# Teacher Delivery Guide Statistics: Continuous Random Variables

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| **Specification** | **Ref.** | **Learning outcomes** | | **Notes** | **Notation** | | | **Exclusions** |
| **Y422 STATISTICS MAJOR: CONTINUOUS RANDOM VARIABLES (b)**  **Y416 STATISTICS b: CONTINUOUS RANDOM VARIABLES** | | | | | | | | |
| The probability density function (pdf) of a continuous random variable | R19 | Be able to use a continuous random variable as a model. | Learners are expected to be familiar with the use of the (continuous) uniform and Normal distributions as models. They should be aware that other distributions underpin some work e.g.  and that other distributions, such as the exponential distribution, are useful models; knowledge of these is not expected and any necessary details will be provided in the examination. | | | Continuous uniform distribution also known as rectangular distribution. | Mixed discrete and continuous random variables. | |
| R20 | Understand the meaning of a pdf and be able to use one to find probabilities. | Unfamiliar pdf’s, including piecewise pdf’s, may be given in examination questions.  In numerical cases learners are expected to write down the relevant definite integral, and may then use a calculator to evaluate it. | | | or other lower case letter for the function. | Using formula for pdf of Normal distribution. | |
| R21 | Know and use the properties of a pdf.  Be able to sketch the graph of a pdf. | and . | | |  | Evaluation of improper integrals. | |

***DISCLAIMER***

This resource was designed using the most up to date information from the specification at the time it was published. Specifications are updated over time, which means there may be contradictions between the resource and the specification, therefore please use the information on the latest specification at all times.If you do notice a discrepancy please contact us on the following email address: [resources.feedback@ocr.org.uk](mailto:resources.feedback@ocr.org.uk)

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| The probability density function (pdf) of a continuous random variable  (cont) | R22 | Be able to find the mean and variance from a given pdf. | Learners are expected to write down the relevant definite integral, and may then use a calculator to evaluate it. In examination questions any integrations to be performed will be over a finite domain.  Standard deviation = .  For a continuous uniform distribution over : , . Formulae will be given but derivations may be required. |  | Deriving mean and variance of the Normal distribution from the pdf.  Evaluation of improper integrals. |
| R23 | Be able to find the mode and median from a given pdf. | Mode only where it exists. |  | Mode for bimodal distributions. |
| The cumulative distribution function (cdf) | R24 | Understand the meaning of a cdf and be able to obtain one from a given pdf.  Be able to sketch a cdf. |  | or other upper case letter for the function. | Normal distribution.  Evaluation of improper integrals. |
| R25 | Be able to obtain a pdf from a given cdf. |  |  |  |
| R26 | Use a cdf to calculate the median and other percentiles. |  |  |  |
| Expectation algebra | R27 | Be able to find the mean of any linear combination of random variables and the variance of any linear combination of independent random variables. |  |  | Proofs. |

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | | **Exclusions** |
| The Normal distribution | R28 | †Be able to use the Normal distribution as a model, and to calculate and use probabilities from a Normal distribution. | Calculations of probabilities are to be done using statistical functions on a calculator.  Relate calculations of probabilities to the graph of the Normal distribution. |  | |  |
| R29 | Be able to use linear combinations of independent Normal random variables in solving problems. | Use the fact that if  and , with  and  independent, then Extend to more than two random variables. | |  | Proof. |
| R30 | Know that the Normal distribution is useful as a model in its own right, and as an approximating distribution in the context of the Central Limit Theorem (CLT). | Includes recognising when the Normal distribution is not appropriate.  Details of the CLT are in SI1 to SI6. |  | |  |
| R31 | Interpret a Normal probability plot to decide whether a Normal model might be appropriate1.  Interpret software output, including *p*-value, from the Kolmogorov-Smirnov test, to decide whether a Normal model might be appropriate. | Learners should know that tests other than the test of goodness of fit are often applied to the Normal distribution. The null hypothesis for the given test is ‘H0: the Normal distribution fits the data’. |  | | cid:image002.png@01D233A7.9F851620test for goodness of fit of Normal distribution. Calculations for Kolmogorov-Smirnov test. |
| R32 | Be able to use the Normal distribution, when appropriate, in the construction of confidence intervals. | See SI 7 to SI 14 below for details. |  | |  |
| 1There are different conventions for how Normal probability plots are drawn, and different features about the underlying distribution, for example skewness, can be inferred from the plot. Learners are only expected to know that the closer the points are to a straight line, the more likely it is that a Normal distribution fits the data; this is to be judged by eye. They are not expected to be able to draw Normal probability plots, nor do any calculations.  In the examination the sample data will be shown on one axis and the other will show expected Normal values. | | | | | | |

# Thinking Conceptually

### General approaches

It would be useful to introduce this topic with a recap of the ideas of discrete random variables and then consider continuous variables in parallel; making the point that we replace summation with integration for the continuous case. This could be a good opportunity to revise discrete random variables as well as to introduce the new work.

### Common misconceptions

Students may find it difficult to understand why the probability of an exact value within a continuous scale will be zero. Discussion about this would be useful near to the beginning of their work on continuous variables. It would also be useful to make the distinction between the exact value of a random variable and a data value; the latter must be given to a certain degree of accuracy and so it represents an interval rather than a single value. For example, a height of 1.76 m could be anywhere in the interval from 1.755 m to 1.765 m.

### Conceptual links to other areas of the specification

This work links into some of the later sections of further statistics such as the sample mean and the Central Limit Theorem, which manipulates continuous random variables in its proof.

# Thinking Contextually

There are many real life contexts which are modelled with continuous probability distributions. Naturally occurring phenomena are generally continuous: such as temperatures, wind speeds, measures of time and size. Many potential questions are set outside of any context but the new Maths and Further Maths A Levels will contain more problem solving and modelling content; so it is vital that all students explore questions in context and learn to choose appropriate models without always being provided with scaffolding by either their teachers or within a test question. Experience of grappling with these kinds of questions will help students to improve and feel more confident in their ability to tackle using models for real life scenarios.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
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| [Continuous Random Variables - Introduction](https://www.youtube.com/watch?v=7mvBrRvoLPg) | Maths AcademyUK3 | 4-minute video introduction to continuous random variables. Suitable for use as a starter resource. | R19, R20 and R21 |
| [Probability Density Function](https://www.youtube.com/watch?v=XgEUQ7LCUmE) | slcMath@pc | 6-minute video introducing pdfs. Good synopsis for plenary. | R19, R20 and R21 |
| [Random variables – notes and real world examples](http://www.utdallas.edu/~scniu/OPRE-6301/documents/Random_Variables_Applications.pdf) | UT Dallas | Notes covering random variables and distributions – selections will be useful here. | R19, R20 and R21 |
| [Lecture: Exponential Distribution](https://www.youtube.com/watch?v=bM6nFDjvEns) | Harvard | 18 minutes duration. Introduces the exponential distribution. Professor Joe Blitzstein, Professor of Statistics at Harvard University. | R19, R20 and R21 |
| [Difference between Poisson and Exponential Distributions](https://www.youtube.com/watch?v=Z-8FtjZNlb4) | Wendy Arnold | 9 minute video with useful examples. | R19, R20 and R21 |
| [Uniform Distributions](https://onlinecourses.science.psu.edu/stat414/node/135) | Penn State | Introductory notes on the Uniform Distribution of a continuous random variable. | R19, R20 and R21 |
| [The Uniform Distribution](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&ved=0ahUKEwiZisaVrq7WAhXKAMAKHb7WABkQFgg3MAI&url=http%3A%2F%2Fwww.personal.soton.ac.uk%2Fjav%2Fsoton%2FHELM%2Fworkbooks%2Fworkbook_38%2F38_2_uniform_dist.pdf&usg=AFQjCNF2WHp-FGDHTAql29UvZ3_EnwgMfw) | HELM | Notes, examples and guided exercise. | R19, R20 and R21 |
| [Revision Video with worked past paper questions](https://www.youtube.com/watch?v=nWNeuC0T-zs) | Further Maths Support Programme | Useful revision tool. Contains lots of exam technique tips. | R19, R20, R21 and R22 |
| [Notes on Continuous Random Variables](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=20&ved=0ahUKEwjF96KSqK7WAhXiD8AKHX3hD84QFgiOATAT&url=http%3A%2F%2Fwww.math.ucsd.edu%2F~jschwein%2Fcontdist.pdf&usg=AFQjCNFrzM11kMrs1nE48RZs1-EK1xlS2Q) | UC San Diego | Set of notes with examples and key points. | R19, R20, R21 and R22 |
| [Lecture which introduces continuous probability distributions](https://www.youtube.com/watch?v=Tci---bVs60) | Harvard University | 50 minute lecture given by Joe Blitzstein who is the Professor of the Practice in Statistics in the Department of Statistics at Harvard University. He gives a thorough introduction to continuous probability distributions. All common ideas are dealt with plus additional depth which would be useful for both students and teachers. Accessible. Towards the end of the lecture there is also some theoretical coverage of the Continuous Uniform Distribution. Contains stretch. | R19, R20, R21, R22, R23, R24, R25 and R26 |
| [Continuous and Mixed Random Variables](https://www.probabilitycourse.com/chapter4/4_0_0_intro.php) | Introduction to Probability, Statistics and Random Processes | Set of notes and questions (with solutions). | R19, R20, R21, R22, R23, R24, R25, R26 and R27 |
| [Probability Density Function/Continuous Random Variables](https://www.youtube.com/watch?v=szjL60gAweE) | patrickJMT | Video covering a brief discussion of probability density functions and continuous random variables. Could be used for flipped learning. | R20 and R21 |
| [Cumulative distribution function](https://www.youtube.com/watch?v=4BswLMKgXzU) | MrNichollTV | 26-minute video on cdfs. Possibly better to select sections of the video than to show the whole thing in one session. | R24, R25 and R26 |
| [Finding the median and quartiles for a continuous probability distribution](http://edshare.soton.ac.uk/2123/1/MA175ex10_3.pdf) | University of Southampton | Worked example including ideas of mode, mean, median and quartiles. | R24, R25 and R26 |
| [Finding Percentiles](https://onlinecourses.science.psu.edu/stat414/node/125) | Penn State | Notes on finding the percentiles of continuous random variable. Includes videos which show how to use both pdf and cdf to obtain, for example, quartiles and median. | R24, R25 and R26 |
| [Finding a CDF from a pdf](https://www.youtube.com/watch?v=WDGyFeuk9oI) | Mr Camilleri | Video demonstration of finding CDF from a pdf. | R24 |
| [Sum of two independent r.v.s](https://onlinecourses.science.psu.edu/stat414/node/165) | Penn State | Notes on sum of two independent random variables using binomial example (could be useful as an introduction) | R27 |
| [Combining two Normal r.v.s](https://www.geogebra.org/m/wt5qdsKB) | GeoGebra | Visualising sum and difference of two independent Normal random variables. | R29 |
| [Normal probability plots](http://www.ocr.org.uk/Images/208950-the-normal-distribution-topic-exploration-pack-.pdf) | OCR | An introduction to Normal probability plots written for Core Maths | R31 |
| [Kolmogorov-Smirnov Test](http://www.itl.nist.gov/div898/handbook/eda/section3/eda35g.htm) | NIST/SEMATECH e-Handbook of Statistical Methods | Some information about the Kolmogorov-Smirnov test | R31 |
| [Assessing Normality](https://www.nbt.nhs.uk/sites/default/files/filedepot/incoming/Normal%20Distribution.pdf) | NHS | Notes on assessing whether a distribution is Normal from the NHS so many contexts are medical. | R31 |

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