Section 5: Sequences and recurrence relations

Solutions to Exercise

1. (i) \( a_1 = 3 \times 1 - 1 = 2 \)
   \( a_2 = 3 \times 2 - 1 = 5 \)
   \( a_3 = 3 \times 3 - 1 = 8 \)
   \( a_4 = 3 \times 4 - 1 = 11 \)

(ii) \( a_1 = 2 \times 3^1 = 6 \)
   \( a_2 = 2 \times 3^2 = 18 \)
   \( a_3 = 2 \times 3^3 = 54 \)
   \( a_4 = 2 \times 3^4 = 162 \)

(iii) \( a_1 = 1^2 = 1 \)
    \( a_2 = 2^2 = 4 \)
    \( a_3 = 3^2 = 9 \)
    \( a_4 = 4^2 = 16 \)

(iv) \( a_1 = (-1)^1 \cdot 2^1 = -2 \)
    \( a_2 = (-1)^2 \cdot 2^2 = 4 \)
    \( a_3 = (-1)^3 \cdot 2^3 = -8 \)
    \( a_4 = (-1)^4 \cdot 2^4 = 16 \)

(v) \( a_1 = 2 \)
    \( a_2 = 2a_1 + 1 = 2 \times 2 + 1 = 5 \)
    \( a_3 = 2a_2 + 1 = 2 \times 5 + 1 = 11 \)
    \( a_4 = 2a_3 + 1 = 2 \times 11 + 1 = 23 \)

(vi) \( a_1 = 3 \)
    \( a_2 = 1 - a_1 = 1 - 3 = -2 \)
    \( a_3 = 1 - a_2 = 1 - (-2) = 3 \)
    \( a_4 = 1 - a_3 = 1 - 3 = -2 \)

2. (i) \( u_1 = 0 \), and then next terms are 1, 2, 3

(ii) \( u_1 = 2 \), and then next terms are \( \frac{2}{5}, \frac{2}{15}, \frac{2}{45} \)
Additional Mathematics (OCR): Algebra

(iii) \( u_s = -\frac{1}{32} \), and then next terms are \( +\frac{1}{64} \), \( -\frac{1}{128} \), \( +\frac{1}{256} \).

(iv) \( u_3 = -1, u_4 = -3 \), and so \( u_5 = -4 \) and then next terms are \(-7, -11, -18\) (this is an example of a Fibonacci sequence)

3. Since the value of the car reduces by 10% each year. The value each year is 0.9 times the value the previous year. Therefore \( a_{k+1} = 0.9a_k \).

4. Since the value of the train set increases by 2% each year, if the original value is £150, the value one year later, \( a_1 \), is \( 150 \times 1.02 \).
   The value the next year \( a_2 \), is \( 150 \times 1.02 \times 1.02 = 150 \times (1.02)^2 \).
   This pattern continues and so the value at the end of the next year \( a_3 \) is \( 150 \times (1.02)^3 \).
   In general \( a_k = 150 \times (1.02)^k \).

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