



AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

- 1 (i) Calculate the relative error in using the approximation $\pi \approx \frac{355}{113}$. [2]

The diameter of a circle is 226.3 cm. It is decided to use the approximation $\pi \approx \frac{355}{113}$ and to round the diameter to the nearest whole number. Find the magnitude of the relative error in using these approximations to calculate

- (ii) the circumference of the circle, [2]

- (iii) the area of the circle. [2]

- 2 The following data were collected during an experiment.

x	1	2	4
y	0	4.5	29.7

- (i) State, with a reason, whether or not it is possible to use Newton's forward difference interpolation formula to construct a polynomial of degree 2 for these data. [1]

- (ii) Use a suitable method to construct an interpolating polynomial of degree 2 for this data. Give your answer in the form $ax^2 + bx + c$. [5]

- (iii) Use your answer to part (ii) to estimate the value of y when $x = 2.5$. [2]

- 3 (i) Show that the equation $x^5 - 3x - 1 = 0$ has a root α , such that $-1 < \alpha < 0$. [2]

- (ii) Use the iteration

$$x_{r+1} = \frac{x_r^5 - 1}{3},$$

to find α correct to five decimal places. Start at $x = 0$. [3]

You are given that there is another root to the equation, β , such that $\beta \approx 1.39$.

- (iii) Determine whether the iteration in (ii) may be used to find β to greater accuracy. [3]

- 4 The formula $I = \frac{(Z_2 + Z_1)^2}{(Z_2 - Z_1)^2}$ occurs in physics.

You are given that $Z_1 = 21.1$ and $Z_2 = 20.9$, correct to one decimal place.

- (i) Calculate the range of possible values of I . [4]

- (ii) Explain why this range is so large. [2]

- 5 The function $f(x)$ has the values shown in the table.

x	0.3	0.35	0.375	0.425	0.45	0.5
$f(x)$	0.897 31	0.879 32	0.871 16	0.856 80	0.850 70	0.840 90

- (i) Explain why it is not possible to use the forward difference method to estimate $f'(0.4)$ using these data. [1]
- (ii) Calculate three estimates of $f'(0.4)$ using the central difference method. Give your answers correct to 4 decimal places. Hence state the value of $f'(0.4)$ to the accuracy which seems justified. [5]
- (iii) Given that $f(0.4) = 0.863\ 63$, find an approximation to $f(0.41)$. [2]

Section B (36 marks)

- 6 (i) By calculating $0.1^{(0.1^3)}$, $0.01^{(0.01^3)}$ and $0.001^{(0.001^3)}$ show that $x^{(x^3)}$ tends to 1 as x tends to 0. [2]
- (ii) Identify the difficulty with using the trapezium rule to evaluate $\int_0^1 x^{(x^3)} dx$.
Use your answers to part (i) to suggest a way of addressing this difficulty. [2]
- (iii) Use the mid-point rule with $h = 1$, $h = 0.5$ and $h = 0.25$ to estimate the value of $\int_0^1 x^{(x^3)} dx$.
Hence give the value of the integral to the accuracy that appears justified. [5]

Further estimates of the integral using the trapezium rule for different values of h are given in the following table.

h	T
1	1
0.5	0.958 502
0.25	0.945 321
0.125	0.941 606
0.062 5	0.940 643

- (iv) Obtain four Simpson's Rule estimates of the integral. Give your answers correct to six decimal places. Hence give the value of the integral to the accuracy that appears justified. [5]
- (v) State the theoretical value for the ratio of differences of a sequence of estimates to a definite integral using Simpson's Rule. Use this value to obtain an improved approximation to $\int_0^1 x^{(x^3)} dx$. State the value of the integral as accurately as you can. [4]

- 7 (i) Show that the equation $3x^5 - 5x^3 - 1 = 0$ has a root α , such that $1 < \alpha < 2$, and a root β such that $-1 < \beta < 0$. [3]

- (ii) Obtain the Newton-Raphson iteration

$$x_{r+1} = x_r - \frac{3x_r^5 - 5x_r^3 - 1}{15x_r^4 - 15x_r^2}. \quad (*) \quad [2]$$

- (iii) Use (*) with $x_0 = 2$ to obtain the value of α correct to 5 decimal places. [3]

- (iv) Explain why it is not possible to use (*) with $x_0 = 0$ or $x_0 = -1$ to obtain a value for β . [2]

- (v) Show numerically that using (*) with $x_0 = -0.3$ leads to a third root, γ . Obtain the value of γ correct to 5 significant figures. [3]

- (vi) Given that $-0.9 < \beta < -0.3$, use interval bisection to obtain a value for β with a maximum possible error of 0.018 75. [4]

- (vii) How many further applications of bisection are needed to obtain a value for β with a maximum possible error of less than 5×10^{-5} ? [1]

END OF QUESTION PAPER

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