OCR
Oxford Cambridge and RSA

Wednesday 14 June 2017 – Morning

AS GCE MATHEMATICS

4736/01 Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4736/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **8** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 The following list is to be arranged into **increasing** order, from smallest to largest.

$$12 \qquad 10 \qquad 3 \qquad 6 \qquad 12 \qquad 4 \qquad 5$$

(i) Use **bubble sort** to arrange the list into increasing order starting at the left-hand side of the list. Record the whole list at the end of each pass. There is no need to show the individual comparisons or swaps. **[3]**

A different list was arranged into increasing order using bubble sort. The following list is what was obtained after the first pass.

$$27 \qquad 8 \qquad 14 \qquad 40$$

(ii) **(a)** Write down two possible orders for the original list. **[2]**

(b) How many more passes will be needed to complete the sort (excluding the first)? **[1]**

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself. A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex. A *simply connected* graph is one that is both simple and connected.

Josh has drawn a simply connected graph that has exactly six vertices and exactly eight arcs. Four of the vertex orders are 2, 2, 4 and 5.

(i) Without doing any calculations and without drawing any graphs, how do the given vertex orders show that:

(a) Josh's graph is not Eulerian, **[1]**

(b) Josh's graph is semi-Eulerian. **[1]**

(ii) Without drawing any examples, calculate the values of the two missing vertex orders. Briefly explain your reasoning, by referring to properties of the graph and their consequences on the vertex orders. **[2]**

(iii) Argue in words, without drawing any examples, why any graph that fits these requirements has the same structure (in the sense of which vertices are directly connected to one another) as the one that Josh has drawn. **[3]**

3 The following algorithm displays three values for each positive integer input.

Line 10	Input a positive integer, N
Line 20	If N is an odd number jump to line 80
Line 30	Let $P = 1$
Line 40	Let $M = N$
Line 50	Divide the value of M by 2 and make this the new value of M
Line 60	If M is an odd number jump to line 90
Line 70	Increase P by 1 and go back to line 50
Line 80	Let $P = 0$ and let $M = N$
Line 90	Let $A = (M^2 + 1) \times 2^{(P-1)}$
Line 100	Let $B = A - 2^P$
Line 110	Display N, B, A and stop

(i) Apply the algorithm to the input $N = 5$. You only need to write down values when they change. You should record the line numbers but there is no need to record passing through lines 20 and 60. **[4]**

(ii) Apply the algorithm to the input $N = 20$. **[5]**

(iii) Explain what happens when $N = 2^x$, for some positive integer x. **[4]**

4 The table below shows the travel times (in minutes) between eight railway stations, A to H. Only travel times for direct routes (that do not involve changing trains) are shown, although an indirect route (changing trains) may be quicker. The travel times for seventeen direct routes are shown.

For this question you must work directly from the values in the table and <u>not</u> from a diagram.

	A	B	C	D	E	F	G	H
A	–	6	22	13	27	–	14	12
B	6	–	19	7	–	–	–	–
C	22	19	–	25	5	–	8	10
D	13	7	25	–	–	16	–	–
E	27	–	5	–	–	8	13	15
F	–	–	–	16	8	–	–	–
G	14	–	8	–	13	–	–	2
H	12	–	10	–	15	–	2	–

The sum of all the values in the table is 444.

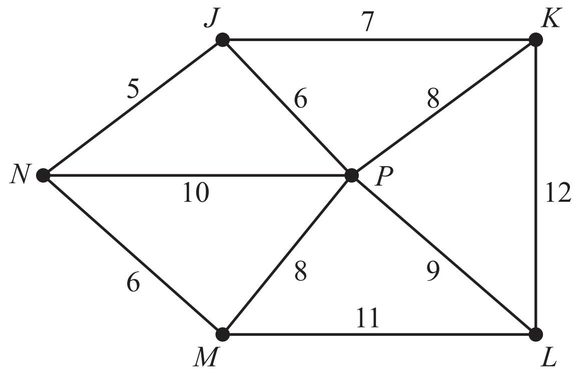
(i) Using only the direct routes shown in the table, and without drawing the network, list the stations that are connected to an odd number of other stations (the odd vertices). **[1]**

(ii) Find the shortest travel time from A to F that only involves changing trains once. **[2]**

(iii) Without drawing the network, use Dijkstra's algorithm to find the shortest travel times (direct or indirect) **from *B*** to each of the other stations. **[4]**

(iv) Deduce the shortest travel time for a closed route (starting and finishing at A) that involves travelling each of the seventeen direct routes shown in the table. Write down the arcs that are repeated and find how many times station C is travelled through when using this route. **[4]**

5 The network below represents the road routes between six villages. The weights on the arcs represent distances in miles.



Phoebe needs to deliver parcels to each village. She lives in village P. She wants to find a route that starts at P, visits each of the other villages (in some order) and finishes at P. She wants her route to be as short as possible.

(i) Use the nearest neighbour method to find a suitable route that starts at P, visits each of the other villages (in some order) and finishes at P. Write down your route and its length in miles. **[2]**

(ii) Find a route that passes through every vertex, starting and ending at P, that is shorter than the route found in part **(i)**. Write down your route and its length in miles. **[2]**

(iii) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should show the order in which you consider the arcs. Draw your tree and give its total weight. **[4]**

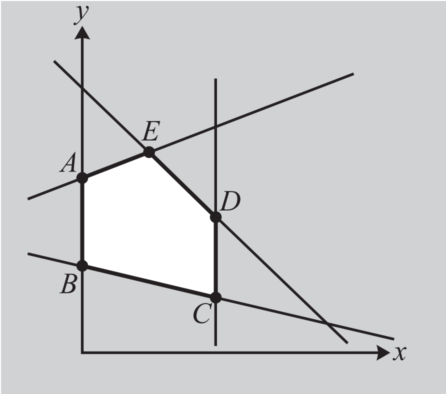
The road joining L to M is damaged. It is replaced by a new road that joins L to M of length k miles. The new road is **not** part of any minimum spanning tree for the new network.

(iv) Find the possible values of k, explaining your reasoning. **[2]**

Although not being part of any minimum spanning tree for the new network, the new road **is** part of the shortest route through every vertex, starting and ending at P.

(v) Find the possible values of k, explaining your reasoning. **[3]**

6 The graph below shows the feasible region (unshaded plus boundaries) for the linear programming (LP) problem below. The vertices of the feasible region are marked as A, B, C, D and E.



LP problem

Minimise $3x + 13y$

Subject to $x \leqslant 4$

$x + 4y \geqslant 8$

$-x + 3y \leqslant 12$

$2x + y \leqslant 11$

$x \geqslant 0,\ y \geqslant 0$

(i) Find the coordinates of the vertices A, B, C, D and E. **[4]**

The objective of the problem is to minimise $3x + 13y$.

(ii) Find the minimum feasible value of $3x + 13y$. **[2]**

The constraint $x \leqslant 4$ is removed, but otherwise the problem is unchanged.

(iii) Find the minimum feasible value of $3x + 13y$ for the new problem. **[3]**

Suppose now that there is an additional restriction that x and y must be integers.

(iv) Show that (5, 1) is feasible but does not give the minimum feasible value of $3x + 13y$ for the new problem with the additional restriction. **[2]**

7 Each day a potter makes dishes, mugs and pots.

Let x = the number of dishes
y = the number of mugs
z = the number of pots
that are made each day.

The dishes, mugs and pots have to be baked together in a kiln which is only used once each day.

There is enough space in the kiln for 15 dishes, with no mugs or pots, **or** 50 mugs, with no dishes or pots, **or** 200 pots, with no dishes or mugs. Usually the kiln holds a mixture of dishes, mugs and pots.

(i) Show that this leads to a constraint of the form $ax + by + cz \leqslant d$, where a, b, c and d are integers to be determined. **[3]**

Other constraints come from the amount of clay required, the potter's time and customer demand.

The potter wants to maximise the profit, £P, from selling the dishes, mugs and pots.

This leads to a linear programming problem, which is represented as a Simplex tableau with four slack variables, s, t, u and v. After one iteration the resulting tableau is:

P	x	y	z	s	t	u	v	RHS
1	0	0.5	–0.75	0	0	0	1.25	50
0	0	0	–0.5	1	0	0	–2.5	20
0	0	0.5	0.25	0	1	0	–0.75	6
0	0	–8	–7	0	0	1	–10	200
0	1	0.5	0.25	0	0	0	0.25	10

(ii) Which column was the first pivot chosen from? **[1]**

(iii) Carry out a second iteration. **[4]**

The final tableau is:

P	x	y	z	s	t	u	v	RHS
1	1	2	0	0	2	0	0	72
0	4	1	0	1	–2	0	0	48
0	3	2	1	0	1	0	0	36
0	31	6	0	0	–3	1	0	492
0	1	0	0	0	–1	0	1	4

(iv) **(a)** Use this tableau to state the number of dishes, mugs and pots that should be made each day and the potter's profit. **[2]**

(b) If the potter makes this number of dishes, mugs and pots, why might the profit be less than the amount given in the answer to part **(iv)(a)**? **[1]**

END OF QUESTION PAPER