# Mapping Guide: Legacy AS and A2 units 3896 to H635

## Content of Core Pure (Y410) – mandatory paper

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | **Legacy Unit & Ref.** | **Notes** |
| --- | --- | --- | --- | --- |
| **CORE PURE : PROOF (a)** | | | | |
| Proof | \* | Be able to prove mathematical results by deduction and exhaustion, and disprove false conjectures by counter example.  Notes  Includes proofs of results used in this specification, where appropriate. | C3p1, C3p2 | The expectation that proof is required of results in the reformed specification is new. |
| Induction | Pp4 | Be able to construct and present a proof using mathematical induction for given results for a formula for the *n*th term of a sequence, the sum of a series or the *n*th power of a matrix.  Notes  The result to be proved will be given.  E.g. for the sequence given by ,  prove that .  Notation  , | FP1p4 | Applications to powers of matrices are new. |

***DISCLAIMER***

This resource was designed using the most up to date information from the specification at the time it was published. Specifications are updated over time, which means there may be contradictions between the resource and the specification, therefore please use the information on the latest specification at all times.If you do notice a discrepancy please contact us on the following email address: [resources.feedback@ocr.org.uk](mailto:resources.feedback@ocr.org.uk)

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | **Legacy Unit & Ref.** | **Notes** |
| --- | --- | --- | --- | --- |
| **CORE PURE: COMPLEX NUMBERS (a)** | | | | |
| Language of complex numbers | Pj1 | Understand the language of complex numbers.  Notes  Real part, imaginary part, complex conjugate, modulus, argument, real axis, imaginary axis.  Notation | FP1j2 | Notation i used rather than j, where . |
| Complex numbers and polynomial equations with real coefficients | j2 | Be able to solve any quadratic equation with real coefficients.  Notation | FP1j1 |  |
| j3 | Know that the complex roots of polynomial equations with real coefficients occur in conjugate pairs. Be able to solve cubic or quartic equations with real coefficients.  Notes  Use of the factor theorem once a real root has been determined.  Sufficient information will be given to deduce at least one complex root or quadratic factor for quartics.  Exclusions  Equations with degree > 4. | FP1j5, FP1j6 |  |
| Arithmetic of complex numbers | j4 | Be able to add, subtract, multiply and divide complex numbers given in the form ,  and  real.  Notes  Division using complex conjugates. | FP1j3 |  |
| j5 | Understand that a complex number is zero if and only if both the real and imaginary parts are zero. | FP1j4 |  |
| Modulus-argument form | j6 | Be able to use radians in the context of complex numbers.  Notes  Use exact values of trigonometric functions for multiples of  and . | FP1j9 |  |
| j7 | Be able to represent a complex number in modulus-argument form. Be able to convert between the forms  and  where *r* is the modulus and *θ* is the argument of the complex number.  Notes    Notation  is the modulus of *z*.  arg *z* for principal argument, where .  Radian measure. | FP1j9, FP2j1 |  |
| j8 | Be able to multiply and divide complex numbers in modulus-argument form.  Notes    The identities for  and may be assumed in the derivation of these results. | FP2j2 |  |
| The Argand diagram | j9 | Be able to represent and interpret complex numbers and their conjugates on an Argand diagram. | FP1j7 |  |
| j10 | Be able to represent the sum, difference, product and quotient of two complex numbers on an Argand diagram. | FP2j8 | Product and quotient are not included in FP1: perhaps implicit in FP2j9 and other FP2j statements. |
| j11 | Be able to represent and interpret sets of complex numbers as loci on an Argand diagram.  Notes  Circles of the form .  Half lines of the form .  Lines of the form .  Regions defined by inequalities based on the above e.g. . Intersections and unions of these.  Notation  For regions defined by inequalities learners must state clearly which regions are included and whether the boundaries are included. No particular shading convention is expected.  Exclusions  for . | FP1j10 | ‘Interpret’ is new, but implicit in the FP1 statement.  Lines are new.  Regions are implicit in FP1j10. |
| **CORE PURE: MATRICES AND TRANSFORMATIONS (a)** | | | | |
| Matrix addition and multiplication | Pm1 | Be able to add, subtract and multiply conformable matrices, and to multiply a matrix by a scalar.  Notes  With and without a calculator for matrices up to 3×3.  Notation  . | FP1m1 | The expectation that calculator use is expected is new. |
| m2 | Understand and use the zero and identity matrices, understand what is meant by equal matrices.  Notation (zero)  (identity). | FP1m2 |  |
| m3 | Know that matrix multiplication is associative but not commutative. | FP1m3 |  |
| Linear transform-ations and their associated matrices | m4 | Be able to find the matrix associated with a linear transformation and vice-versa.  Notes  2-D transformations include the following.   * Reflection in the *x* and *y* axes and in . * Rotation centre the origin through an angle  (counter clockwise positive) * Enlargement centre the origin * Stretch parallel to *x* or *y* axis * Shear *x* or *y* axis fixed, shear factor1   3-D transformations will be confined to reflection in one of *x* = 0, *y* = 0, *z* = 0 or rotation of multiples of 90° about *x,* *y* or *z* axis2  Learners should know that any linear transformation may be represented by a matrix.  Notation  Matrices will be shown in bold type, transformations in non-bold type.  The image of the column vector **r** under the transformation associated with matrix  is . | FP1m4 | Stretches and shears are new.  3-D transformations are new. |
| 1A shear may be defined by giving the fixed line and the image of a point. (The fixed line of a shear is a line of invariant points.) The shear factor is the distance moved by a point divided by its perpendicular distance from the fixed line. Learners should know this, but the shear factor should not be used to define a shear as there are different conventions about the sign of a shear factor.  2Positive angles counter clockwise when looking towards the origin from the positive side of the axis of rotation. | | | |
| m5 | Understand successive transformations in 2-D and the connection with matrix multiplication.  Notes  Describe a transformation as a combination of two of those above.  Exclusions  More than 2 dimensions. | FP1m5 |  |
| \* | Understand the language of vectors in two dimensions and three dimensions.  Notes  Scalar, vector, modulus, magnitude, direction, position vector, unit vector, cartesian components, equal vectors, parallel vectors. Notation  , | C4v1 |  |
| Invariance | m6 | Know the meaning of, and be able to find, invariant points and invariant lines for a linear transformation.  Exclusions  More than 2 dimensions. | FP1m6 | Lines of invariant points were in the legacy specification, but not invariant lines. |
| Determinant of a matrix | m7 | Be able to find the determinant of a 2×2 matrixand a 3×3 matrix.  Know the meaning of the terms singular and non-singular as applied to matrices.  Notes  With a calculator for 3×3 matrices.  A singular square matrix is non-invertible and therefore has determinant zero.  Notation  or det **M**  or . | FP1m7, FP2m1 | FP1m7 covers the 2-D case.  FP2m1 includes without a calculator case (and inverses).  Language ‘singular’ comes in FP1m8. |
| m8 | Know that the magnitude of the determinant of a 2×2 matrix gives the area scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.  Notes  E.g. Quadrilateral ABCD is labelled clockwise and transformed in 2-D; a negative determinant for the transformation matrix means that the labelling on the image A’B’C’D’ is anticlockwise.  Exclusions  Proof | FP1m8 | Interpreting the sign of a determinant is (perhaps) implicit in the legacy specification. |
| m9 | Know that the magnitude of the determinant of a 3×3 matrix gives the volume scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.  Notes  The sign of the determinant determines whether the associated transformation preserves or reverses orientation (‘handedness’).  E.g. If a triangle ABC is labelled clockwise when seen from point S, then for a negative determinant, the triangle A’B’C’ is anti-clockwise when seen from S’.  Exclusions  Proof |  | This is new. |
| m10 | Know that det(**MN**)=det **M** ×det **N** and the corresponding result for scale factors of transformations.  Notes  Scale factors in 2-D only.  Exclusions  Algebraic proof. |  | This is new. |
| Inverses of square matrices | m11 | Understand what is meant by an inverse matrix.  Notes  Square matrices of any order.  Notation | FP1m9 |  |
| m12 | Be able to calculate the inverse of a non-singular 2×2 matrixor 3×3 matrix.  Notes  With a calculator for 3×3 matrices. | FP1m10, FP2m1 | FP2m1 covers the 3-D case, though in the reformed specification the inverse can be found with a calculator. |
|  | m13 | Be able to use the inverse of a non-singular 2×2 or 3×3 matrix. Relate the inverse matrix to the corresponding inverse transformation.  Notes  E.g. to solve a matrix equation and interpret in terms of transformations: find the pre-image of a transformation. |  | This is new, but perhaps implicit in FP1m5 in the 2-D case. |
|  | m14 | Understand and use the product rule for inverse matrices.  Notes | FP1m11 |  |
| **CORE PURE: VECTORS AND 3-D SPACE (a)** | | | | |
| Scalar products and the equations of planes | Pv1 | Know how to calculate the scalar product of two vectors, and be able to use the two forms of the scalar product to find the angle between two vectors.  Notes  Including test for perpendicular vectors.  Notation    Exclusions  Proof of equivalence of two forms in general case. | C4v3 |  |
| v2 | Be able to form and use the vector and cartesian equations of a plane. Convert between vector and cartesian forms for the equation of a plane.  Notes  Plane:   where .  Exclusions  The form | C4v6 |  |
| v3 | Know that a vector which is perpendicular to a plane is perpendicular to any vectorin the plane.  Notes  If a vector is perpendicular to two non-parallel vectorsin a plane, it is perpendicular to the plane. | C4v7 |  |
| Intersection of planes | v4 | Know the different ways in which three distinct planes can be arranged in 3-D space.  Notes  If two planes are parallel the third can be parallel or cut the other two in parallel lines; if no pair is parallel the planes can intersect in a point, form a sheaf or form a prismatic intersection.  Notation  A sheaf is where three planes share a common line. A prismatic intersection is where each pair of planes meets in a line; the three lines are parallel. | FP2m5 | The content is more clearly stated in the reformed specification. |
|  | v5 | Be able to solve three linear simultaneous equations in three variables by use of the inverse of the corresponding matrix.  Interpret the solution or failure of solution geometrically in terms of the arrangement of three planes.  Be able to find the intersection of three planes when they meet in a point.  Notes  Inverse obtained using a calculator.  If the corresponding matrix is singular, learners should know the possible arrangements of the planes; they will be given extra information or guidance if required to distinguish between these arrangements.  Exclusions  Finding equation of lines of intersection of two planes. | FP1m12, FP2m5 | In FP1m12 enough information is given so that the inverse matrix is known. In the reformed specification a calculator is used to find the matrix. |
|  | v6 | Know that the angle between two planes can be found by considering the angle between their normals.  Notes  The angle between two non-perpendicular planes is the acute angle between them. | C4v8 |  |
| **CORE PURE: ALGEBRA (a)** | | | | |
| Relations between the roots and coefficients of polynomial equations | Pa1 | Understand and use the relationships between the roots and coefficients of quadratic, cubic and quartic equations.  Notation  Roots.  Exclusions  Equations of degree 5. | FP1a5 |  |
| a2 | Be able to form a new equation whose roots are related to the roots of a given equation by a linear transformation.  Notes  For a cubic or quartic equation.  Exclusions  Non-linear transformations of roots. | FP1a6 |  |
| **CORE PURE: SERIES (a)** | | | | |
| Summation of series | Ps1 | Be able to use standard formulae for  and  and the method of difference to sum series. Notes  Formulae for  and will be given but proof could be required, e.g. by induction.  Including the method of differences.  Notation | FP1a2 |  |

## Content of Mechanics a (Y411) – option

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | **Legacy Unit & Ref.** | **Notes** | |
| --- | --- | --- | --- | --- | --- |
| **MECHANICS a: DIMENSIONAL ANALYSIS** | | | | | |
| Dimensional consistency | Mq1 | Be able to find the dimensions of a quantity in terms of M, L, T.  Notes  Know the dimensions of angle and frequency. Work out without further guidance the dimensions of density (mass per unit volume), pressure (force per unit area) and other quantities in this specification.  Other kinds of density will be referred to as e.g. mass per unit area.  Deduce the dimensions of an unfamiliar quantity from a given relationship.  Notation  M, L, T, [ ] | M3q1 | The reformed specification is clearer about what should be known. | |
| q2 | Understand that some quantities are dimensionless. | M3q2 |  | |
| q3 | Be able to determine the units of a quantity by reference to its dimensions.  Notes  And vice versa. | M3q3 |  | |
|  | q4 | Be able to change the units in which a quantity is given.  Notes  E.g. density from kg m–3 to g cm–3. | M3q4 |  | |
| q5 | Be able to use dimensional analysis to check the consistency of a relationship. | M3q5 |  | |
| Formulating and using models by means of dimensional arguments | q6 | Use dimensional analysis to determine unknown indices in a proposed formula.  Notes  E.g. for the period of a pendulum. | M3q6 |  | |
| q7 | Use a model based on dimensional analysis.  Notes  E.g. to find the value of a dimensional constant.  E.g. to investigate the effect of a percentage change in some of the variables. |  | This is implicit in M3q1 to M3q6. See also M1p1 to M1p8. | |
| **MECHANICS a: FORCES** | | | | |
| The language of forces | \* | Understand the language relating to forces.  Understand that the value of the normal reaction depends on the other forces acting and why it cannot be negative.  Notes  Weight, tension, thrust (or compression), normal reaction (or normal contact force), frictional force, resistance. Driving force, braking force1. NB weight is not considered to be a resistive force. | M1d1 |  |
|  |  | 1The driving force of a car, bicycle, train engine etc. is modelled as a single external force. Similarly for a braking force. These are actually frictional forces acting at the point(s) of contact with the road or track. The internal processes which cause these forces are not considered. | | |
| Friction | Md1 | † Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to represent the situation in an appropriate force diagram.  Notes  Smooth is used to mean frictionless. | M2d1 |  |
| d2 | † Understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal contact force (normal reaction). | M2d2 |  |
| d3 | † Understand that the frictional force may be modelled by  and that friction acts in the direction to oppose sliding. Model friction using  when sliding occurs.  Notes  Limiting friction. The definition of  as the ratio of the frictional force to the normal contact force.  Notation  Coefficient of  friction is .  Exclusions  The term angle of friction. | M2d3 |  |
| d4 | Be able to derive and use the result that a body on a rough slope inclined at an angle  to the horizontal is on the point of slipping if . |  | This is new as an explicit requirement. The derivation could be asked under M2d3 to M2d5 and M2d8 to M2d9. Permission to quote the result is a clarification. |
| d5 | † Be able to apply Newton's Laws to situations involving friction. | M2d4 |  |
| Vector treatment of forces | d6 | † Be able to resolve a force into components and be able to select suitable directions for resolution.  Notes  E.g. horizontally and vertically, or parallel and perpendicular to an inclined plane. | M1d3 |  |
| d7 | † Be able to find the resultant of several concurrent forces by vector addition.  Notes  Graphically or by adding components. | M1d4 |  |
| Equilibrium of a particle | d8 | † Know that a particle is in equilibrium under a set of concurrent forces if and only if their resultant is zero. | M1d5 |  |
|  | d9 | † Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.  Notes  E.g. a triangle of forces. | M1d6 |  |
|  | d10 | † Be able to formulate and solve equations for equilibrium by resolving forces in suitable directions, or by drawing and using a polygon of forces.  Notes  Questions will not be set that require Lami’s theorem but learners may quote and use it where appropriate. | M1d7 |  |
| Equilibrium of a rigid body | d11 | Be able to draw a force diagram for a rigid body.  Notes  In cases where the particle model is not appropriate. | M2d5 |  |
| d12 | Understand that a system of forces can have a turning effect on a rigid body.  Notes  E.g. a lever. | M2d6 |  |
|  | d13 | Know the meaning of the term couple.  Notes  A couple is not about a particular axis. |  | This is new. |
|  | d14 | Be able to calculate the moments about a fixed axis of forces acting on a body.  Be able to calculate the moment of a couple.  Notes  Both as the product of force and perpendicular distance of the axis from the line of action of the force, and by first resolving the force into components.  Take account of a given couple when taking moments.  Exclusions  Vector treatment. | M2d7 | Moment of a couple is new. |
|  | d15 | Understand and be able to apply the conditions for equilibrium of a rigid body.  Notes  The resultant of all the applied forces is zero and the sum of their moments about any axis is zero. Three forces in equilibrium must be concurrent or parallel.  Situations may involve uniform 3D objects, such as a cuboid, whose centre of mass can be written down by considering symmetry.  E.g. infer the existence of a couple at a hinge by consideration of equilibrium and calculate the size of the couple. | M2d8 |  |
|  | d16 | Be able to identify whether equilibrium will be broken by sliding or toppling.  Notes  E.g. a cuboid on an inclined plane. | M2d9 |  |
| **MECHANICS a: WORK, ENERGY AND POWER** | | | | |
| The language of work, energy and power | Mw1 | Understand the language relating to work, energy and power.  Notes  Work, energy, mechanical energy, kinetic energy, potential energy, conservative force, dissipative force, driving force, resistive force  Power of a force, power developed by a vehicle1. |  | This is implicit in M2w1 to M2w9. |
|  | 1In an examination question ‘the power developed by a car’ (or a bicycle or train engine) means the useful, or available, power. It is the power of the driving force; it is not the power developed by the engine, some of which is lost in the system. | | | |
| Concepts of work and energy | w2 | Be able to calculate the work done by a force which moves along its line of action.  Exclusions  The use of calculus for variable forces. | M2w1 |  |
| w3 | Be able to calculate the work done by a force which moves at an angle to its line of action.  Notes  Zero work is done by a force acting perpendicular to displacement.  Exclusions  Use of scalar product . | M2w1 |  |
| w4 | Be able to calculate kinetic energy.  Notation | M2w2 |  |
| w5 | Be able to calculate gravitational potential energy.  Notes  Relative to a defined zero level.  Notation | M2w6 |  |
| The work-energy principle | w6 | Understand when the principle of conservation of energy may be applied and be able to use it appropriately.  Notes  E.g. the maximum height of a projectile, a particle sliding down a smooth curved surface, a child swinging on a rope. | M2w7 |  |
| w7 | Understand and use the work-energy principle.  Notes  The total work done by all the external forces acting on a body is equal to the increase in the kinetic energy of the body.  E.g. a particle sliding down a rough curved surface | M2w4 |  |
| Power | w8 | Understand and use the concept of the power of a force as the rate at which it does work.  Notes  Power = (force) × (component of velocity in the direction of the force).  The concept of average power as (work done) ÷ (elapsed time).  E.g. finding the maximum speed of a vehicle. | M2w8 |  |
| **MECHANICS a: MOMENTUM and IMPULSE** | | | | |
| Momentum and impulse treated as vectors | Mi1 | Be able to calculate the impulse of a force as a vector and in component form.  Notes  Impulse = force × time over which it acts.  Exclusions  The use of calculus for variable forces. | M2i1 |  |
| i2 | Understand and use the concept of linear momentum and appreciate that it is a vector quantity. | M2i2 |  |
| i3 | Understand and use the impulse-momentum equation.  Notes  The total impulse of all the external forces acting on a body is equal to the change in momentum of the body. Use of relative velocity in one dimension is required. | M2i3 |  |
| Conservation of linear momentum | i4 | Understand and use the principle that a system subject to no external force has constant total linear momentum and that this result may be applied in any direction.  Notes  The impulse of a finite external force (e.g. friction) acting over a very short period of time (e.g. in a collision) may be regarded as negligible.  Application to collisions, coalescence and a body dividing into one or more parts. | M2i4 |  |
| Direct impact | i5 | Understand the term direct impact and the assumptions made when modelling direct impact collisions1.  Notes  E.g. a collision between an ice hockey puck and a straight rink barrier: puck moving perpendicular to barrier.  E.g. a collision between two spheres moving along their line of centres.  E.g. a collision between two railway trucks on a straight track.  Exclusions  Any situation with rotating objects. |  | This is new. |
| **1Assumptions when modelling direct impact collisions** | | | | |
| This note explains the implicit assumptions made in examination questions when modelling direct impact collisions. Learners may be asked about these assumptions. An *object* means a real-world object. It may be modelled as a *particle* or a *body*.   * If the non-fixed objects involved in collisions may be modelled as particles, then all the motion and any impulses due to the collisions act in the same straight line. * If the non-fixed objects involved in collisions may be modelled as bodies then these bodies will be uniform bodies with spherical or circular symmetry. * The impulse of any collision between such bodies acts on the line joining their centres, and the motion takes place along this line.   These assumptions ensure that the collision happens at a point and that no angular momentum is created, hence none of the objects starts to rotate.   * The impulse of any collision between such a body, or a particle, and a plane (e.g. a wall or floor) acts in a direction perpendicular to the plane.   For a direct impact the motion of the object is also in the direction perpendicular to the plane.   * Objects do not rotate before or after the collision. Rotating objects are beyond this specification. | | | | |
| Direct impact (continued) | i6 | Be able to apply the principle of conservation of linear momentum to direct impacts within a system of bodies. | M2i6 |  |
|  | i7 | Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to a direct impact.  Notes  Newton's Experimental Law is:  the speed of separation is  the speed of approach  where  is known as the coefficient of restitution.  Notation  Coefficient of restitution is . | M2i7 |  |
|  | i8 | Understand the significance of .  Notes  The bodies coalesce.  The collision is inelastic. |  | This is implicit in M2i7. |
|  | i9 | Be able to apply Newton's Experimental Law in modelling direct impacts.  Notes  E.g. between a particle and a wall.  E.g. between two discs. | M2i7 |  |
| i10 | Be able to model situations involving direct impact using both conservation of linear momentum and Newton's Experimental Law. | M2i8 |  |
|  | i11 | Understand the significance of *.*  Notes  The collision is perfectly elastic. Kinetic energy is conserved. | M2i9 |  |
|  | i12 | Understand that when  kinetic energy is not conserved during impacts and be able to find the loss of kinetic energy. | M2i9 |  |
| **MECHANICS a: CENTRE OF MASS** | | | | |
| Locating a centre of mass | MG1 | Be able to find the centre of mass of a system of particles of given position and mass.  Notes  In 1, 2 and 3 dimensions.  Notation      Exclusions  Non-uniform bodies. | M2G1 |  |
|  | G2 | Know how to locate centre of mass by appeal to symmetry.  Notes  E.g. uniform circular lamina, sphere, cuboid | M2G2 |  |
| G3 | Know the positions of the centres of mass of a uniform rod, a rectangular lamina and a triangular lamina. |  | This is new. |
|  | G4 | Be able to find the centre of a mass of a composite body by considering each constituent part as a particle at its centre of mass.  Notes  Composite bodies may be formed by the addition or subtraction of parts.  Where a composite body includes parts whose centre of mass the learner is not expected to know, or be able to find, the centre of mass will be given. | M2G3 |  |
| Applications of the centre of mass | G5 | Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body.  Notes  For the purpose of calculating its moment, the weight of a body can be taken as acting through its centre of mass.  E.g. a suspended object  E.g. does an object standing on an inclined plane slide or topple? | M2G4 |  |

## Content of Statistics a (Y412) – option

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | | **Legacy Unit & Ref.** | | **Notes** |
| --- | --- | --- | --- | --- | --- | --- |
| **STATISTICS a: SAMPLING** | | | | | | |
| Sampling | Sx1 | Be able to explain the importance of sample size in experimental design.  Notes  E.g. an informal explanation of how the size of a sample affects the interpretation of an effect size. | |  | | Explicitly stated, this is new. |
| x2 | Be able to explain why sampling may be necessary in order to obtain information about a population, and give desirable features of a sample.  Notes  Population too large or it is too expensive to take a census.  Sampling process may be destructive.  Sample should be unbiased, representative of the population; data should be relevant, not changed by the act of sampling.  Notation  A sample may also be considered as *n* observations from a random variable. | |  | | Explicitly stated, this is new. |
| x3 | Be able to explain the advantage of using a random sample when inferring properties of a population.  Notes  A random sample enables proper inference to be undertaken because the probability basis on which the sample has been selected is known. | | S3I2 | | This is more explicit in the reformed specification. |
| **STATISTICS a: DISCRETE RANDOM VARIABLES** | | | | | | |
| Probability distributions | SR1 | Be able to use probability functions, given algebraically or in tables.  Be able to calculate the numerical probabilities for a distribution.  Be able to draw and interpret graphs representing probability distributions.  Notes  Other than the Poisson and geometric distributions, the underlying random variable will only take a finite number of values.  An understanding that probabilities are non-negative and sum to 1 is expected.  Notation | | S1R1, S1R2 | |  |
| Expectation and variance | R2 | Be able to calculate the expectation (mean), , and understand its meaning.  Notation | | S1R3 | |  |
|  | R3 | Be able to calculate the variance, , and understand its meaning.  Notes  Knowledge of .  Standard deviation = .  Notation | | S1R4 | |  |
|  | R4 | Be able to use the result  and understand its meaning. | | S3a1 | |  |
|  | R5 | Be able to use the result  and understand its meaning. | | S3a2 | |  |
| Expectation and variance  (continued) | R6 | Be able to find the mean of any linear combination of random variables and the variance of any linear combination of independent random variables.  Notes    Exceptions  Proofs. | | S3a3 | |  |
| The discrete uniform distribution | R7 | Recognise situations under which the discrete uniform distribution is likely to be an appropriate model.  Notes  E.g. *X* has a uniform distribution over the values {4, 5, 6, 7, 8, 9}.  E.g. a fair spinner with six equally-sized sections, labelled 4, 5, 6, 7, 8, 9. | |  | | This is new. |
| R8 | Be able to calculate probabilities using a discrete uniform distribution. | |  | | This is new. |
| R9 | Be able to calculate the mean and variance of any given discrete uniform distribution.  Notes  If *X* has a uniform distribution over the values {1, 2, … *n*} then  and . The formulae for this particular uniform distribution will be given but their derivations may be asked for. | |  | | This is new. |
| The binomial distribution | R10 | Recognise situations under which the binomial distribution is likely to be an appropriate model, and be able to calculate probabilities to use the model.  Know and be able to use the mean and variance of a binomial distribution,  and . Prove these results in particular cases.  Notes  E.g. prove results by considering a binomial random variable as the sum of  independent Bernoulli random variables:  where each  takes the value 1 with probability and 0 with probability  This proof assumes the relationship about variance in SR6.  Notation | | S1H1, S1H3, S1H6 | | Knowledge of variance, and proof of results for mean and variance, are new. This work is based on the introduction to the Binomial distribution in AS Mathematics. |
| The Poisson distribution | R11 | Recognise situations under which the Poisson distribution is likely to be an appropriate model.  Notes  Modelling the number or events occurring in a fixed interval (of time or space) when the events occur randomly at a constant average rate, and independently of each other.  It is expected that these conditions can be applied to the particular context.  If the mean and variance of the data do not have a similar value then the Poisson model is unlikely to be suitable.  Notation | | S2P1 | |  |
| R12 | Recognise situations in which both the Poisson distribution and the binomial distribution might be appropriate models.  Notes  In a situation where the binomial model is appropriate, if  is large and  is small, then the conditions for a Poisson distribution to be appropriate are approximately satisfied. In the absence of guidance either model can be used.  Exclusions  Formal criteria. Using the Poisson distribution as a numerical approximation for calculating binomial probabilities. | |  | | This is new and different from but linked to S2P3, which is about using the Poisson distribution as an approximation to the Binomial distribution as a calculation aid, in certain circumstances. |
| R13 | Be able to calculate probabilities using a Poisson distribution.  Notes  Including use of a calculator to access Poisson probabilities and cumulative Poisson probabilities. | | S2P2 | |  |
| R14 | Know and be able to use the mean and variance of a Poisson distribution.  Notes    Exclusions  Proof. | | S2P4 | |  |
| R15 | Know that the sum of two or more independent Poisson distributions is also a Poisson distribution.  Notes  and  when  and  are independent.  Exclusions  Proof. | | S2P5 | |  |
| The geometric distribution | R16 | Recognise situations under which the geometric distribution is likely to be an appropriate model.  Notes  Link with corresponding binomial distribution.  Notation  , wherenumber of Bernoulli trials up to and including the first success.  Exclusions  The alternative definition which counts the number of failures. | |  | | This is new. |
| R17 | Be able to calculate the probabilities within a geometric distribution, including cumulative probabilities.  Notes  where  probability of success and .  .  An understanding of the calculation is expected. | |  | | This is new. |
| R18 | Be able to use the mean and variance of a geometric distribution.  Notes  .  Exclusions  Proof. | |  | | This is new. |
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| **STATISTICS a: BIVARIATE DATA** | | | | | | |
| There are two kinds of bivariate data considered in A level Mathematics and Further Mathematics and it is important to distinguish between them when considering correlation and regression. This note explains the reason for the distinction; learners will only be assessed on what appears under a specification reference below.  Case A: Only **one of the variables** may be considered as **a random variable**. Often this occurs when one of the variables, the independent variable, is controlled by an experimenter and the other, the dependent variable, is measured. An example of this would be (weight, extension) in an investigation of Hooke’s Law for a spring. In this case certain fixed weights are used; this variable is *not* a random variable, any errors in measuring the weights are negligible. The extension *is* a random variable. There will be deviations from the ‘true’ value that a perfect experimenter would observe from a perfect spring as well as errors in the measurement. This case is referred to as **‘random on non-random’**. The points on the scatter diagram are restricted to lie on certain vertical lines corresponding to the values of the controlled variable.  Case B: The **two variables may both** be considered as **random variables**. An example of this would be (height, weight) for a sample from a population of individuals. For any given value of height there is a distribution of weights; for any given value of weight there is a distribution of heights. That is, there is no ‘true’ weight for a given height or ‘true’ height for a given weight. This case is referred to as **‘random on random’**. The scatter diagram appears as a ‘data cloud’.  If a linear relationship between the variables is to be investigated and modelled using correlation and regression techniques then the two cases must be treated differently.  If it is desired to test the significance of Pearson’s product moment correlation coefficient then, as with all parametric hypothesis tests, probability calculations have to be performed to calculate the *p*-value or the critical region. These calculations rely on certain assumptions about the underlying distribution – **these assumptions can never be met in the ‘random on non-random case’** – because one of the variables does not have a probability distribution – so **such a test is never valid in this case**. In fact the pmcc is not used in this case. In the ‘random on random’ case the distributional assumptions **may** be met – see the specification below for details.  If it is desired to calculate the equation of a line of best fit then the least-squares method is often used in both cases. However its interpretation is different in the two cases. In the example of the random on non-random case, (weight, extension), the line of regression is modelling the ‘true’ value of the extension for a given weight – the value that a perfect experimenter would observe from a perfect spring. In the example of the **random on random case**, (height, weight), the two **lines of regression are modelling the mean value of the distribution of weights for a given height and the mean value of the distribution of heights for a given weight**. | | | | | | |
| **STATISTICS a: BIVARIATE DATA** | | | | | | |
| Scatter diagrams | Sb1 | Understand what bivariate data are and know the conventions for choice of axis for variables in a scatter diagram.  Notes  In the random on non-random case the independent variable is often one which the experimenter controls; the dependent variable is the one which is measured. The independent variable is usually plotted on the horizontal axis.  In the random on random case (where both variables are measured), it may be that one is more naturally seen as a function of the other; this determines which variable is plotted on which axis. | | S2b2 | |  |
| b2 | Be able to use and interpret a scatter diagram.  Notes  To look for outliers (by eye). To gain insight into the situation, for example to decide whether a test for correlation or association might be appropriate.  Learners may be asked to add to a given scatter diagram in order to interpret a new situation. | | (S2b1) | | The emphasis in the reformed specification is on interpretation rather than drawing. |
| b3 | Interpret a scatter diagram produced by software.  Notes  Including where the software draws a trendline and gives a value for pmcc or (pmcc)². | | (S2b1) | | The emphasis in the reformed specification is on interpretation rather than drawing. |
| Pearson’s product moment correlation coefficient (pmcc) | b4 | Be able to calculate the pmcc from raw data or summary statistics.  Notes  Only the use of a calculator is expected for calculation from raw data. Summary statistics formulae will be given.  Notation  Sample value *r.* | | S2b3 | | Use of a calculator is expected in the reformed specification. |
| b5 | Know when it is appropriate to carry out a hypothesis test using Pearson’s product moment correlation coefficient.  Notes  The data must be random on random i.e. both variables must be random. There must be a modelling assumption that the data are drawn from a bivariate Normal distribution. This may be recognised on a scatter diagram by an approximately elliptical distribution of points. Learners will not be required to know the formal meaning of bivariate Normality but will be expected to know that where one or both of the distributions is skewed, bimodal, etc., the procedure is likely to be inappropriate.  The test is for correlation, a linear relationship, so a scatter diagram is helpful to check that the data cloud does not indicate a non-linear relationship. | | (S2b4) | | In the legacy specification, only random on random data are considered in the context of correlation, and random on non-random in the context of regression. In the reformed specification learners are expected to know when calculating a pmcc is appropriate, and the interpretation of the line(s) of regression, depending on whether the data are random on random or random on non-random. This is a new feature of the reformed specification. |
|  | b6 | Be able to carry out hypothesis tests using the pmcc and tables of critical values or the *p*-value from software.  Notes  Only ‘H0: No correlation in the population’ will be tested.  Both one-sided and two-sided alternative hypotheses will be tested.  Learners should state whether there is sufficient evidence or not to reject H0 and then give a non-assertive conclusion in context e.g. ‘There is sufficient evidence to suggest that there is positive correlation between … and …’  Notation  Null hypothesis, alternative hypothesis  H0, H1 | | (S2b4) | | The use of a *p*-value from software is new. |
| b7 | Use the pmcc as an effect size1.  Notes  Sensible informal comments about effect size are expected, either alongside or instead of a hypothesis test.  Exclusions  Any formal rules for judging effect size will be given. | |  | | This is new. |
| 1**Note on effect size for correlation**  For a large set of random on random bivariate data a small non-zero value of the pmcc is likely to lead to a rejection of the null hypothesis of no correlation in the population; the test is uninformative. In some contexts it is more important to consider the size of the correlation rather than test whether the population correlation is non-zero. The phrase ‘effect size’ is sometimes used in this context for the value of the pmcc. In social sciences Cohen’s guideline is often used: small effect size 0.1; medium effect size 0.3, large effect size 0.5. Learners are not expected to know this rule; this or any other formal rule will be given if necessary.  Effect sizes for other situations, e.g. for the difference of two means, are beyond the scope of this specification. | | | | | | |
| Spearman’s rank correlation coefficient | Sb8 | Be able to calculate Spearman's rank correlation coefficient from raw data or summary statistics.  Notes  Use of a calculator on the ranked data is expected.  Notation  Sample value .  Exclusions  Tied ranks. | | S2b5 | |  |
| b9 | Be able to carry out hypothesis tests using Spearman's rank correlation coefficient and tables of critical values or the output from software.  Notes  Hypothesis tests using Spearman’s rank correlation coefficient require no modelling assumptions about the underlying distribution.  Only ‘H0: No association in the population’ will be tested.  Both one-sided and two-sided alternative hypotheses will be tested.  Learners should state whether there is sufficient evidence or not to reject H0 and then give a non-assertive conclusion in context e.g. ‘There is insufficient evidence to suggest that there is an association between … and …’ | | S2b6 | | Output from software is new. |
| Comparison of tests | b10 | Decide whether a test based on *r* or *rs* may be more appropriate, or whether neither is appropriate.  Notes  Considerations include the appearance of the scatter diagram, the likely validity of underlying assumptions, whether association or correlation is to be tested for.  Spearman’s test is not appropriate if the scatter diagram shows no evidence of a monotonic relationship i.e one variable tends to increase (or decrease) as the other increases.  Understanding that ranking data loses information, which may affect the outcome of a test. | | S2b4 | | More explicit in the reformed specification. |
| Regression line for a random variable on a non-random variable | b11 | Be able to calculate the equation of the least squares regression line using raw data or summary statistics.  Notes  The goodness of fit of a regression line may be judged by looking at the scatter diagram.  In this case examination questions will be confined to cases in which a random variable,  and a non-random variable, , are modelled by a relationship in which the ‘true’ value of is a linear function of .  Only the use of a calculator is expected for calculation from raw data. Summary statistics formulae will be given.  Exclusions  Derivation of the least squares regression line. | | S2b7 | | See note on Sb5. |
| b12 | Be able to use the regression line as a model to estimate values and know when it is appropriate to do so.  Know the meaning of the term residual and be able to calculate and interpret residuals.  Notes  residual = observed value – value from regression line  Informal checking of a model by looking at residuals.  Notation  Interpolation extrapolation. | | S2b8 | | Use of the regression line as a model was implicit in S2b7 and S2b8. |
| Regression lines for a random variable on a random variable | b13 | Be able to calculate the equation of the two least squares regression lines,  on  and  on *,* using raw data or summary statistics.  Be able to use either regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so.  Notes  In the  on  case, the least squares regression line estimates , that is the expected value of  for a given value of . Conversely for the  on  case.  Only the use of a calculator is expected for calculation from raw data.  Exclusions  Derivation of the least squares regression lines. | |  | | This is new. See note on Sb5. |
| b14 | Check how well the model fits the data.  Notes  Informal checking only of a model by visual inspection of a scatter diagram or consideration of (pmcc) 2.  Exclusions  Residuals in this case. | |  | | This is new. |
| b15 | Know the relationship between the two regression lines and when to use one rather than the other.  Be able to use the correct regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so.  Notes  Both lines pass through . Choice of line to use depends on which variable is to be estimated.  Notation  Interpolation extrapolation. | |  | | This is new. |
| **STATISTICS a: CHI-SQUARED TESTS** | | | | | | |
| Contingency tables | Sb16 | Be able to interpret bivariate categorical data in a contingency table.  Notes  Numerical data can be put into categories, but this loses information. |  | | This is implicit in S2H1 and S2H2. | |
| test for a contingency table | SH1 | Be able to apply the  test (chi-squared) to a contingency table.  Notes  Only ‘H0: No association between the factors’ or H0: ‘variables are independent’ will be tested.  Calculating degrees of freedom is expected.  Knowing how to calculate observed values and contributions to the test statistic are expected, but repetitive calculations will not be required.  Learners should state whether there is sufficient evidence or not to reject H0 and then give a non-assertive conclusion in context e.g. ‘There is not sufficient evidence to believe that there is association between … and …’.  Exclusions  Yates’ continuity correction is not expected, though its appropriate use will not be penalised. | S2H1 | |  | |
| H2 | Be able to interpret the results of a  test using tables of critical values or the output from software.  Notes  Output from software may be given as a *p*-value.  Interpretation may involve considering the individual cells in the table of contributions to the test statistic. | S2H2 | | Output from software is new. | |
| test for goodness of fit | H3 | Be able to carry out a  test for goodness of fit of a uniform, binomial, geometric or Poisson model.  Notes  Only ‘H0: the given model fits the data’ or ‘H0; the given model is suitable’ will be tested.  Calculating degrees of freedom is expected.  Knowing how to calculate observed values and contributions to the test statistic is expected, but repetitive calculations will not be required.  Learners should be aware that cells are often combined when there are small expected frequencies, but will not have to make such decisions in examination questions.  Learners should state whether there is sufficient evidence or not to reject H0 and then give a non-assertive conclusion in context e.g. ’It is reasonable to believe that the … model is suitable.’ | S3I13 | | The Normal model is not included in the reformed specification but it does include the geometric model. | |
| H4 | Be able to interpret the results of a  test using tables of critical values or the output from software.  Notes  Output from software may be given as a *p*-value. | S3I13 | | Output from software is new. | |

## Content of Modelling with Algorithms (Y413) – option

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | **Legacy Unit & Ref.** | **Notes** |
| --- | --- | --- | --- | --- |
| **MODELLING WITH ALGORITHMS: ALGORITHMS** | | | | |
| Algorithms | A1 | Understand that an algorithm is a finite sequence of operations for carrying out a procedure or solving a problem.  Understand that an algorithm can be the basis for a computer program.  Notes  Initial state; input; output; variable.  ‘Finite’ means that the procedure terminates.  Exclusions  Algorithms with a random element. |  | This is new, but implicit in D1A1. |
|  | A2 | Be able to interpret and apply algorithms presented in a variety of formats.  Notes  Formats include flowcharts; written English; pseudocode.  E.g. in pseudocode,  Let *i* = *i* + 1 means that the number in location *i* is replaced by its current value plus 1.  Questions will not be set requiring unduly repetitive calculations.  Notation  Loop, pass.  ‘if … then…’  ‘Go to step …’  Iterative process.  Exclusions  Any particular version of pseudocode or programming language. | D1A1 |  |
|  | A3 | Be able to repair, develop and adapt simple algorithms. | D1A2 | ‘Repair’ is new. |
|  | A4 | Understand and be able to use the basic ideas of algorithmic complexity and be able to analyse the complexity of given algorithms.  Know that complexity can be used, among other things, to compare algorithms.  Notes  Worst case; size of problem; effect on solution time of multiplying the size of a large problem by a given factor and/or repeatedly applying an algorithm.  Notation  Order notation e.g. for quadratic complexity.  Exclusions  Analysis leading to non-polynomial complexity. | D1A3, D1A4 |  |
|  | A5 | Understand that algorithms can sometimes be proved correct or incorrect.  Notes  Proof by exhaustion and disproof by counter-example. |  | This is new. |
| Algorithms  (continued) | A6 | Understand and know the importance of heuristics.  Notes  A heuristic (sometimes called a heuristic algorithm) is a method which finds a solution efficiently, with no guarantee that it is optimal.  It is important when classic methods are inefficient or fail.  Notation  E.g packing algorithms.  E.g. find a solution to a linear problem which requires an integer solution by exploring around the solution to the corresponding LP. |  | This is new. |
| Sorting algorithms | A7 | Know and be able to use the quick sort algorithm.  Be able to apply other sorting algorithms which are specified.  Notation  Pivot values.  Pass. Ascending, descending. | (D1A2) | A more explicit spelling out of what is required in the Notes for D1A2. |
|  | A8 | Be able to count the number of comparisons and/or swaps needed in particular applications of sorting algorithms, and relate this to complexity.  Notes  Quick sort algorithm has (worst case) complexity .  Exclusions  Average complexity. | (D1A4) | A more explicit statement of what is required in analysis of complexity in this case. |
|  | A9 | Be able to reason about a given sorting algorithm.  Notes  E.g. explain why it will always work. |  | This is new. |
| Packing algorithms | A10 | Know and be able to use first fit and first fit decreasing packing algorithms and full bin strategies.  Notes  Know that these are not guaranteed to be optimal.  Notation  Bin. | (D1A2) | A more explicit spelling out of what is required in the Notes for D1A2. |
|  | A11 | Be able to count the number of comparisons needed in particular applications of packing algorithms, and relate this to complexity.  Notes  First fit and first fit decreasing algorithms have (worst case) complexity . | (D1A4) | A more explicit statement of what is required in analysis of complexity in this case. |
| **MODELLING WITH ALGORITHMS: NETWORKS** | | | | |
| Networks and graphs | N1 | Understand and be able to use graphs and associated language.  Notes  Node/vertex; arc/edge; tree; order of a node; simple, complete, connected and bipartite graphs; trees; digraphs.  Notation  Incidence matrix. | D1g1 | A more explicit statement of what is required. |
|  | N2 | Be able to model problems by using graphs.  Notes  E.g. river crossing problems.  E.g. matching problems. | D1g2 |  |
|  | N3 | Understand that a network is a graph with weighted arcs.  Notes  Directed and undirected networks. | D1N1 |  |
|  | N4 | Be able to model problems by using networks.  Notes  E.g. shortest path, maximum flow.  E.g. allocation and transportation problems. | D1N2 |  |
| Kruskal’s, Prim’s and Dijkstra’s algorithms | N5 | Be able to solve minimum connector problems using Kruskal’s and Prim’s algorithms.  Notes  Kruskal’s algorithm in graphical form only. Prim’s algorithm in graphical or tabular form.  Notation  Minimum spanning tree. | D1N3 |  |
| N6 | Model shortest path problems and solve using Dijkstra’s algorithm. | D1N4 |  |
|  | N7 | Know and use the fact that Kruskal’s, Prim’s and Dijkstra’s algorithms have quadratic complexity. | (D1A4), D2N3 | A more explicit statement of what is required in analysis of complexity in this case. |
| Critical path analysis | N8 | Model precedence problems with an activity-on-arc network. | D1X1 |  |
|  | N9 | Use critical path analysis and be able to interpret outcomes, including implications for criticality.  Be able to analyse float (total, independent and interfering), resourcing and scheduling.  Notes  E.g show how to use the minimum number of people to complete a given project in the minimum time.  Notation  Critical activities, critical path(s), forward and backward passes, longest path. | D1X1, D1X2, D1X3, D1X4 |  |
| Network flows | N10 | Be able to use a network to model a transmission system.  Notes  Single and super sources and sinks.  Flow in = flow out for other nodes.  Notation  Source: S.  Sink: T. | DCN1 |  |
|  | N11 | Be able to specify a cut and calculate its capacity.  Notes  *Either* split the vertices into two sets, one containing S and the other T, *or* specify the arcs that are cut. | DCN2 |  |
|  | N12 | Understand and use the maximum flow/minimum cut theorem.  Notes  If an established flow is equal to the capacity of an identified cut, then the flow is maximal and the cut is a minimum cut.  Exhaustive testing of cuts will not be assessed.  Exclusions  Flow augmentation. Labelling algorithm. | DCN3 |  |
| Solving network problems using technology | N13 | Understand that network algorithms can be explored, understood and tested in cases in which the algorithm can be run by hand, but for practical problems the algorithm needs to be formulated in a way suitable for computing power to be applied.  Notes  Formulations will be restricted to LPs. Questions may be set about the time taken by computer software to implement an algorithm when its complexity is known. |  | This is new. |
| **MODELLING WITH ALGORITHMS: LINEAR PROGRAMMING** | | | | |
| Formulating a problem | L1 | Understand and use the language associated with linear programming.  Notes  Linear programming, objective, maximisation, minimisation, optimisation, constraints.  Notation  LP is an abbreviation for linear program. |  | This is implicit in D1L1 to D1L5 and D2L1 to D2L5. |
| L2 | Be able to identify and define variables from a given problem.  Be able to formulate a problem as a linear program.  Notes  Variables should be clearly identified as representing numerical values.  e.g. ‘Let *x* be the number of …’.  Problem may be given in context. | D1L3 | Defining variables is implicit in D1L3. |
| L3 | Be able to recognise when an LP is in standard form.  Notes  A linear function to be maximised, constraints with “… constant” and non-negative, continuous variables. |  | This is new, but implicit in D1L3. |
| L4 | Be able to use slack variables to convert an LP in standard form to augmented form.  Notes  Also called slack form.  As standard form, but using non-negative slack variables to convert inequalities to equalities.  Notation  State variables. Slack variables.  Basic and non-basic variables. |  | This is new, but implicit in D2L1 and D2L5. |
| L5 | Recognise when an LP requires an integer solution.  Notes  E.g. when a variable is discrete.  E.g. a shortest path problem, because the variables take the values 1 or 0, depending on whether the corresponding arc is in the path or not.  If an LP requires an integer solution this should be stated in the formulation.  Notation  ILP is an abbreviation for integer LP. |  | This is new, but implicit in D1L4. |
| L6 | Be able to formulate a range of network problems as LPs.  Notes  Shortest path problems; network flows; critical path (longest path) problems; matching, allocation and transportation problems.  See after L18 for examples. | DCL1, DCN5, DCM6 | This is not precisely the same list of network problems. |
| Graphical solution of an LP | L7 | Be able to graph inequalities in 2-D and identify feasible regions.  Be able to recognise infeasibility.  Notes  No particular shading convention is expected, but learners must make clear which is the feasible region.  Exclusions  Drawing diagrams in more than 2-D. | D1L2 |  |
|  | L8 | Be able to solve a 2-D LP graphically.  Notes  By finding at least one optimal feasible point and the value of the objective function at this point.  Using the gradient of the objective function or by enumeration. | D1L4 |  |
|  | L9 | Be able to consider the effect of modifying constraints or the objective function.  Notation  Post-optimal analysis. | D1L5 |  |
|  | L10 | Be able to solve simple 2-D integer LP problems graphically.  Notes  The optimal lattice point may or may not be near the LP solution. | D1L4 |  |
|  | L11 | Be able to use a visualisation of a 3-D LP to solve it.  Be able to reduce a 3-D LP to a 2-D LP when one constraint is an equality.  Notes  Diagram will be given. Regions will be defined by an inequality based on the cartesian equation of a plane. |  | This is new. |
| Simplex method | L12 | Be able to use the simplex algorithm on an LP in augmented form.  Notes  Setting up an initial tableau, choosing a pivot, transforming the tableau, interpreting a tableau, recognising when a tableau represents an optimal solution.  Problems may be infeasible or have multiple solutions (degeneracy).  Notation  Initial, intermediate, final tableau. Slack variables.  Pivot. Basic/non-basic variables.  Exclusions  Knowledge of complexity of the simplex algorithm. | D2L1 |  |
|  | L13 | Understand the geometric basis for the simplex method.  Notes  Interpret a tableau in terms of the vertex and value of the objective function. | D2L2 |  |
| Simplex and non-standard form | L14 | Recognise that if an LP includes  constraints then the two-stage simplex method may be used; understand how this method works and be able to set up the initial tableau in such cases.  Exclusions  Big-M method. | D2L3 | big-M is required in D2L3 but not in the reformed specification. |
|  | L15 | Be able to reformulate an equality constraint as a pair of inequality constraints.  Notes  E.g. replace *x* = 4 by *x*  4 and *x*  4. | D2L4 |  |
|  | L16 | Recognise that if an LP has variables which may take negative values or requires the objective function to be minimised then some initial reformulation is required before the simplex algorithm may be applied.  Notes  Learners need only know that such reformulation is possible.  Exclusions  Be able to apply simplex in these situations. |  | This is new. |
| Use of software | L17 | Understand that simple LPs can be solved using graphical techniques or the simplex method, but for practical problems computing power needs to be applied.  Know that a spreadsheet LP solver routine, or other software, can solve an LP given in standard form or, in some cases, in non-standard form. |  | This is new. |
|  | L18 | Be able to interpret the output from a spreadsheet optimisation routine, or other software, for the simplex method or ILPs.  Notes  Select the appropriate information to solve the original problem.  This may lead to further analysis of the problem. | (DCL2) | Requirement to interpret solutions remains, but in the reformed specification it is output from software which is interpreted. |

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | **Legacy Unit & Ref.** | **Notes** |
| --- | --- | --- | --- | --- |
| **Examples of reformulating network problems as LPs**  These examples show how six types of network problems can be reformulated as LPs. They illustrate the sort of notation that will be used in questions. They do not show the level of difficulty of problem that will be examined. | | | | |
| A  C  D  B  1  4  4  2  2 |  | Shortest path  Find a shortest path from A to D.  Variables take the value 1 if the corresponding arc is used in a shortest path, and 0 otherwise. |  | Minimise  2AB + 4BD + 4AC + 2CD + BC + CB  subject to  AB + AC=1  AB + CB ­­– BC – BD = 0  AC + BC – CB – CD = 0  BD + CD = 1 |
| S  2  C  T  B  1  4  2  4 |  | Network flow  Find a maximum flow from S to T through the network. |  | Maximise  SB + SC  subject to  SB + CB – BC – BT = 0  SC + BC – CB – CT = 0  SB  2  BT  4  SC  4  CT  2  BC  1  CB  1 |
| A  2  C  D  B  1  4  2  4 |  | Longest path  Find a longest path from A to D.  Variables take the value 1 if the corresponding arc is used in a shortest path, and 0 otherwise.  This can be used to solve critical path problems on a directed network. |  | Maximise  2AB + 4BD + 4AC + 2CD + BC + CB  subject to  AB + AC = 1  AB + CB – BC – BD = 0  AC + BC – CB – CD = 0  BD + CD = 1  AB  1  BD  1  AC  1  CD  1  BC  1  CB  1 |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | 3 | 4 | | A | x |  |  | x | | B | x |  | x |  | | C |  | x | x |  | | D |  |  | x |  |   "x" indicates a possible matching |  | Matching problem  Possible associations between elements of {A, B, C, D} and {1, 2, 3, 4} are shown in the table. In a matching each element of one set is associated with at most one element of the other. The LP tries to find a maximal matching, i.e. a matching with as many associations as possible.  Each variable (e.g. C3) takes the value 1 (if C and 3 are associated) or 0. | "x" indicates a possible matching | Maximise  A1 + A4 + B1 + B3 + C2 + C3 + D3  subject to  A1 + A4  1  B1 + B3  1  C2 + C3  1  D3  1  A1 + B1  1  C2  1  B3 + C3 + D3  1  A4  1 |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | 3 | 4 | | A | 5 | 2 | 3 | 6 | | B | 1 | 7 | 2 | 4 | | C | 5 | 8 | 3 | 1 | | D | 4 | 4 | 2 | 6 | |  | Allocation problem  This is like a matching problem, except that (usually) every association is possible, and each association has a cost. The LP minimises the total cost for a maximal matching.  Each variable (e.g. A1) takes the value 1 or 0, depending on whether A is associated with 1 or not in the matching. |  | Minimise  5A1 + 2A2 + 3A3 + 6A4 + B1 + 7B2  + 2B3 + 4B4 + 5C1 + 8C2 + 3C3 + C4  + 4D1 + 4D2 + 2D3 + 6D4  subject to  A1 + A2 + A3 + A4 = 1  B1 + B2 + B3 + B4 = 1  C1 + C2 + C3 + C4 = 1  D1 + D2 + D3 + D4 = 1  A1 + B1 + C1 + D1 = 1  A2 + B2 + C2 + D2 = 1  A3 + B3 + C3 + D3 = 1  A4 + B4 + C4 + D4 = 1 |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 5 | 5 | 5 | 5 | |  |  | 1 | 2 | 3 | 4 | | 3 | A | 5 | 2 | 3 | 6 | | 6 | B | 1 | 7 | 2 | 4 | | 9 | C | 5 | 8 | 3 | 1 | | 2 | D | 4 | 4 | 2 | 6 | |  | Transportation problem  The body of the table shows the costs per item of transporting from one set of locations {A, B, C, D} to another {1, 2, 3, 4}.  The margins show the availability of items at locations A, B, C and D and the demands at 1, 2, 3 and 4.  The LP minimises the total cost of delivering all the required items. |  | Minimise  5A1 + 2A2 + 3A3 + 6A4 + B1 + 7B2  + 2B3 + 4B4 + 5C1 + 8C2 + 3C3 + C4  + 4D1 + 4D2 + 2D3 + 6D4  subject to  A1 + A2 + A3 + A4 = 3  B1 + B2 + B3 + B4 = 6  C1 + C2 + C3 + C4 = 9  D1 + D2 + D3 + D4 = 2  A1 + B1 + C1 + D1 = 5  A2 + B2 + C2 + D2 = 5  A3 + B3 + C3 + D3 = 5  A4 + B4 + C4 + D4 = 5 |

## Content of Numerical Methods (Y414) – option

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | **Legacy Unit & Ref.** | **Notes** |
| --- | --- | --- | --- | --- |
| **NUMERICAL METHODS: USE OF TECHNOLOGY** | | | | |
| Use of spreadsheets and calculators | NQ1 | Be able to use a spreadsheet to implement the methods and to explore associated ideas.  Be able to interpret the output from a spreadsheet.  Notes  Learners are expected to be familiar with a spreadsheet; no particular one is expected.  In the examination the spreadsheet facility available on some calculators may be used, but this is not expected.  Learners will be given output from a spreadsheet and may be asked to explain what certain cells represent, to explain or give formulae for certain cells, to give solutions and justify their accuracy, to comment on errors, convergence or order.  Notation  Cell B4 will mean the cell in column B, row 4.  Simple spreadsheet functions will be used, including  =IF(condition, value\_if\_true, value\_if\_false)  Learners may give formulae from any spreadsheet with which they are familiar.  Exclusions  Use of a computer in the examination. |  | This is new, but the use of a spreadsheet is implicit in the objectives and the coursework requirements for the legacy specification; this statement explains the requirements in the reformed specification where there is no coursework. |
|  | Q2 | Be able to use the iterative capability of a calculator.  Notes  In the examination learners are expected to use the iterative capabilities of their calculators (e.g. the ANS button) to generate values of iterative sequences.  Any permitted calculator may be used, but capabilities such as numerical differentiation, numerical integration and equation solvers should not be used in the examination; learners must show sufficient working to make their method clear.  Lengthy calculations will not be required. |  | This is new, but implicit in the legacy specification. |
| **NUMERICAL METHODS: ERRORS** | | | | |
| Absolute and relative error | NU1 | Know how to calculate errors in sums, differences, products and quotients. Know the meaning of absolute and relative error. Notes  Exact value:  Approximate value:  Absolute error:  Relative error: .  Notation  Absolute error will be used as a signed quantity. Another convention defines absolute error to be the magnitude of this quantity; this usage will not be penalised. | NMv1 |  |
| Error propagation by arithmetical operations and by functions | U2 | Know how to calculate the error in  when there is an error in .  Exclusions  Functions of more than one variable. | NMv2 |  |
| U3 | Understand the effects on errors of changing the order of a sequence of operations. | NMv3 |  |
| Errors in the representation of numbers: rounding; chopping | U4 | Understand that computers represent numbers to limited precision. | NMv4 |  |
| U5 | Understand the consequences of subtracting nearly equal quantities.  Notes  The subtraction might be embedded within a more complicated calculation e.g. in a fraction or in solving simultaneous equations. | NMv5 |  |
| U6 | Understand rounding and chopping and their consequences, including for calculations.  Notes  e.g. 7.86 rounded to 1d.p. is 7.9;  7.86 chopped to 1d.p. is 7.8;  5.7 chopped to the nearest integer is 5.  e.g. 200 numbers are each expressed to 1 dp. Each number is chopped to the nearest integer, and then they are added.  Maximum error in any one number is 0.9; maximum error in sum is 200 × 0.9 = 180.  Average error in one number is 0.45, so expected error for sum is 90  Notation  Maximum, average and expected error. |  | This is new but mentioned in the ‘specification’ column for NMv4 and NMv5. |
| Order of convergence and order of method | U7 | Understand convergence and divergence when applied to sequences.  Understand the order of convergence of an iterative sequence and the order of a method. Be able to comment on these given output from a spreadsheet.  Notes  An iterative sequence (e.g. a sequence produced by the Newton-Raphson method) has *k*th order convergence if the sequence of errors  satisfy the approximate relationship .  For a method with a ‘step-length’ *h*, (e.g. central difference method), the order of the method is the value *k* such that, approximately, . (For such a method a sequence of approximations can be produced by using a sequence of values of *h*; the sequence of errors will have an order of convergence, but this is **not**, in general, the order of the method.)  Exclusions  Formal analysis e.g. using Taylor expansions. |  | A more explicit bringing together of ideas in the legacy specification e.g. NM34, NMc2, NMc4. |
| Improving a solution | U8 | Be able to use error analysis to produce an improved solution.  Notes  Learners may be expected to calculate or identify the ratio of differences of a sequence of approximations to, for example, a definite integral. This may be presented as part of a spreadsheet output. They should be able to use an appropriate value for the ratio of differences to obtain an improved approximation by extrapolation – including to infinity - and should be able to quote and justify an appropriate level of precision in their final answer. |  | An explicit statement of what was expected in coursework and is implicit elsewhere. |
| **NUMERICAL METHODS: SOLUTION OF EQUATIONS** | | | | |
| Bisection method; False Position (linear interpolation); Secant method; Fixed point iteration; Newton-Raphson method | Ne1 | Understand the graphical interpretations of these methods.  Notes  Including staircase and cobweb diagrams.  Learners should be able to comment on suitability of starting point.  Exclusions  Proofs of orders of convergence. | NMe1 |  |
| e2 | Be able to solve equations to any required degree of accuracy using these methods.  Notes  Justify the accuracy claimed. | NMe2 |  |
| e3 | Understand the relative computational merits and possible failure of these methods.  Notes  Learners should recognise situations in which fixed point iteration and Newton-Raphson methods will fail. | NMe3 |  |
| e4 | Know that fixed point iteration generally has first order convergence, Newton-Raphson generally has second order convergence.  Notes  Learners should be able to comment on failure of the method or lower-order convergence in simple cases from graphical considerations or from spreadsheet output: e.g. the relationship between the order of convergence and the gradient of at the root in the iteration .  Exclusions  Formal proofs of convergence.  Formal analysis of failure or lower-order convergence. | NMe4 |  |
|  | e5 | Understand and be able to apply relaxation to a fixed point iteration: to accelerate convergence; to convert a divergent sequence to a convergent sequence.  Notes  For the iteration the relaxed iteration is . Formula will be given.  Different values of  have different effects on convergence.  Exclusions  Calculus to find optimal choice for . |  | This is new but partly in NCe1. |
| **NUMERICAL METHODS: NUMERICAL DIFFERENTIATION** | | | | |
| Forward difference method; Central difference method | Nc1 | Be able to estimate a derivative using the forward and central difference methods with a suitable value (or sequence of values) of *h*.  Notes  Use a suitable sequence of values of *h* to observe when the limitation of a spreadsheet’s accuracy is reached, to analyse errors and to justify the accuracy of a solution.  Notation    Exclusions  Second derivatives. | NMc1 |  |
| c2 | Have an empirical and graphical appreciation of the greater accuracy of the central difference method.  Know that the forward difference method is generally a first order method and that the central difference method is generally a second order method.  Exclusions  Proofs of order of method. | NMc2 |  |
| **NUMERICAL METHODS: NUMERICAL INTEGRATION** | | | | |
| Midpoint rule; trapezium rule; Simpson's rule | Nc3 | Be able to evaluate a given definite integral to any desired degree of accuracy using these methods. Notes  To estimate :  These formulae will be given. Lengthy calculations will not be required in the examination.  Any of the rules may be applied more than once, e.g. with *h* halving each time.  Learners are expected to be able to consider properties of the function - e.g. the graph is concave upwards over the given interval – to determine whether the rule over- or under- estimates.  Notation  N.B. The commonly used notation for Simpson’s rule, , shown in the formula, leads to an inconsistent definition of .  will be referred to as the midpoint rule based on  strips;  will be referred to as the trapezium rule based on  strips; the concept of strips will not be applied to Simpson’s rule. | NMc3 |  |
| The relationship between methods | c4 | Know that, generally, the midpoint and trapezium rules are second order methods and Simpson’s rule is a fourth order method.  Understand the development of Simpson's rule from the midpoint and trapezium rules.  Notes    Formulae will be given. | NMc4 |  |
| **NUMERICAL METHODS: APPROXIMATIONS TO FUNCTIONS** | | | | |
| Newton's forward difference interpolation method | Nf1 | Be able to use Newton's forward difference interpolation formula to reconstruct polynomials and to approximate functions.  Notes  Functions tabulated at equal intervals; learners should recognise when this is not the case and the method is not suitable.  Formula will be given.  Learners should be able to construct and use a difference table and know that *n*th differences are constant for an *n*th degree polynomial.  Notation | NMf1 |  |
| Lagrange’s form of the interpolating polynomial | f2 | Be able to construct the interpolating polynomial of degree *n* given a set of  data points.  Notes  Formula will be given. | NMf2 |  |

## Content of Mechanics b (Y415) – option

| **Spec. Content** | | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | | **Legacy Unit & Ref.** | **Notes** |
| --- | --- | --- | --- | --- | --- | --- |
| **MECHANICS b: MOMENTUM and IMPULSE** | | | | | | |
| Oblique impact | | Mi13 | | Understand the term oblique impact and the assumptions made when modelling oblique impact collisions1.  Notes  E.g. a collision between a sphere and a surface when the sphere is moving in a direction which is not perpendicular to the surface.  E.g. a collision between two discs not moving along their lines of centres.  Exclusions  Any situation with rotating objects. |  | This is new but partly covered in M2i10.  The content of this section (Mi13 to Mi17), which is oblique impact between a particle and a plane, is covered in M2i10 to M2i14. The content referring to oblique impact between bodies is new. |
| i14 | | Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to an oblique impact.  Notes  The coefficient of restitution is the ratio of the components of the velocities of separation and approach, in the direction of the line of impulse. | M2i12 | This is new for bodies colliding. |
|  | | i15 | | Be able to model situations involving oblique impact between an object and a smooth plane by considering the components of its motion parallel and perpendicular to the line of impulse. | M2i10, M2i11, M2i12, M2i13, M2i14 |  |
|  | | i16 | | Be able to model situations involving oblique impact between two bodies by considering the components of their motion in directions parallel and perpendicular to the line of the impulse. |  | This is new. |
|  | | i17 | | Be able to calculate the loss of kinetic energy in an oblique impact. | M2i13 | This is new for bodies colliding. |
| **1Assumptions when modelling oblique impact collisions** | | | | | | |
| This note explains the implicit assumptions made in examination questions when modelling oblique impact collisions. Learners may be asked about these assumptions. When two objects collide obliquely they cannot be modelled as particles; with two particles there is no preferred direction to act as the line of impulse. If an object collides with a plane (e.g. a wall or a floor) then it may be modelled as a particle or as a body, as appropriate.   * If the non-fixed objects involved in collisions may be modelled as bodies then these bodies will be uniform bodies with spherical or circular symmetry. * The impulse of any collision between such bodies acts on the line joining their centres.   These assumptions ensure that the collision happens at a point and that no angular momentum is created, hence none of the objects starts to rotate. An oblique impact collision occurs when the line of relative motion of the bodies is not the same as the line joining their centres at the point of collision.   * The impulse of any collision between such a body, or a particle, and a plane (e.g. a wall or floor) acts in a direction perpendicular to the plane.   An oblique impact collision in this situation means that the motion of the object is not in the direction perpendicular to the plane.   * The contact between the surfaces in any collision is smooth.   This is an extra assumption for oblique collisions. It ensures that the linear momentum of each object is conserved in the direction perpendicular to the line of impulse.   * Objects do not rotate before or after the collision. Rotating objects are beyond this specification. | | | | | | |
| **MECHANICS b: CIRCULAR MOTION** | | | | | | |
| The language of circular motion | | Mr1 | | Understand the language associated with circular motion.  Notes  The terms: tangential, radial and angular velocity; radial component of acceleration.  Notation  for angular velocity.  or .  Exclusions  Angular velocity as a vector. | M3r1 |  |
| Modelling circular motion | | r2 | | Identify the force(s) acting on a body in circular motion.  Notes  Learners will be expected to set up equations of motion. | M3r2 |  |
| r3 | | Be able to calculate acceleration towards the centre of circular motion. Notes  Using the expressions  and . | M3r3 |  |
| Circular motion with uniform speed | | r4 | | Be able to model situations involving circular motion with uniform speed in a horizontal plane.  Notes  E.g. a conical pendulum, a car travelling horizontally on a cambered circular track. | M3r4 |  |
| Circular motion with non-uniform speed | | r5 | | Be able to model situations involving circular motion with non-uniform speed.  Notes  E.g. rotation in a horizontal circle with non-uniform angular velocity. | M3r5 |  |
|  | | r6 | | Be able to calculate tangential acceleration.  Notes  Tangential component of acceleration .  Use of Newton’s 2nd law, , in the tangential direction. | M3r6 |  |
|  | | r7 | | Be able to model situations involving motion in a vertical circle.  Notes  The use of conservation of energy, and of  in the radial and tangential directions.  E.g. sliding on the interior or exterior surface of a sphere. | M3r7 |  |
|  | | r8 | | Identify the conditions under which a particle departs from circular motion.  Notes  E.g. when a string becomes slack, when a particle leaves a surface.  Questions may ask about the subsequent motion. | M3r8 |  |
| **MECHANICS b: HOOKE'S LAW** | | | | | |
| The language of elasticity | Mh1 | | Understand the language associated with elasticity.  Notes  Modulus of elasticity, stiffness, natural length, string, spring, equilibrium position. |  | This is implicit in M3h1 to M3h5. |
| h2 | | Understand that Hooke’s Law models the extension/compression of a material as a linear function of tension/thrust.  Notation  where  is the stiffness. |  | This is implicit in M3h1 to M3h5. |
| Extension of an elastic string and extension or compression of a spring. | h3 | | Be able to calculate the stiffness or modulus of elasticity in a given situation. | M3h1 |  |
| h4 | | Be able to calculate the tension in an elastic string or spring.  Notation  where  is the modulus of elasticity and  the natural length. | M3h2 |  |
| h5 | | Be able to calculate the equilibrium position of a system involving elastic strings or springs.  Notes  E.g. a heavy object suspended by a spring. | M3h3 |  |
| h6 | | Be able to calculate energy stored in a string or spring. Notes  The proof of this result may include the use of calculus.  (This is an exception to the exclusion in Mw2.)  Notation  or . | M3h4 | Proof may be required; this is new in the reformed specification. |
|  | h7 | | Be able to use energy principles to model a system involving elastic strings or springs including determining extreme positions.  Notes  Applications to maximum extension for given starting conditions in a system, whether horizontal or vertical. | M3h5 |  |
|  | h8 | | Understand when Hooke’s Law is not applicable.  Notes  Hooke’s Law does not apply when the relationship between extension/ compression and tension/thrust for a material is not linear. Many materials obey Hooke’s Law for a limited range of tensions/thrusts but extend/compress in a non-linear way for high values of tension/thrust. |  | This is new. |
| **MECHANICS b: CENTRE OF MASS** | | | | | | |
| Use of calculus to find centre of mass | | MG6 | | † Be able to calculate the volume generated by rotating a plane region about an axis.  Notes  Rotation about the *x*- and *y*- axes only.  Exclusions  Non-cartesian coordinates. | M3g1 |  |
| G7 | | Be able to use calculus methods to calculate the centre of mass of solid bodies formed by rotating a plane region about an axis.  Notes  E.g. hemisphere, cone.  Exclusions  Variable density.  Pappus’ theorem. | M3g2 |  |
| G8 | | Be able to find the centre of mass of a compound body, parts of which are solids of revolution.  Notes  By treatment as equivalent to a finite system of particles. | M3g3 |  |
| G9 | | Be able to use calculus methods to calculate the centre of mass of a plane lamina.  Exclusions  Pappus’ theorem. | M3g4 |  |
|  | | G10 | | Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body.  Notes  For the purpose of calculating its moment, the weight of a body can be taken as acting through its centre of mass.  E.g. a suspended object  E.g. does an object standing on an inclined plane slide or topple? | M3g5 |  |
| **MECHANICS b: VECTORS AND VARIABLE FORCES** | | | | | | |
| The language of kinematics | | Mk1 | | † Understand the language of kinematics appropriate to motion in 2- and 3- dimensions.  Know the distinction between displacement and distance, between velocity and speed, and between acceleration and magnitude of acceleration.  Know the distinction between distance from and distance travelled.  Notes  Position vector, relative position.  Notation        Exclusions  Vector form of . | M1k1, M1k2, M1k3, M1k10 |  |
| Velocity and position vector | | k2 | | † Be able to extend the scope of techniques from motion in 1-dimension to that in 2- and 3- dimensions by using vectors.  Notes  Using calculus and constant acceleration formulae. | M1k9 |  |
| Mv1 | | Be able to find the acceleration, velocity and position vector of a particle subject to a constant or variable force in 1-, 2- and 3-dimensions.  Notes  In terms of time or other parameters of a situation. | M1k8, M1k9 | More general cases than in M1k8 and M1k9. |
| v2 | | Be able to use the acceleration, velocity and position vector of a particle to model situations in 1-, 2- and 3- dimensions.  Notes  Including inferring the force acting. | M1v5 | More general than M1v5. |
| The equation of the path of a particle in 2D | | v3 | | Be able to eliminate a parameter from the expressions for the position vector of a particle, thereby forming a single equation.  Notes  E.g. elimination of time.  Questions will be restricted to cases where the elimination is straightforward. | (M1k11) | In M1k11 the parameter is time, but it need not be in the reformed specification. |
| v4 | | Be able to interpret the equation resulting from the elimination of a parameter from the terms of a position vector.  Notes  E.g. a bounding parabola.  E.g. solving for  or solving for  and interpreting. |  | This is new. |
| v5 | | Derive the cartesian equation of the path of a particle in 2-dimensions from an expression for its position vector.  Notes  E.g. the trajectory of a projectile. | (M1y4) | More general than M1y4. |
| v6 | | Be able to find the range of a projectile up or down a uniform slope.  Notes  Only cases where the projectile's initial position is on the slope.  Appropriate use of coordinates parallel and perpendicular to the slope, or horizontal and vertical.  Standard modelling assumptions for projectile motion are as follows:   * No air resistance. * The projectile is a particle. * Horizontal distance travelled is small enough to assume that gravity is always in the same direction. * Vertical distance travelled is small enough to assume that gravity is constant. |  | This is new. |
| v7 | | Be able to find the maximum range of a projectile up or down a uniform slope, and the associated angle of projection.  Notes  Only cases where the projectile's initial position is on the slope. |  | This is new. |
| Differential equations | | v8 | | Be able to formulate differential equations for motion under variable acceleration in 1- and 2-dimensions.  Notes  E.g. use Hooke’s Law for a particle on a spring or a particle attached to two springs to establish simple harmonic motion (SHM).  E.g. establish that SHM applies to part of the motion of a particle.  E.g. establish approximate SHM for a simple pendulum.  E.g. when resistance is a given function of velocity.  Including use of . | M4d2 | Solving DE is required in M4d2, but not in the reformed specification (see Mv9). |
| v9 | | Be able to verify a general or particular solution of a differential equation for motion under variable acceleration.  Notes  In the particular case this requires both showing that the solution is compatible with the equation and also that it conforms to the boundary or initial conditions for the situation when they are known.  Exclusions  Solving a differential equation, other than writing down a solution to SHM as in v12. |  | This is from Core Pure Pc7. |
|  | | v10 | | Be able to use the boundary or initial conditions to produce a particular solution from a general solution. |  | This is from Core Pure Pc7. |
| v11 | | Be able to recognise and formulate the simple harmonic motion equation expressed in non-standard forms and to transform it into the standard form by means of substitution.  Notes  E.g. ,  where  can represent a variable such as an angle. | M3o1, M3o2, M3o3 | This is also in Core Pure Pc15. |
| v12 | | Be able to solve the equation for simple harmonic motion, , and be able to relate the solution to the context.  Notes  Learners may state that they recognise the differential equation is that for SHM, and quote the solution in an appropriate form (e.g.  ,  or )  Notation  amplitude.  period |  | This is from Core Pure Pc15. |

## Content of Statistics b (Y416) – option

| **Spec. Content** | **Ref.** | **Learning Outcomes, Notes, Notation, Exclusions** | | **Legacy Unit & Ref.** | **Notes** |
| --- | --- | --- | --- | --- | --- |
| **STATISTICS b: CONTINUOUS RANDOM VARIABLES** | | | | | |
| The probability density function (pdf) of a continuous random variable | SR  19 | Be able to use a continuous random variable as a model.  Notes  Learners are expected to be familiar with the use of the (continuous) uniform and Normal distributions as models. They should be aware that other distributions underpin some work in this unit e.g.  and that other distributions, such as the exponential distribution, are useful models; knowledge of these is not expected and any necessary details will be provided in the examination.  Notation  Continuous uniform distribution also known as rectangular distribution.  Exclusions  Mixed discrete and continuous random variables. | | S3R1 |  | |
|  | R20 | Understand the meaning of a pdf and be able to use one to find probabilities.  Notes  Unfamiliar pdf’s, including piecewise pdf’s, may be given in examination questions.  In numerical cases learners are expected to write down the relevant definite integral, and may then use a calculator to evaluate it.  Notation  or other lower case letter for the function.  Exclusions  Using formula for pdf of Normal distribution. | | S3R2 |  | |
|  | R21 | | Know and use the properties of a pdf.  Be able to sketch the graph of a pdf.  Notes  and .  Exclusions  Evaluation of improper integrals. | S3R3 | Graphing is implicit in S3R3. |
| The probability density function (pdf) of a continuous random variable  (continued) | R22 | | Be able to find the mean and variance from a given pdf.  Notes  Learners are expected to write down the relevant definite integral, and may then use a calculator to evaluate it. In examination questions any integrations to be performed will be over a finite domain.  Standard deviation = .  For a continuous uniform distribution over : , . Formulae will be given but derivations may be required.  Exclusions  Deriving mean and variance of the Normal distribution from the pdf.  Evaluation of improper integrals. | S3R4 |  |
|  | R23 | | Be able to find the mode and median from a given pdf.  Notes  Mode only where it exists.  Exclusions  Mode for bimodal distributions. | S3R5 |  |
| The cumulative distribution function (cdf) | R24 | | Understand the meaning of a cdf and be able to obtain one from a given pdf.  Be able to sketch a cdf.  Notes    Notation  or other upper case letter for the function.  Exclusions  Normal distribution.  Evaluation of improper integrals. | S3R6 |  |
| R25 | | Be able to obtain a pdf from a given cdf.  Notes | S3R7 |  |
| R26 | | Use a cdf to calculate the median and other percentiles. | S3R8 |  |
| Expectation algebra | R27 | | Be able to find the mean of any linear combination of random variables and the variance of any linear combination of independent random variables.  Notes    Exclusions  Proofs. | S3a3 |  |
| The Normal distribution | R28 | | †Be able to use the Normal distribution as a model, and to calculate and use probabilities from a Normal distribution.  Notes  Calculations of probabilities are to be done using statistical functions on a calculator.  Relate calculations of probabilities to the graph of the Normal distribution. | S2N1 |  |
| R29 | | Be able to use linear combinations of independent Normal random variables in solving problems.  Notes  Use the fact that if  and , with  and  independent, then  Extend to more than two random variables.  Exclusions  Proof. | S3a4 |  |
| R30 | | Know that the Normal distribution is useful as a model in its own right, and as an approximating distribution in the context of the Central Limit Theorem (CLT).  Notes  Includes recognising when the Normal distribution is not appropriate.  Details of the CLT are in SI1 to SI6. |  | This is new. |
| R31 | | Interpret a Normal probability plot to decide whether a Normal model might be appropriate1. Interpret software output, including *p*-value, from the Kolmogorov-Smirnov test, to decide whether a Normal model might be appropriate.  Notes  Learners should know that tests other than the  test of goodness of fit are often applied to the Normal distribution. The null hypothesis for the given test is ‘H0: the Normal distribution fits the data’.  Exclusions  test for goodness of fit of Normal distribution. Calculations for Kolmogorov-Smirnov test. |  | This is new. |
| R32 | | Be able to use the Normal distribution, when appropriate, in the construction of confidence intervals.  Notes  See SI7 to SI14 below for details. | (S3I7) |  |
| 1There are different conventions for how Normal probability plots are drawn, and different features about the underlying distribution, for example skewness, can be inferred from the plot. Learners are only expected to know that the closer the points are to a straight line, the more likely it is that a Normal distribution fits the data; this is to be judged by eye. They are not expected to be able to draw Normal probability plots, nor do any calculations.  In the examination the sample data will be shown on one axis and the other will show expected Normal values. | | | | | |
| **STATISTICS b: INFERENCE** | | | | | |
| Distribution of sample mean and the Central Limit Theorem (CLT) | SI1 | | Be able to estimate population mean from sample data.  Notes  This is a point estimate; see confidence intervals below for interval estimates.  Notation    Exclusions  Proof. | S3I3 |  |
| I2 | | Be able to estimate population variance using the sample variance.  Notes  Sample variance given by  Exclusions  Proof. | S3I4 |  |
| I3 | | Understand that the sample mean is a random variable with a probability distribution.  Notes  If  independent observations  are taken from a distribution with mean  and variance  then  is a random variable with a probability distribution. Using the results of SR6 and SR27 the mean of this distribution is and its variance is .  Notation  Sampling distribution of the mean. |  | This is new, but implicit in S3I5. |
| I4 | | Be able to calculate and interpret the standard error of the mean.  Notes  The ‘standard error of the mean’ is the standard deviation of the sampling distribution of the mean. It is equal to . If is not known then it may be estimated from a particular sample as ; this estimate is also sometimes referred to as the ‘standard error of the mean’. | S3I6 |  |
| I5 | | Know that if the underlying distribution is Normal then the sample mean is Normally distributed.  Notes  Using the results of SR29. |  | This is new, but implicit in S3a4. |
| I6 | | Understand how and when the Central Limit Theorem may be applied to the distribution of sample means. Use this result in probability calculations, using a continuity correction where appropriate.  Be able to apply the CLT to the sum of *n* identically distributed independent random variables.  Notes  Ifhas a mean and finite variance, whatever its underlying distribution, the distribution of the sample meanmay be approximated by a Normal distribution if is sufficiently large;  is often used as a rule of thumb for ‘sufficiently large’.  Exclusions  Formal statement and derivation of the CLT.  Distributions for which the CLT does not apply. | S3I5 | Applying the CLT to the sum is new. |
| Confidence intervals using the Normal and *t* distributions | SI7 | | Know the meaning of the term confidence interval for a parameter and associated language.  Notes  A *confidence interval* is an interval estimate for a population parameter, based on a sample. If the confidence interval is constructed a large number of times, based on independent samples, then the *confidence level* is the long run proportion of confidence intervals which contain the true value of the parameter.  Exclusions  Asymmetric confidence intervals.  One-sided confidence intervals. | S3I7 | This statement corresponds only to part of S3I7. |
| I8 | | Understand the factors which affect the width of a confidence interval.  Notes  Sample size, confidence level, population variability. |  | This is new. |
| I9 | | Be able to construct and interpret a confidence interval for a single population mean using the Normal or *t* distributions and know when it is appropriate to do so.  Notes  Use the Normal distribution when the sample size is large, using *s*² as an estimate for  if necessary.  For a small sample from an underlying Normal distribution:   * if the population variance is known use the Normal distribution; * if the population variance is unknown use the *t* distribution, with *s*² as an estimate for  with *n* –1 degrees of freedom. | S3I7 |  |
| I10 | | Know when samples from two populations should be considered as paired. | S3I8 | This statement corresponds only to part of S3I8. |
| Confidence intervals using the Normal and *t* distributions  (cont) | I11 | | Be able to construct and interpret a confidence interval for the difference in mean of two paired populations using a paired sample and a Normal or *t* distribution; know when it is appropriate to do so.  Notes  Treat the differences as a single distribution and construct a confidence interval for the mean of the differences using the same procedure as for a single mean.  It is rarely the case that the population variance for the differences is known; this possibility will not be examined. | S3I8 |  |
| I12 | | Interpret confidence intervals given by software.  Notes  May include confidence intervals from distributions with which learners are not familiar; necessary details will be given. |  | This is new. |
| I13 | | Use a confidence interval for a population parameter to make a decision about a hypothesised value of that parameter.  Notes  By checking whether the confidence interval contains the hypothesised value. |  | This is new. |
| Hypothesis testing for an average using Wilcoxon, Normal or *t* tests | SH5 | | Be able to carry out a hypothesis test for a single population median using the Wilcoxon signed rank test and know when it is appropriate to do so.  Notes  Learners are expected to know that this is an example of a non-parametric (or distribution-free) hypothesis test, and when such tests may be useful.  Underlying distribution needs to be symmetrical.  H0: population median is given value.  Both one-sided and two-sided alternative hypotheses will be tested.  Learners should state whether there is sufficient evidence or not to reject H0 and then give a non-assertive conclusion in context e.g. ‘There is not sufficient evidence to believe that the median … has changed’.  Notation  the sum of the ranks for positive differences.  the sum of the ranks for negative differences. | S2N6, S3I11 |  |
| SH6 | | Be able to carry out a hypothesis test for a single population mean using the Normal or *t* distributions and know when it is appropriate to do so.  Notes  Use the Normal distribution when the sample size is large, using *s*² as an estimate for  if necessary.  For a small sample from an underlying Normal distribution:   * if the population variance is known use the Normal distribution; * if the population variance is unknown use the *t* distribution, with *s*² as an estimate for  with *n* –1 degrees of freedom.   H0: population mean is given value.  Both one-sided and two-sided alternative hypotheses will be tested.  Learners should state whether there is sufficient evidence or not to reject H0 and then give a non-assertive conclusion in context e.g. ‘There is sufficient evidence to believe that the mean… is not ...’. | S3I9 |  |
| **STATISTICS b: SIMULATION** | | | | | |
| Simulation of random variables | SZ1 | | Know that spreadsheets can be used to simulate probability distributions, and be able to do so for discrete and continuous uniform distributions and Normal distributions.  Notes  Learners are expected to appreciate the variation which repeated sampling produces. |  | This is new. |
|  | Z2 | | Know that simulations can be used to approximate probability distributions and to estimate probabilities, including in situations where the theory may be technically difficult. Be able to interpret output from spreadsheets investigating such situations.  Notes  E.g. if  and  are independent random numbers from  , estimate .  E.g. investigate the CLT for the sample mean from a continuous uniform distribution for various values of the sample size,  .  E.g. investigate whether the result about linear combinations of independent Normal random variables holds in different cases.  E.g. estimate the probability that 10 dice give a total score of greater than 50.  E.g. I commute to and from work on trains which run every 15 minutes. If I arrive at the station at a random time between trains, what is the probability that I have to wait for more than 20 minutes in total on any one day?  Learners will be expected to interpret output from spreadsheets to investigate such scenarios. |  | This is new, but see D1Z1. |

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| **Content from Legacy Units (FP1) which does not appear in the reformed AS Level specification (H635):**  FP1j6 – Complex Numbers: Be able to solve polynomial equations of higher degree (than 4) with real coefficients in simple cases  FP1C1 – Curve Sketching: Be able to sketch the graph of *y* = f(*x*) obtaining information about symmetry, asymptotes parallel to the axes, intercepts with the coordinate axes, behaviour near *x* = 0 and for numerically large *x*  FP1C2 – Curve Sketching: Be able to ascertain the direction from which a curve approaches an asymptote  FP1C3 – Curve Sketching: Be able to use a curve to solve an equality  FP1p3 – Proof: Be able to find the unknown constants in an identity (perhaps implicit in work on partial fractions)  FP1a1 – Algebra: Know the difference between a sequence and a series  FP1a2 – Algebra: Be able to sum a simple series  FP1a3 – Algebra: Know the meaning of the word *converge* when applied to either a sequence or a series  FP1a4 – Algebra: Be able to manipulate simple algebraic inequalities, to deduce the solution of such an inequality  FP1m13 – Matrices: Be able to give a geometrical interpretation of a case where the matrix is singular for simultaneous equations in 2-D |

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