OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4768**: Statistics 3

Advanced GCE

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures

are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao"

even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)	(*A*)	Population standard deviation unknown. Small sample.	M1 A1 **[2]**	Allow $n < 30$. Just "small sample" scores M0A0 Needs "population" for M1, but "unknown variance and small sample" is SC B1 SC "Population not normal" B1	
1	(i)	(*B*)	$H_0: \mu = 30$ $H_1: \mu > 30$ where μ(mg) is the mean amount of salt per jar. $\bar{x} = 30.75$ $s^2 = 3.8$ (**OR** $s = 1.949$) Test statistic is $\frac{30.75-30}{\sqrt{\frac{3.8}{16}}}$ $= 1.54$ Using t_{15}. One-tail 5% point is 1.753 (**OR** $P(t_{15} > 1.54) = 0.0722$) 1.753 > 1.54 (**OR** $0.0722 > 0.05$) Not significant. Insufficient evidence that the mean amount of salt per jar is greater than 30(mg).	B1 B1 B1 B1 M1 A1 M1 A1 M1 A1 ft **[10]**	both hypotheses. Not using $\bar{x}$ Adequate verbal definition. If not using μ must say "population" ft c's mean and sd. no ft if wrong. no ft if wrong. Explicit comparison seen. consistent with c's mean and sd. If the comparison is not explicit (e.g. the two numbers written close to each other) can get SC B1 for the correct conclusion.	
1	**(ii)**		CI is $30.75 \pm 2.131 \times \sqrt{\frac{3.8}{16}}$ $= 30.75 \pm 0.4873 = (29.7, 31.8)$	M1 B1 M1 A1 **[4]**	Using c's mean using c's sd c.a.o. Must be expressed as an interval. 3 or 4 sf only.	

1	**(iii)**		The (population) mean is fixed, we can't talk about the probability of it being in a given interval. 95% of the confidence intervals created by repeated sampling will contain the true mean.	B1 B1 **[2]**	"It either is or isn't" is ok	

<table>
<tr><td>2</td><td>(i)</td><td>(A)</td><td>The (population) distribution of differences of marks is (not known to be) normal.</td><td>B1
[1]</td><td>Must refer to differences (not just “the underlying population”</td><td></td></tr>
<tr><td>2</td><td>(i)</td><td>(B)</td><td>H_0: $m = 0$, H_1: $m > 0$
where m is the population median of the difference between Judge 1’s and Judge 2’s marks.

<table><tr><td>difference</td><td>2.5</td><td>-2.3</td><td>1.6</td><td>-4.1</td><td>3.0</td><td>3.1</td><td>0.8</td><td>-2.2</td></tr><tr><td>rank</td><td>5</td><td>4</td><td>2</td><td>8</td><td>6</td><td>7</td><td>1</td><td>3</td></tr></table>
$W_- = 4 + 8 + 3 = 15$
Refer to Wilcoxon tables with n=8.
5% critical value is 5.
15 < 5 so result not significant

Insufficient evidence to suggest that judge 1 awards higher marks on the whole.</td><td>B1
B1

M1
M1
A1

A1
M1
A1
M1

A1

[10]</td><td>Both hypotheses
For adequate definition of m, in context; must refer to <u>population</u> median.
(not “difference in medians”
For differences.
For ranks.
ft from here if ranks wrong.

or $W_+ = 21$
no ft if wrong
or 31 no ft if wrong
ft c’s test statistic. Must compare this or W_+ to 31. No ft if comparing wrong tail.
ft c’s test statistic. Must include ‘evidence’ and ‘on the whole’ or oe.</td><td></td></tr>
<tr><td>2</td><td>(ii)</td><td></td><td>H_0: Judges awarded the same number of each mark.
H_1: Judges did not award the same number of each mark.

<table><tr><td>observed</td><td>5</td><td>6</td><td>10</td><td>9</td><td>14</td><td>16</td><td>14</td><td>6</td></tr><tr><td>expected</td><td>10</td><td>10</td><td>10</td><td>10</td><td>10</td><td>10</td><td>10</td><td>10</td></tr></table>
$\chi^2 = 2.5 + 1.6 + 0 + 0.1 + 1.6 + 3.6 + 1.6 + 1.6$

$= 12.6$
Refer to χ^2_7
The 10% critical value is 12.02.
12.6 > 12.02 so significant
There is sufficient evidence that judges have not been awarding the same number of each mark.</td><td>B1

B1

M1

A1
M1
A1
M1
A1

[8]</td><td>Both hypotheses. Must be in correct context. Allow ‘uniform distribution’ or ‘in equal proportions’.
‘Model fits data’ or ‘belief is justified’ is ok.
Do not accept ‘data fits model’ oe’.

For expected frequencies.

Calculation of χ^2. (if 12.6 not seen, must see evidence of calculation)
cao.
No ft if wrong.
No ft if wrong.
ft their test statistic
Must be in context and mention ‘evidence’. (‘organiser’s belief’ is sufficient context)</td><td></td></tr>
</table>

3	**(i)**		Negative cubic through the origin, positive x-intercept Only the part between 0 and 6 shown; gradient 0 at the origin and correct shape between max and 6.	M1 A1 **[2]**	A0 if there is a point of infelction between 4 and 6.	
3	**(ii)**		$f'(x) = \frac{4x-x^2}{36} = 0$ Mode is $x = 4$	M1 A1 **[2]**	Needs both attempt at differentiation and =0. No need to justify this is max. SC B1 for 'x = 4 is the maximum point on the graph'.	
3	**(iii)**		$E(X) = \int_0^6 \frac{1}{108}x^3(6-x)\,dx$ $= \left[\frac{x^4}{72} - \frac{x^5}{540}\right]_0^6$ $= \frac{18}{5}\ (= 3.6)$ $Var(X) = \int_0^6 \frac{1}{108}x^4(6-x)\,dx - \left(\frac{18}{5}\right)^2$ $\text{Integral} = \left[\frac{x^5}{90} - \frac{x^6}{648}\right]_0^6 \left(= \frac{72}{5}\right)$ $Var(X) = \frac{72}{5} - \left(\frac{18}{5}\right)^2 = \frac{36}{25}$ $\text{sd}\left(= \sqrt{\frac{36}{25}}\right) = \frac{6}{5}$	M1 A1 A1 M1 B1 A1 A1 ft B1 **[8]**	Limits needed somewhere. For correct integration. Can be implied by correct answer. c.a.o. (3.6 seen implies all three marks) For the integral, needs limits. For -(their mean)^2 Correct integrated expression. Using their mean www	
3	**(iv)**		$\bar{X}$ is approximately normal mean = 3.6 sd = $\frac{1.2}{\sqrt{50}}\ (= 0.1697)$ or var = 0.028798 $P(\bar{X} > 4) = 1 - P\left(Z < \frac{4-3.6}{0.1697} = 2.357\right)$ $= 0.00921$ The distribution of X is not normal.	M1 B1 ft B1 M1 A1 B1 **[6]**	s.o.i. Allow c's mean from (ii) For standardising using their mean, sd or var divided by $\sqrt{50}$. Requires 1- (Note: can't get this M1 if no $\sqrt{50}$) c.a.o. (ans 0.0092 from tables is ok, 2 to 4 sf)	

4	**(i)**		$P(M > 0.5) = 1 - P\left(Z < \frac{0.5-0.468}{0.067} = 0.4776\right)$ $= 0.316$	M1 A1 A1 **[3]**	For standardising. Award here or elsewhere. For 0.4776. Can be implied by correct 0.316. answer between 0.316 and 0.3165, 3 or 4 sf.	
4	**(ii)**		Require $P(M - T) < 0$ $M - T \sim N(0.073, 0.0131)$ $P\left(Z < \frac{0-0.073}{0.1146} = -0.637\right) = 0.262$	M1 B1 B1 A1 **[4]**	 Mean Variance. Accept sd (=0.1146) c.a.o, 3 or 4 sf (0.263 comes from early rounding and is A0)	
4	**(iii)**		Cost $C = 3.50M + 4.00T_1 + 4.00T_2$ $\sim N(4.798, 0.332)$ $P(C > 5) = 1 - P\left(Z < \frac{5 - 4.798}{0.576} = 0.351\right)$ $= 0.363$	M1 B1 B1 A1 **[4]**	Recognising that the two T's are different (if incorrect variance, need an explicit calculation showing that T_1 and T_2 were used rather than 2T) Mean Variance. Accept sd (0.576) c.a.o, 3 or 4 sf.	
4	**(iv)**	**(*A*)**	mean = $4.68w$ sd = $0.067\sqrt{10}w$ ($= w\sqrt{0.00489}$ = $0.212w$)	B1 B1 **[2]**		
4	**(iv)**	**(*B*)**	$\frac{14-4.68w}{0.067\sqrt{10}w} \leq -1.282$ $14 - 4.68w \leq -0.272w$ $w \geq 3.18$	M1 B1 M1 A1 **[4]**	Standardising with their mean and sd (allow variance) -1.282 seen (must be the correct sign, so can be 4.68w-14 and +1.282; allow = Attempting to solve, some working needed, but can be implied by correct answer; allow =. 3 or 4 sf; allow w = 3.18	

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