GCE

Mathematics (MEI)

Unit 4753: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2017
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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners’ meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates’ scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

<table>
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<tr>
<th>Annotation in scoris</th>
<th>Meaning</th>
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<tr>
<td>✓ and ✗</td>
<td></td>
</tr>
<tr>
<td>BOD</td>
<td>Benefit of doubt</td>
</tr>
<tr>
<td>FT</td>
<td>Follow through</td>
</tr>
<tr>
<td>ISW</td>
<td>Ignore subsequent working</td>
</tr>
<tr>
<td>M0, M1</td>
<td>Method mark awarded 0, 1</td>
</tr>
<tr>
<td>A0, A1</td>
<td>Accuracy mark awarded 0, 1</td>
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<td>B0, B1</td>
<td>Independent mark awarded 0, 1</td>
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<tr>
<td>SC</td>
<td>Special case</td>
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<tr>
<td>^</td>
<td>Omission sign</td>
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<tr>
<td>MR</td>
<td>Misread</td>
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<tr>
<td>Highlighting</td>
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**Other abbreviations in mark scheme**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
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<tr>
<td>E1</td>
<td>Mark for explaining</td>
</tr>
<tr>
<td>U1</td>
<td>Mark for correct units</td>
</tr>
<tr>
<td>G1</td>
<td>Mark for a correct feature on a graph</td>
</tr>
<tr>
<td>M1 dep*</td>
<td>Method mark dependent on a previous mark, indicated by *</td>
</tr>
<tr>
<td>cao</td>
<td>Correct answer only</td>
</tr>
<tr>
<td>oe</td>
<td>Or equivalent</td>
</tr>
<tr>
<td>rot</td>
<td>Rounded or truncated</td>
</tr>
<tr>
<td>soi</td>
<td>Seen or implied</td>
</tr>
<tr>
<td>www</td>
<td>Without wrong working</td>
</tr>
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</table>
Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>Guidance</th>
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</table>
| 1 | \[
\frac{1}{(5-2x^3)^2} = (5-2x^3)^{-2}
\]
\[
\frac{d}{dx}(5-2x^3)^{-2} = (-6x^2)(-2)(5-2x^3)^{-3}
\]
\[
= 12x^2(5-2x^3)^{-3}
\] isw | M1 | Chain rule on \((5-2x^3)^{-2}\)
correct expression, allow
\((-6x^2)(-2)u^{-3}\) o.e. | or quotient (or product) e.g.
\((5-2x^3)^{-2}\cdot 0 - 1.2(5-2x^3)(-6x^2)
\]
\[
= (5-2x^3)^{1-3}
\] [must have correct denom for M1]
| A1 | A1cao | or \[
\frac{12x^2}{(5-2x^3)^3}
\] isw |
| 2 | ![Diagram](image-url) | M1 | inverted ‘v’ shape
through \((-1, 0)\), \((1, 0)\) and \((0, 2)\)
correct domain \((-1 \leq x \leq 1)\) |
| A1 | A1 | |
| 3 (i) | \[
y = \ln(1-x) \quad x \leftrightarrow y
\]
\[
x = \ln(1 - y)
\]
\[
\Rightarrow e^x = 1 - y
\]
\[
\Rightarrow y = 1 - e^x [\text{so } f^{-1}(x) = 1 - e^x]
\] domain \(x < \ln 2\)
range \(-1 < y < 1\) | M1 | or \(e^x = 1 - x\)
y = 1 - e^x or \(f^{-1}(x) = 1 - e^x\)
allow \(x < 0.693\) or better, \(-\infty < x < \ln 2\)
or \(-1 < f^{-1}(x) < 1\). Must use \(x\) for
domain, \(y\) or \(f^{-1}(x)\) for range. |
| A1 | A1 | can interchange \(x\) and \(y\) at any stage |
| B1 | B1 | \(x \leq \ln 2\) is B0, \(\ln 0 < x < \ln 2\) is B0
allow \((-1, 1)\) but not \([-1, 1]\). If not labelled, take
inequality with \(x\) as domain and with \(y\) or \(f^{-1}(x)\) as range |
| | [3] | |
| 3 (ii) | \[
f(-x) = \ln(1 + x)
\]
\[
fg(x) = \ln(1 - x^2)
\]
\[
\ln(1-x) + \ln(1+x) = \ln(1-x)(1+x)
\]
\[
= \ln(1-x^2)
\] | B1 | soi e.g. from \(\ln(1-x) + \ln(1+x) = \ldots\) |
| B1 | B1 | must include brackets |
| B1 | | must include brackets |
### 4

#### (i)

\[ 2x^{\frac{1}{3}} + \frac{2}{3} y^{\frac{2}{3}} \frac{dy}{dx} = 0 \]

\[ \Rightarrow \frac{dy}{dx} = -3x^{-\frac{2}{3}} y^{-\frac{2}{3}} \text{ o.e.} \]

- M1
- A1
- A1

\[ \frac{d}{dx} \left( y^{\frac{1}{3}} \right) = \frac{1}{3} y^{-\frac{2}{3}} \frac{dy}{dx} \] seen correct equation

must simplify \( 2 / (2/3) = 3 \)

mark final answer

#### (ii)

when \( x = 1, y = 8 \), \( \Rightarrow \frac{dy}{dx} = -3 \times 4 = -12 \)

- M1
- A1cao

substituting both \( x = 1 \) and \( y = 8 \) into their \( dy/dx \)

NB check power of \( x \) is correct in part (i)

### 5

#### (i)

Initial temperature is 10.5 \(^{[\circ C]}\)

boiling point is 80\(^{[\circ C]}\)

- B1
- B1

#### (ii)

\[ 30 = 10.5 + 69.5(1 - e^{-kt}) \]

\[ \Rightarrow e^{-kt} = 1 - 19.5/69.5 \]

\[ \Rightarrow -k = \ln(0.7194...) \]

\[ \Rightarrow k = -\ln(0.7194...) = 0.3293... \]

- B1
- M1
- A1

re-arranging and taking \( \ln \) (correctly)

art 0.33 or \( \ln(139/100) \) o.e.

#### (iii)

\[ 79 = 10.5 + 69.5(1 - e^{-kt}) \]

\[ \Rightarrow e^{-kt} = 1 - 0.9856.. = 0.014388489... \]

\[ \Rightarrow t = -\ln(0.014388489...)/0.3293 [= 12.879... \text{ mins}}\]

- M1
- M1
- A1cao

substituting \( \theta = \) their \((80-1)\) into the eqn and re-arranging for \( e^{-kt} \)

taking \( \ln \) correctly

Trial and error: e.g. \( t = 12, \theta = 78.66 \)

\( t = 13, \theta = 79.04, \) so 13 mins SCB2

### 6

Suppose the polygon has \( n \) sides.

Then \( 180(n-2) = 155n \)

\[ \Rightarrow 25n = 360 \Rightarrow n = 14.4 \]

which is impossible as \( n \) is an integer

So no regular polygon has interior angle 155\(^{\circ}\)

or

When \( n = 14 \), int angle = \( 180 \times 12/14 = 154.29^{\circ} \)

When \( n = 15 \), int angle = \( 180 \times 13/15 = 156^{\circ} \)

So no \( n \) which gives an interior angle 155\(^{\circ}\).

- M1
- A1

or sum of ext angles = 360\(^{\circ}\) so \( 25n = 360 \)

or 72/5

- A1cao

clear statement of conclusion

accept 154\(^{\circ}\)
| 7 | (i) | \( \frac{1}{2} \) \( x \) = \( \sin y \)  
\[ \Rightarrow x = 2 \sin y \]  
\[ \frac{dx}{dy} = 2 \cos y \]  
\[ \text{B1cao} \]  
\[ \text{B1cao} \]  
\[ \text{[2]} \] |
| 7 | (ii) | \[ \frac{dx}{dt} = \frac{dx}{dy} \frac{dy}{dt} \]  
\[ \text{M1} \]  
\[ \text{o.e.} \]  
\[ \text{substituting} \ x = 1 \ \text{into} \ y = \arcsin \left( \frac{1}{2} \right) \frac{\pi}{6} \]  
\[ \text{condone} \ 30^\circ \]  
\[ \text{sin} \ y = \frac{1}{2} \ \Rightarrow \ \cos y = \sqrt{1 - \sin^2 y} \]  
\[ = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} \]  
\[ \Rightarrow \frac{dx}{dy} = \frac{\sqrt{3}}{2} \]  
\[ \text{M1} \]  
\[ \text{A1} \]  
\[ \text{sin}, \ e.g. \ \frac{dy}{dx} = \frac{2}{\sqrt{3}} \]  
\[ \text{A1} \]  
\[ \text{or} \]  
\[ \frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{1}{4} x^2}} \frac{1}{2} \]  
\[ \text{M1A1} \]  
\[ \text{A1} \]  
\[ \text{when} \ x = 1, \ \frac{dy}{dx} = \frac{1}{\sqrt{3}} \]  
\[ \Rightarrow 2 = \frac{\sqrt{3}}{3}, \frac{dy}{dt}, \frac{dy}{dt} = \frac{2}{\sqrt{3}} \]  
\[ \text{A1} \]  
\[ \text{[5]} \]  
\[ \text{or} \]  
\[ \frac{2\sqrt{3}}{3} \]  
\[ \text{must be exact, but isw if approximated} \]
### Question 8

#### (i)

\[ \left( \frac{\pi}{2}, 0 \right), \, (0, \frac{1}{2}) \]

- **B1B1** [2]: or \( y = 0 \Rightarrow x = \pi/2 \); \( x = 0 \Rightarrow y = 1/2 \)

Ignore incorrect labelling

#### (ii)

\[ \frac{d}{dx} y = \frac{2 \cos x - \sin x - \cos x}{2 \sin x} \]

- **M1 A1** correct quotient or product rule
- **M1** correct expression (isw)

\[ \text{when } \frac{d}{dx} y = 0 \]

\[ (2 - \sin x)(-\sin x) - \cos x(-\cos x) \]

\[ = 0 \]

\[ \Rightarrow 1 - 2\sin x = 0 \]

\[ \Rightarrow x = \frac{\pi}{6}, \, y = \frac{\sqrt{3}}{3} \quad \text{o.e.} \]

- **A1** must be exact, isw

\[ \text{denom must be correct at some stage missing brackets may be inferred from subsequent work not denominator} \]

#### (iii)

(a) 

\[ \int_{0}^{\frac{\pi}{2}} \cos x \frac{d}{dx} \left( \frac{\sin x}{2 - \sin x} \right) \]

- **B1ft** correct integral and limits
- **M1** \( c \ln (2 - \sin x) \)
- **A1** setting (only) their numerator to zero

\[ = \ln 2 \]

or

\[ = \ln \left( \frac{\sin x}{1} \right) \quad \text{not denominator} \]

(b) 

\[ \int_{0}^{k} \cos x \frac{d}{dx} \left( \frac{\sin x}{2 - \sin x} \right) \]

- **M1** setting \( k \) to \( \pi/2 \)
- **A1** eliminating logarithms correctly

\[ = \ln 2 \]

\[ = \ln \left( \frac{2 - \sin k}{2} \right) \]

\[ \Rightarrow k = \arcsin (2 - \sqrt{2})^{*} \]

**NB AG**

**or**

\[ \text{at some stage missing brackets may be inferred from subsequent work not denominator} \]

- **A1cao**

- **N.B AG**

---

**SC:** verifying (**max 3 marks out of 5**):

- **M1** attempt to find integral from 0 to \( k \) to \( \pi/2 \) to \( \pi/2 \) their area
- **A1** o.e. e.g. \( \sin (2 - \sin k) \)
- **M1** eliminating logarithms correctly
- **A1** o.e. e.g. \( (2 - \sin k)^{2} = 2 \)

or from \( \arcsin (2 - \sqrt{2}) \) to \( \pi/2 \)
**Mark Scheme**  
**June 2017**

| 9 (i) | \( f(x) = (-x)^{3}e^{(-x)^{2}} \)  
        | Rotational symmetry of order two about the origin. | M1 | substituting \(-x\) for \(x\) in \(f(x)\)  
        | must have \(f(-x) = (-x)^{3}e^{(-x)^{2}}\) for A1  
        | or point or half-turn \((180^\circ)\) symmetry about \(O\) | A1 | at least once  
        | allow description of symmetry, e.g. \(\text{fits its outline if rotated etc…}\) | A1 |  
| 9 (ii) | \( f(x) = 3x^2e^{-x^2} + x^3(-2x)e^{-x^2} \)  
        | \(f'(x) = 0\) when \(3x^2e^{-x^2} - 2x^4e^{-x^2} = 0\)  
        | \(\Rightarrow 3x^2 = 2x^4\)  
        | \(\Rightarrow x = 0, \sqrt{1.5}, -\sqrt{1.5}\)  
        | \(y = 0, 0.41, -0.41\)  
        | So \((0, 0)\), \((1.22, 0.41)\), \((-1.22, -0.41)\) | M1 |  
        | A1* |  
        | product rule  
        | correct expression | A1 |  
        | their deriv = 0  
        | taking out or dividing by \(e^{-x^2}\) | A1 |  
        | dep A1*  
        | or \(x = \pm\sqrt{1.5}\) o.e. dep A1* | A2 dep A1* |  
        | consistent with their derivatives - condone deriv of \(e^{-x^2}\) is \(e^{-x^2}\) for M1  
        | must be 2 terms  
        | must be 2 terms | A1 |  
| 9 (iii) |  
        | \(\text{correct shape for } -2 \leq x \leq 2\) with 2 TPs, through \(O\), reasonable half turn symmetry  
        | coords of TPs and stationary inflexion at origin shown dep \((0,0)\) given as a stationary point in part (ii) | M1 |  
        | A1 dep |  
        | need not show stationary inflexion at \(O\). Ignore shape outside \(-2 \leq x \leq 2\)  
        | condone plotting beyond \([-2, 2]\) provided shape is correct | A1 |  
| 9 (iv) (A) | \(\text{let } t = x^2, \frac{dt}{dx} = 2x \Rightarrow x\frac{dx}{dt} = \frac{1}{2} dt\) o.e.  
        | \(\int x^3e^{-x^2}dx = \int x^2e^{-x^2}x dx = \int \frac{1}{2} te^{-t} [dt]\) | M1 |  
        | A1 |  
        | \(k = \frac{1}{2}\) | A1 |  
| 9 (iv) (B) | \(\int_0^2 x^3e^{-x^2}dx = k \int_0^1 te^{-t} [dt]\)  
        | let \(u = t, \frac{du}{dt} = e^{-t}, u' = 1, v = -e^{-t}\)  
        | \[= [k]\left(t(-e^{-t}) \right)_0^1 - \int_0^1 (-e^{-t}) [dt]\]  
        | \[= [k]\left(-e^{-1} - te^{-1} \right)_0^1\]  
        | \[= \frac{1}{2} e^{-4} - 2e^{-4} + \frac{1}{2} = \frac{1}{2} - \frac{5}{2e^2}\] | M1 |  
        | A1 |  
        | \(\text{correct parts on } \int te^{-t}[dt] \text{ or } \int kt e^{-t}[dt]\)  
        | \(\text{ft their } k\)  
        | \(\text{ignore limits, ft their } k\)  
        | \(\text{limits must be correct here, ft their } k\)  
        | \(\text{oe but must evaluate } e^0 = 1\) and combine \(e^{-4}\) terms | A1 |  
        | A1 cao |  
        | [4] |  

**Diagram**

- At point (1.22, 0.41), \(y = 0.41\)
- At point (-1.22, -0.41), \(y = -0.41\)
- Origin, \((0, 0)\)
- Range \([-2, 2]\)