# Teacher Delivery Guide Core Pure: Vectors and 3-D Space

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **CORE PURE: VECTORS AND 3-D SPACE (a)** |
| Scalar products and the equations of planes | Pv1 | Know how to calculate the scalar product of two vectors, and be able to use the two forms of the scalar product to find the angle between two vectors. | Including test for perpendicular vectors. |  | Proof of equivalence of two forms in general case. |
| v2 | Be able to form and use the vector and cartesian equations of a plane. Convert between vector and cartesian forms for the equation of a plane. | Plane: where . |  | The form  |
| v3 | Know that a vector which is perpendicular to a plane is perpendicular to any vectorin the plane. | If a vector is perpendicular to two non-parallel vectorsin a plane, it is perpendicular to the plane. |  |  |

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| **CORE PURE: VECTORS AND 3-D SPACE (a)** |
| Intersection of planes | v4 | Know the different ways in which three distinct planes can be arranged in 3-D space.  | If two planes are parallel the third can be parallel or cut the other two in parallel lines; if no pair is parallel the planes can intersect in a point, form a sheaf or form a prismatic intersection. | A sheaf is where three planes share a common line. A prismatic intersection is where each pair of planes meets in a line; the three lines are parallel. |  |
|  | v5 | Be able to solve three linear simultaneous equations in three variables by use of the inverse of the corresponding matrix.Interpret the solution or failure of solution geometrically in terms of the arrangement of three planes.Be able to find the intersection of three planes when they meet in a point. | Inverse obtained using a calculator.If the corresponding matrix is singular, learners should know the possible arrangements of the planes; they will be given extra information or guidance if required to distinguish between these arrangements. |  | Finding equation of lines of intersection of two planes. |
|  | v6 | Know that the angle between two planes can be found by considering the angle between their normals. | The angle between two non-perpendicular planes is the acute angle between them. |  |  |

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| **CORE PURE: VECTORS AND 3-D SPACE (b)** |
| Vector product | Pv7 | Be able to use the vector product in component form to give a vector perpendicular to two given vectors. | Vectors with numerical components only.When a vector perpendicular to two others is required learners should indicate that they are using the vector product but no further working need be shown. Formula will be given; a calculator may be used. |   |  |
| Vector products  | v8 | Be able to use the alternative form for the vector product. Know the significance of . |  where , in that order, form a right-handed triple.Formula will be given. | The vectors , in that order, form a right-handed triple. |  |

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| **CORE PURE: VECTORS AND 3-D SPACE (b)** |
| Lines | v9 | Be able to form and use the equation of a line in 3-D. | In vector and cartesian form.Direction vector. | Line:  |  |
|  | v10 | Be able to calculate the angle between two lines. | The angle between two non-perpendicular lines (which may be skew) is the acute angle between their direction vectors. |  |  |
|  | v11 | Know the different ways in which two lines can intersect or not in 3-D space. | Two lines intersect at a point or are parallel or skew. |  |  |
|  | v12 | Be able to determine whether two lines in three dimensions are parallel, skew or intersect, and to find the point of intersection if there is one. |  |  |  |
|  | v13 | Be able to find the distance between two parallel lines and the shortest distance between two skew lines. | Formula for skew lines will be given, but questions may expect understanding of the underlying principles. |  | Proof of formula. |

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **CORE PURE: VECTORS AND 3-D SPACE (b)** |
| Points, lines and planes | Pv14 | Be able to find the intersection of a line and a plane. |  |  |  |
|  | v15 | Be able to calculate the angle between a line and a plane. | If they are not perpendicular, the angle between a line and a plane is the acute angle between the line and its orthogonal projection onto the plane. |  | The language ‘orthogonal projection’ is not expected. |
|  | v16 | Be able to find the distance from a point to a line in 2 or 3 dimensions. | The distance between a point and a line means the shortest distance between them.Formula will be given in 2-D case, but questions may expect understanding of the underlying principles. |  | Proof of formula. |
|  | v17 | Be able to find the distance from a point to a plane. | The distance between a point and a plane means the shortest distance between them.Formula will be given, but questions may expect understanding of the underlying principles. |  | Proof of formula. |

# Thinking Conceptually

### General approaches

This section builds off the work done on matrices earlier, and it may well be worth recapping this work briefly before introducing new concepts and/or making connections with the matrices work. It is probably a good idea to introduce the scalar and vector products at the point when they are needed, but it is worth looking at their properties as a whole, rather than just defining them and using them ad hoc.

In common with other topics looking at geometric elements, it will be useful for students to have access to some high-quality diagrams. These could be either provided on a digital projector, or as a handout to which students can refer as they are working on different topics. The use of technology such as Geogebra or Desmos to visualise can be very useful.

### Common misconceptions or difficulties learners may have

It is very easy and common for students to confuse the vector product with simple multiplication, so that they try to compute it component-wise, similar to adding and subtracting vectors. The anti-commutative nature of the vector product can also confuse students; although this aspect is not part of the content of the Pure Core it is a property many will come across simply by experimentation; their calculator may well calculate vector products. Students can make the assumption that ‘multiplication’ is still commutative as the symbol is familiar

Students may also not realise that in 3-D there is the ‘skew’ orientation of lines – not parallel but not intersecting. This shows that we cannot just treat 3-D as a simple extension of the work done in 2-D earlier. Again visualisation of this using technology or physical manipulatives is a good idea.

### Conceptual links to other areas of the specification

Vectors are introduced in A-level Mathematics, and students should recall the basic language for vectors from there. Distances between points are looked at there, and this can be drawn on when looking at other distance calculations.

Vectors are used in the mechanics option (if this option is studied by students).

# Thinking Contextually

Scalar products are used in mechanics to calculate work done by a force.

The vector product has a number of uses in more advanced mathematics. It can be used to calculate the distance between two skew lines and is used like this in calculating with computer graphics, for example when representing 3-D objects and wishing to know if they collide.

It is also used in applied mathematics to calculate angular momentum and torque and appears in the mathematics used by Maxwell to describe electromagnetic radiation, as it is used to define the vector operator curl.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
| --- | --- | --- | --- |
| [The Scalar Product](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-scalarprod-2009-1.pdf) | MathCentre | Notes and questions on the scalar product. | v1 |
| [Angle between two lines](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=1&cad=rja&uact=8&ved=0ahUKEwiDtpWNusjXAhVrDcAKHSH5AZkQtwIIKDAA&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DJSEPDJfl8m8&usg=AOvVaw2bW1Fc1LX8jPr_Q6qUUlpW) | Mario’s Math Tutoring | 3-minute video on finding the angle between lines using the scalar product. | v1, v2, v3 |
| [Dot Product Investigation](https://teacher.desmos.com/activitybuilder/custom/58a48fee8949d816073ea302) | Desmos | An interactive activity looking at the scalar product. | v1, v2, v3 |
| [Angle between a line and a plane](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=1&cad=rja&uact=8&ved=0ahUKEwiUvaqrusjXAhWUOsAKHdVeAK0QtwIIKDAA&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DOlGNSuKXD6c&usg=AOvVaw21rTUA92b9M2TjBdBAHove) | Swinburne Commons | 6 minute video on finding the angle between and line and a plane using the scalar product. | v1, v2, v3 |
| [Equation of a Straight Line](https://www.cut-the-knot.org/Curriculum/Calculus/StraightLine.shtml) | Cut the Knot | Notes on various different ways of writing the equation of a line, together with an app that will show the equivalence of forms for particular lines. | v2 |
| [y=mx+c](https://www.youtube.com/watch?v=260KIQxRLWI) | Sasmallmath | 13-minute video on the Cartesian equation of a straight line. | v2 |
| [Vector equation of a straight line](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=1&cad=rja&uact=8&ved=0ahUKEwjJgsyhtMjXAhUMLMAKHTbfAtAQtwIIKDAA&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3Dh3FUPv5AnfM&usg=AOvVaw19QbMOxwOu5CHl7xJn--WJ) | Jon Shiach | 5-minute video on vector equation of a straight line. | v2 |
| [Equations of Planes](http://tutorial.math.lamar.edu/Classes/CalcIII/EqnsOfPlanes.aspx) | Lamar University | Notes on Cartesian and vector equations of planes, with some good diagrams included. | v2 |
| [The Equation of a Plane](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=2&cad=rja&uact=8&ved=0ahUKEwiosbXXtsjXAhWLB8AKHRZoDmUQtwIILjAB&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3Dng8uqyxjliM&usg=AOvVaw3jnGF5x9mTHQjx9ApkSdTa) | AlRichards314 | A 21-minute video on Cartesian and vector equations of planes. | v2 |
| [Vector Equation of a Plane](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=9&cad=rja&uact=8&ved=0ahUKEwiosbXXtsjXAhWLB8AKHRZoDmUQtwIIVTAI&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3D9v8u_g0lxJs&usg=AOvVaw2pFiv7w_vJgdmSxXrJwgzT) | OnlineMathsClass | A 12-minute video, looking specifically at vector equations of planes. | v2 |
| [Lines and Planes](https://math.oregonstate.edu/home/programs/undergrad/CalculusQuestStudyGuides/vcalc/lineplane/lineplane.html) | Oregon State University | Notes and examples on equations of lines and planes. | v2 |
| [The Vector Product](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-vectorprod-2009-1.pdf) | MathCentre | Notes and questions on the vector product. | v8 |
| [Multiplication of Vectors](https://nrich.maths.org/2393) | NRICH | An introduction to Vector and Scalar Products. | v8 |
| [Intersection of lines in 3-D](https://www.youtube.com/watch?v=LxDM5-wRAzI) | Swinburne Commons | 13-minute video on finding the point of intersection of two lines. | v9, v10, v11, v12 and v13 |
| [Skew, Parallel or Coincident?](https://www.youtube.com/watch?v=bJ56Xr9081k) | patrickJMT | 5-minute video on determining the relations between a pair of lines. | v9, v10, v11, v12 and v13 |
| [Intersecting and Skew Lines](https://www.youtube.com/watch?v=C--Fa7Su3HI) | Exam Solutions | 13-minute video looking at lines in 3-D and some exam questions. | v12 and v13 |
| [Distance between 2 Lines](http://2000clicks.com/mathhelp/GeometryPointsAndLines3D.aspx) | Graeme McRae | Notes and examples on how to calculate the distance between lines. | v13 |
| [Shortest Distance Between To Skew Lines in 3D Space](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=1&cad=rja&uact=8&ved=0ahUKEwja94e5wsrXAhViIcAKHaevC54QtwIIKDAA&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DHC5YikQxwZA&usg=AOvVaw29RNKzwMYLqV5EOQy3Lo1v) | DLBmaths | 12-minute video on calculating the shortest distance. | v13 |
| [Distance from a point to a line](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=1&cad=rja&uact=8&ved=0ahUKEwjgnabawsrXAhUBL8AKHS7sAEIQtwIIKDAA&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3D9wznbg_aKOo&usg=AOvVaw0J80yUqnRHm3mEddRUOQ5Q) | DLBmaths | An 18-minute video looking at calculating the distance from a point to a line using three different methods. | v13 |
| [Distance](https://brilliant.org/wiki/3d-coordinate-geometry-distance/) | Brilliant | Notes and examples on all distances – between lines and between a point and a line. Some useful diagrams to help illustrate teaching points. | v13 |
| [Intersections of Lines and Planes](http://geomalgorithms.com/a05-_intersect-1.html) | iSurfer | Online notes and examples on the intersection of lines and planes. | v14 |
| [Distance from a point to a plane](https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=video&cd=1&cad=rja&uact=8&ved=0ahUKEwio7-Wgx8rXAhVmLMAKHcBEBzAQtwIIKDAA&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DdB6jyyKhwZE&usg=AOvVaw0MORQuiN8cYKSzgKf-lnZu) | DLBmaths | 12-minute video looking at how to calculate this distance and giving examples. | v17 |

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