distribution (Discrete): U(N) with PDF and CDF](https://www.geogebra.org/m/DSwmHTVA) | Geogebra | Compare pdf and cdf for the uniform distribution. | R7, R8 and R9 |
| [Video Lecture: Introduction to the Binomial Distribution](https://www.coursera.org/learn/basic-statistics/lecture/sMjTp/4-08-the-binomial-distribution) | University of Amsterdam / Coursera | Short and good introductory lecture about the Binomial Distribution. Roughly 8 minutes long. Could be used in class or for flipped learning. | R10 |
| [Video Lecture Binomial Distribution](https://www.youtube.com/watch?v=k2BB0p8byGA) | Harvard University | Enjoyable and informative 50 minute lecture given by Professor Joe Blitzstein from the Department of Statistics at Harvard University. Accessible with stretch. | R10 |
| [Most Likely Value](http://www.s253053503.websitehome.co.uk/msv/msv-3.html) | MSV | This activity offers a slightly different slant on the standard question, 'What is the most likely value for a Binomial Distribution to give?' | R10 |
| [Binomial Reverse](http://www.s253053503.websitehome.co.uk/msv/msv-11.html) | MSV | Here is a standard (and maybe a little dull?) Binomial question that livens up when reversed. | R10 |
| [Binomial modelling](https://ed.ted.com/lessons/why-do-airlines-sell-too-many-tickets-nina-klietsch) | TEDed | Binomial modelling for airline overbooking. | R10 |
| [Poisson Distribution](https://www.geogebra.org/m/utauAHgF) | Geogebra | Interactive Poisson Distribution generator. | R11, R12, R13 and R13 |
| [Introduction to the Poisson Distribution](http://www.jbstatistics.com/introduction-to-the-poisson-distribution/) | JBstatistics | 9 minute video, good as a starter, which introduces the Poisson Distribution. | R11, R12, R13, R14 and R15 |
| [Introduction to the Poisson Distribution](http://www.stats.ox.ac.uk/~marchini/teaching/L5/L5.notes.pdf) | Oxford University Department of Statistics | Comprehensive notes with examples on the Poisson Distribution. | R11, R12, R13, R14 and R15 |
| [Poisson Distribution / Poisson Curve: Simple Definition](http://www.statisticshowto.com/poisson-distribution/) | Statistics How to | Notes and examples of the Poisson distribution. Includes a short section on the criteria for using Poisson or Binomial. | R13 |
| [Adding Two Poissons](http://www.s253053503.websitehome.co.uk/msv/msv-38.html) | MSV | Investigation of the resulting distribution when you add two independent Poisson variables. | R15 |
| [Geometric Distribution Introduction](http://www.jbstatistics.com/introduction-to-the-geometric-distribution/) | JBstatistics | 11 minute video which introduces the Geometric Distribution. Excellent starter or plenary resource. Very clear. | R16 and R17 |
| [Brief Introduction to the Geometric Distribution with clear guide to notation](http://people.wku.edu/david.neal/382/discrete/Geometric.pdf) | Western Kentucky University | Reference document which introduces the Geometric Distribution with clear examples. | R16, R17 and R18 |
| [Explanation of the Geometric Distribution](http://arnoldkling.com/apstats/geometric.html) | Arnold Kling (Economist) | Concise notes on the Geometric Distribution. Useful as a homework or revision reference resource. | R16, R17 and R18 |
| [Geometric Probability](https://www.geogebra.org/m/BYJtavUB#material/ddyJFGmS) | Geogebra | Set of interesting geometric simulations. | R16, R17 and R18 |

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