Friday 19 May 2017 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of 12 pages. The Question Paper consists of 4 pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer all the questions.

1 Find \( \sum_{r=1}^{n} (r^2 - r - 8) \), giving your answer in a fully factorised form. [5]

2 Use an algebraic method to find the square roots of the complex number \( 43 - (6\sqrt{10})i \). Give your answers in the form \( x + iy \), where \( x \) and \( y \) are exact real numbers. [5]

3 The matrices \( A \) and \( B \) are given by \( A = \begin{pmatrix} 1 & 4 \\ -2 & a \end{pmatrix} \) and \( B = \begin{pmatrix} 7 & 3 \\ 1 & 5 \end{pmatrix} \), where \( a \neq -8 \) and \( I \) is the \( 2 \times 2 \) identity matrix. Find

   (i) \( 7A - I \), [2]

   (ii) \( (A^{-1}B^{-1})^{-1} \). [3]

4 Prove by induction that, for \( n \geq 1 \), \( \sum_{r=1}^{n} \frac{1}{(2r+1)} = \frac{n}{2n+1} \). [5]

5 The matrix \( \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} \) represents the transformation \( P \).

   (i) Describe fully the transformation \( P \). [3]

   Transformation \( Q \) is a stretch, parallel to the \( y \)-axis with scale factor 4.

   (ii) Find the matrix that represents transformation \( Q \). [2]

   Transformation \( T \) is equivalent to transformation \( P \) followed by transformation \( Q \).

   (iii) Find the matrix that represents transformation \( T \). [2]

   (iv) Find the area of the image of the unit square under transformation \( T \). [2]

6 The complex number \( z_1 \) has modulus 3 and argument \( \frac{3}{5} \pi \). The complex number \( z_2 \) has modulus 3 and argument \( -\frac{9}{10} \pi \).

   (i) Sketch on a single argand diagram \( z_1 \), \( z_2 \) and \( z_1 - z_2 \). [3]

   (ii) Find the exact value of \( |z_1 - z_2| \) and the exact value of \( \arg(z_1 - z_2) \). [5]

   (iii) Give a geometrical description of the locus given by \( |z - z_1| = |z - z_2| \). [2]
7 (i) Show that \[ \frac{1}{2r-1} - \frac{1}{2r+5} = \frac{6}{(2r-1)(2r+5)}. \] 

Hence find

(ii) \[ \sum_{r=2}^{30} \frac{6}{(2r-1)(2r+5)}, \] giving your answer correct to 3 decimal places, [5]

(iii) \[ \sum_{r=2}^{\infty} \frac{6}{(2r-1)(2r+5)}, \] giving your answer as a single fraction. [1]

8 In the cubic equation \[ 4z^3 + az^2 + bz + c = 0, \] \(a, b\) and \(c\) are real numbers. One root is \(1 + \frac{1}{2}i\) and the sum of the roots is 6. Find the values of \(a, b\) and \(c\). [7]

9 The matrix \(C\) is given by
\[
C = \begin{pmatrix}
a & 1 & 1 \\ 3 & a & 1 \\ 5 & 3 & 2
\end{pmatrix}.
\]

(i) Find the value of \(a\) for which \(C\) is singular. [5]

In the three simultaneous equations given below, \(p\) is a constant.

\[
ax + y + z = p \\
3x + ay + z = p - 1 \\
5x + 3y + 2z = p - 2
\]

(ii) Write down one value of \(a\) for which these equations have a unique solution, giving a brief reason. [1]

(iii) Using the value of \(a\) found in (i), find the value of \(p\) for which these equations are consistent. [3]

10 The complex number \(a + ib\) is denoted by \(z\) and the complex number \(c + id\) is denoted by \(w\).

It is given that \(z^2 = z^* w\).

(i) Show that \(2ab = ad - bc\). [4]

(ii) Given that the real part of \(w = 0\), find the values of \(b\) in terms of \(a\). [6]

END OF QUESTION PAPER
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