

**Wednesday 28 June 2017 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4798/01** Further Pure Mathematics with Technology (FPT)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator
- Computer with appropriate software

**Duration:** Up to 2 hours

## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

## COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 This question concerns curves with parametric equations

$$x = \cos t, y = \sin nt,$$

where  $n$  is a positive integer and  $0 \leq t < 2\pi$ .

- (i) Sketch the curves in the cases  $n = 2$ ,  $n = 3$  and  $n = 4$ . [4]
- (ii) For the case  $n = 4$ , find the values of  $t$  where the curve crosses the  $x$ -axis. Hence show that the curve crosses itself at three points and state the  $x$ -coordinates of these points. [4]
- (iii) For the case  $n = 3$ , find the coordinates of the points where the tangent to the curve is parallel to the  $x$ -axis. [4]
- (iv) Show that, for any positive integer value of  $n$ , the curve has  $2n$  distinct points where the tangent to the curve is parallel to the  $x$ -axis. [6]
- (v) For the case  $n = 2$ , find a cartesian equation of the curve in the form  $y^2 = f(x)$ , where  $f(x)$  is a polynomial in  $x$ . Hence find the total area enclosed by the curve. [7]

- 2 This question concerns the function  $f(z) = e^z$  for  $z \in \mathbb{C}$ .

- (i) The values of  $f(2+i)$  and  $f(2.01+i)$  are denoted by  $z_1$  and  $z_2$  respectively. Find  $z_1$  and  $z_2$ , giving real and imaginary parts correct to 3 decimal places.

Find the value of  $\frac{z_2 - z_1}{0.01}$  and explain why this is approximately equal to  $z_1$ . [5]

- (ii) Construct a spreadsheet to demonstrate that  $\lim_{h \rightarrow 0} \left( \frac{f(2+i+h) - f(2+i)}{h} \right) = e^{2+i}$  where  $h \in \mathbb{R}$ .

State which values of  $h$  you have used and the expression(s) you have evaluated. Quoting sufficient values from your spreadsheet, explain how the result is demonstrated.

Use your spreadsheet to find, correct to 1 significant figure, the largest value of  $h$  such that

$$\operatorname{Re} \left( \frac{f(2+i+h) - f(2+i)}{h} - e^{2+i} \right) < 0.01. \quad [6]$$

- (iii) Show that  $f(k + \pi i)$  is a negative real number for all  $k \in \mathbb{R}$ . Show also that there are no values of  $z$  such that  $f(z) = 0$ . [6]
- (iv) Show that the points on an Argand diagram representing the roots of the equation  $f(z) = -2$  lie on a straight line and write down the equation of this line.

Now taking  $z$  to denote a general point on this line, find  $f(z)$  and hence describe the locus of points given by  $f(z)$  as  $z$  varies. [7]

- 3** This question investigates those positive integers,  $n$ , which can be expressed as the sum of the squares of two positive integers  $a$  and  $b$ , and those which can be expressed as the sum of the squares of three positive integers  $a$ ,  $b$  and  $c$ .

(i) Show that if  $n = a^2 + b^2$  then  $a < \sqrt{n}$  and  $b < \sqrt{n}$ . [1]

(ii) Create a program that will find all possible values of  $a$  and  $b$  such that  $n = a^2 + b^2$  for a given value of  $n$ , where  $a \leq b$ . You should write out your program in full.

Use your program to find all such ways of expressing  $n$  in the form  $a^2 + b^2$  for  $n = 1009$ ,  $n = 1019$  and  $n = 1037$ . [8]

(iii) By considering all the possible values of  $a^2 \pmod{4}$  and  $b^2 \pmod{4}$ , show that if  $n \equiv 3 \pmod{4}$  then  $n$  cannot be expressed in the form  $a^2 + b^2$ . [5]

(iv) Edit your program so that it will find all possible values of  $a$ ,  $b$  and  $c$  such that  $n = a^2 + b^2 + c^2$  for a given value of  $n$ , where  $a \leq b \leq c$ . You should state the changes you have made to your program.

Use your program to find all such ways of expressing  $n$  in the form  $a^2 + b^2 + c^2$  for  $n = 161$  and  $n = 167$ . [4]

(v) Show that if  $n \equiv 7 \pmod{8}$  then  $n$  cannot be expressed in the form  $a^2 + b^2 + c^2$ . [5]

**END OF QUESTION PAPER**

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