GCE

Mathematics

Unit **4721**: Core Mathematics 1

Advanced Subsidiary GCE

Mark Scheme for June 2017

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

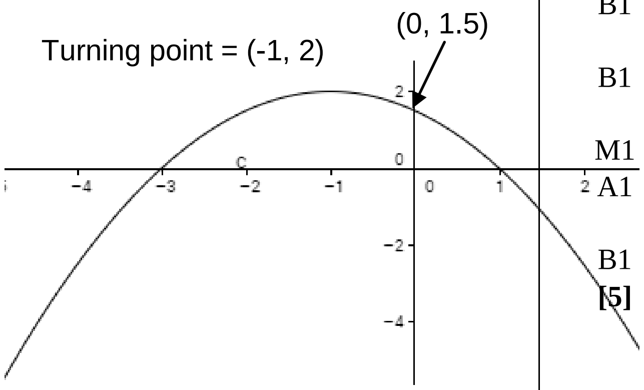
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

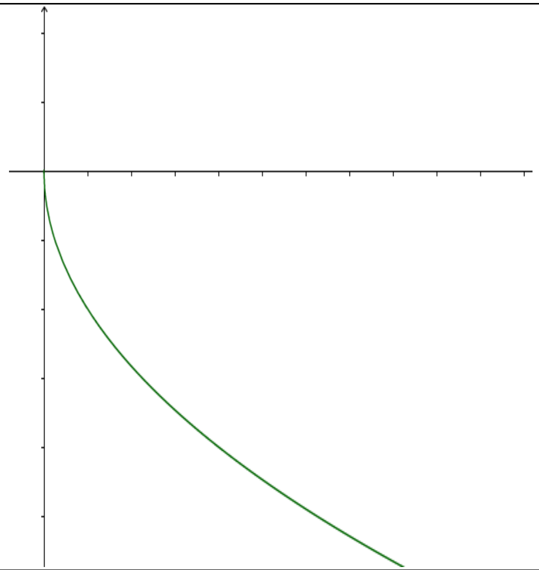
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\frac{2+\sqrt{7}}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$	M1	Attempt to rationalise the denominator – must attempt to multiply. (May use $-\sqrt{7}-2$)	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a+b\sqrt{7}$
			$\frac{11+4\sqrt{7}}{7-4}$	A1	Either numerator or denominator correct and simplified to no more than two terms	**A1** Either a or b correct
			$\frac{11}{3}+\frac{4\sqrt{7}}{3}$	A1 **[3]**	Fully correct and simplified. Allow $\frac{11+4\sqrt{7}}{3}$, terms in any order **Do not ISW** if then incorrect	**A1** Both correct Do not allow $\frac{-11-4\sqrt{7}}{-3}$ for last A1.
2			$2(x^2-6x)+x-6=0$	M1*	Substitute for x/y to eliminate one of the variables	If x eliminated: $y = (6-2y)^2-6(6-2y)$
			$2x^2-11x-6=0$	A1	Correct 2/3-term quadratic in solvable form	$4y^2-13y=0$
			$(2x+1)(x-6)=0$	M1* dep	Attempt to solve resulting quadratic. **See appendix 1.**	$y(4y-13)=0$
			$x=-\frac{1}{2}, x=6$	A1	x values correct	**Spotted solutions:** If **M0 DM0** **SC B1** One correct pair **www**
			$y=\frac{13}{4}, y=0$	A1 **[5]**	y values correct Award **A1 A0** for one pair **correctly found from correctly factorised quadratic**	**SC B1** Second correct pair **www** Must show on both line and curve (Can then get 5/5 if both found **www** and exactly two solutions justified)
3			$\sqrt{x}=x^{\frac{1}{2}}$ seen or implied	B1		Alternative using product rule: **B1** as main scheme
			$3x^{\frac{1}{2}}-21x+x^{\frac{5}{2}}-7x^3$	M1 A1	Attempts to expand brackets with 3/4 terms soi Correct expression for $f(x)$ in index form	**M1*** Clear attempt at $uv' + vu'$ **A1** All terms fully correct
			$\frac{3}{2}x^{-\frac{1}{2}}-21+\frac{5}{2}x^{\frac{3}{2}}-21x^2$	M1 A1 **[5]**	Attempt to differentiate their expression with at least one non-zero term correct Correct expression for $f'(x)$ **cao ISW** any attempts to put back into root form.	**M1*dep** Attempt to expand brackets with at least two terms simplified correctly **A1** Correct expression for $f'(x)$

Question			Answer	Marks	Guidance	
4				B1 B1 M1 A1 B1 [5]	Negative parabola Turning point at $(-1, 2)$; coordinates must be labelled on graph or clearly stated elsewhere Correct method to find roots* Correct x intercepts (1,0) and $(-3, 0)$ Correct y intercept $(0, \frac{3}{2})$ **NB** – Do not award 5/5 if sketch inconsistent with stated values e.g. turning point shown in wrong quadrant etc. Withhold one B1.	For **first mark** must clearly be a parabola – must not stop at or before x axis, do not allow straight line sections drawn with a ruler or tending to extra turning points etc. Must not be a finite plot. * If not using given form to solve, M mark only available for attempt to solve $k\left(-\frac{1}{2}x^2 - x + \frac{3}{2}\right) = 0$. **See appendix 1.**
5			$k = t^{\frac{1}{3}}$ $4k^2 + 17k - 15 = 0$ $(4k-3)(k+5) = 0$ $k = \frac{3}{4}, k = -5$ $t = \frac{27}{64}, t = -125$	M1* M1* dep A1 M1 A1 [5]	Substitute for $t^{\frac{1}{3}}$ to obtain a quadratic expression Rearrange and attempt to solve resulting quadratic equation. **See appendix 1.** Correct values of k Attempt to cube at least one value Final answers correct	Alternative**: M2** Rearrange and factorise into two brackets containing $t^{\frac{1}{3}}$. **See appendix 1.** **SC** If straight to formula with no evidence of substitution at start and no cubing/cube rooting at end, then **B1** for $\frac{-17\pm\sqrt{(17^2-4\times4\times-15)}}{2\times4}$ or better **No marks** if whole equation cubed etc. **Spotted solutions:** If **M0 DM0** or **M1 DM0** **SC B1** $t = \frac{27}{64}$ **www** **SC B1** $t = -125$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)

Question			Answer	Marks	Guidance	
6	(i)		$3(x^2 - \frac{5}{3}x) + 1$ $3[(x - \frac{5}{6})^2 - \frac{25}{36}] + 1$ $3(x - \frac{5}{6})^2 - \frac{13}{12}$	B1 B1 M1 A1 **[4]**	$a = 3$ $b = -\frac{5}{6}$ (not $\frac{-\frac{5}{3}}{2}, \frac{-2.5}{3}$) $1 - 3b^2$ or $3\times(\frac{1}{3} - b^2)$ $c = -\frac{13}{12}$. Allow $-\frac{39}{36}$ etc.	$3(x - \frac{5}{6})^2 + \frac{13}{12}$ **B1 B1 M0 A0** $3(x - \frac{5}{6}) - \frac{13}{12}$ **4/4 BOD** $3(x - \frac{5}{6}x)^2 - \frac{13}{12}$ **B1 B0 M1 A0** $3(x^2 - \frac{5}{6})^2 - \frac{13}{12}$ **B1 B0 M1 A0** $3x(x - \frac{5}{6})^2 - \frac{13}{12}$ **B0 B1 M1 A0** $3(x^2 - \frac{5}{6}) - \frac{13}{12}$ **B1 B0 M1 A0** $3(x + \frac{5}{6})^2 - \frac{13}{12}$ **B1 B0 M1 A0**
	(ii)		$(-5)^2 - 4.\ 3.1 = 13$ So 2 real roots	B1 B1ft **[2]**	ft their discriminant e.g. "– 25 – 12 = – 37 so no roots" scores **B0 B1ft**	Use of $\sqrt{b^2 - 4ac}$ can score **B0 B1**
7	(i)		$\frac{dy}{dx} = 8x^3 - 2x$ At stationary points $8x^3 - 2x = 0$ $x = \frac{1}{2}, x = -\frac{1}{2}, x = 0$	B1 M1 A1 **[3]**	Correct differentiation Sets their derivative to zero Correctly obtains **all three** roots.	**B0 M0** if expression is integrated and equated to zero. Do not accept $\pm\sqrt{\frac{1}{4}}$.
	(ii)		$\frac{d^2y}{dx^2} = 24x^2 - 2$ When $x = \pm\frac{1}{2}$, $\frac{d^2y}{dx^2} > 0$ so minimum, maximum when $x = 0$	M1 A1 **[2]**	Uses correct method to find nature of **at least one** stationary point e.g. substitution into second derivative (at least one term correct from their first derivative in (i)) and consider sign. Correct conclusions for all three points **www**	**Alternate valid methods include:** 1) Determining sign of gradient at either side of stationary point 2) Evaluating *y* at, and either side of, stationary point 3) Correct sketch Working must be fully correct to obtain the **A** mark

Question			Answer	Marks	Guidance	
	(iii)		$x < -\frac{1}{2}, 0 < x < \frac{1}{2}$	B2 **[2]**	Both regions correct (allow B1 for one correct region)	Condone use of ≤ instead of <. Condone e.g. $\sqrt{\frac{1}{4}}$ here.
8	**(i)**			B1 B1 **[2]**	Correct shape in correct quadrant – must intend to go through (0, 0) Sketch must also : • Start at (0,0) • Have fully correct curvature – does not tend to a horizontal asymptote • Not be a finite "plot"	
8	**(ii)**		$y = -2\sqrt{x+3}$	M1 A1 **[2]**	Translates curve by +/– 3 parallel to the *x*-axis Fully correct, must have "*y* ="	
	(iii)		Stretch Scale factor $\frac{3\sqrt{5}}{2}$ parallel to the *y*-axis (Scale factor $\frac{4}{45}$ **oe** parallel to the *x*-axis)	B1 B1 **[2]**	**Must** use stretch/stretched/stretching… Allow "factor" or "SF" for "scale factor" For "parallel to the *y* axis" allow "vertically", "in the *y* direction". Do not accept "in/on/across/up the *y* axis" , "SF 5 units" Apply the same principles to alternative correct answer:	Allow **first B1 only** for multiple transformations **provided all are stretches**. Allow $\frac{\sqrt{45}}{2}, \sqrt{\frac{45}{4}}$ etc. for $\frac{3\sqrt{5}}{2}$ **B0B1** is possible e.g. "Enlarge by scale factor…" etc. but not for (e.g.) "translate by scale factor…" or similar.

Question			Answer	Marks	Guidance	
9	(i)		$(2x+5)(x-2)=0$ $-\frac{5}{2}, 2$ $x < -\frac{5}{2}, x > 2$	M1 A1 M1 A1 **[4]**	Correct method to find roots. **See appendix 1.** Roots correct Chooses the "outside region" for their roots Allow "$x < -\frac{5}{2}, x > 2$", "$x < -\frac{5}{2}$ **or** $x > 2$" but do not allow "$x < -\frac{5}{2}$ **and** $x > 2$"	**NB** e.g. $-\frac{5}{2} > x > 2$ scores **M1A0** if correct answer not previously seen. Must be strict inequalities for **A** mark
9	(ii)		Gradient of line = – 3 $\frac{dy}{dx} = 4x+1$ $4x+1=-3$ $x=-1$ $y=-9$ $-9=-3(-1)+c \Rightarrow c=-12$ **OR** $2x^2+x-10=c-3x$ $2x^2+4x-10-c=0$ Tangent $\Rightarrow b^2-4ac=0$ $\Rightarrow 4^2-4.2.(-10-c)=0$ $c=-12$	B1 B1 M1 A1 A1 **OR** M1 A1 M1 A1 A1 **[5]**	Stated or used. Correct differentiation Equates their derivative with their gradient of line x correct c correct. Could also obtain from substituting $x=-1$ into $2x^2+x-10=c-3x$. **OR** Equates line and curve Obtains correct quadratic = 0 Uses tangency implies $b^2-4ac=0$ Fully correct substitution c correct	Look out for using 3 instead of –3. This gives $x=\frac{1}{2}$ which also leads to y = – 9. **B0B1M1A0A0** Max 2/5

Question			Answer	Marks	Guidance	
10	(i)		$(x-4)^2-16+(y+1)^2-1=0$ $(x-4)^2+(y+1)^2=17$	M1	Correct method to find centre of circle	e.g. $(x\pm 4)^2$ and $(y\pm 1)^2$ seen (or implied by correct answer)
			Centre = (4, – 1)	A1	Correct centre soi.	**M** can be implied by correct centre.
			$m=-\frac{1}{4}$	B1	Gradient of OA correct (could use OC or CA) [A = (8, – 2) is not required for this part, but may be used]	Note: Centre (– 4, 1) leads to "correct" answer. **M1A0B0M1A0** Max 2/5
			$y=-\frac{1}{4}x$	M1	Attempts equation of straight line through O or A or centre of the circle with their calculated gradient.	
			$x+4y=0$	A1 **[5]**	**www** Correct equation in required form i.e. $k(x+4y)=0$ for integer k, allow $0=4y+x$ etc.	<u>Alternative for first three marks:</u> **M1** Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x+2y\frac{dy}{dx}-8+2\frac{dy}{dx}=0$ and substitutes O to obtain $\frac{dy}{dx}=4$ **B1** Find correct negative reciprocal
10	(ii)		A = (8, – 2) $m'=4$	B1ft B1ft	Must be seen/used in (ii); ft their centre ft their gradient in (i)	
			$y+2=4(x-8)$	M1	Attempts equation of perpendicular line through their A. (**Not** (4, –1).)	If centre used here, max **B1B1**, 2/6.
			When $y=0, x=\frac{17}{2}$	M1	Attempt to find x value of point B from their equation of perpendicular line	Equation of line/B may not be seen explicitly.
			Area = $\frac{1}{2}\times\frac{17}{2}\times 2=\frac{17}{2}$	M1	Attempt to find area of OAB e.g. $\frac{1}{2}\times$ their OB $\times$ their 2, or $\frac{1}{2}\times$ their OA$\times$ their AB, or split into two triangles	Must have used a valid method to find B. OA = $\sqrt{68}$, AB = $\sqrt{\frac{17}{4}}$
				A1 **[6]**	Accept 8.5 or equivalent fractions but not unsimplified surds. **www**	Look out for "correct" answer from wrong coordinates – **A0**.

Question			Answer	Marks	Guidance	
11	(i)		Gradient of given line = 6	B1	soi as gradient of the line	Can be implied by use of $-\frac{1}{6}$
			Perpendicular gradient = $-\frac{1}{6}$	M1	Uses product of perpendicular gradients is – 1 at some point; may be implied by later working.	
			$\frac{dy}{dx} = -2kx^{-3}$	M1 A1	Attempt to differentiate (ax^{-3} seen) Fully correct	
			$-\frac{1}{6} = -2k(-3)^{-3}$	M1	Equates their derivative at $x = -3$ with their perpendicular gradient	e.g. $-\frac{27}{2k} = 6$ (implies first M1)
			$k = -\frac{9}{4}$	A1 **[6]**	Correct value of k. Allow $-\frac{27}{12}$ etc.	
	(ii)		When $x = -3$, $y = -\frac{9}{4(-3)^2} = -\frac{1}{4}$	B1	Correct value of y **www**	
			$y + \frac{1}{4} = 6(x + 3)$	M1	Attempts equation of straight line through $(-3, y)$, any non-zero gradient. y must be from their k but allow slips for M mark.	
				A1ft	Correct equation in any form – gradient 6 but ft their value of $\frac{k}{9}$. Allow 6 $(x - -3)$	For the first A mark, allow follow through their value of k – straight line through (–3, their $\frac{k}{9}$) with correct gradient of 6 e.g. k = 81 leads to $y - 9 = 6(x + 3)$
			$24x - 4y + 71 = 0$	A1 **[4]**	Correct equation in required form i.e. $a(24x - 4y + 71) = 0$ for integer a, terms in any order. **cao**	

APPENDIX 1 – this contains a generic mark scheme grid

Allocation of method mark for solving a quadratic

e.g. $2x^2 - x - 6 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x - 3)(x + 2)$	**M1**	$2x^2$ and -6 obtained from expansion
$(2x - 3)(x + 1)$	**M1**	$2x^2$ and $-x$ obtained from expansion
$(2x + 3)(x + 2)$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$$\frac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times 2}$$ earns **M1** (minus sign incorrect at start of formula)

$$\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$$ earns **M1** (6 for *c* instead of −6)

$$\frac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$$ **M0** (2 sign errors: initial sign and *c* incorrect)

$$\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times -6}$$ **M0** (2*c* on the denominator)

Notes – for equations such as $2x^2 - x - 6 = 0$, then $b^2 = 1^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - x - 6 = 0$$

$$2\left(x^2 - \frac{1}{2}x\right) - 6 = 0$$

$$2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6 = 0$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{49}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{1}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last (complete) attempt.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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