OCR
Oxford Cambridge and RSA

Tuesday 19 June 2018 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 A point P moves round the curve with equation $3x^2 + 4y^2 = 4$. At time t, P is at the point (x, y), as shown in Fig. 1.

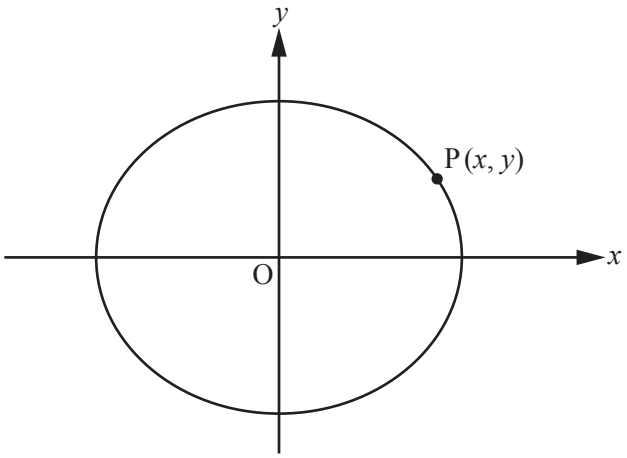

Fig. 1

(i) Find $\frac{dy}{dx}$ in terms of x and y. **[2]**

(ii) When P is at the point on the curve with x-coordinate 1 and positive y-coordinate, $\frac{dx}{dt} = 4$.

Find $\frac{dy}{dt}$ at this point. **[4]**

2 The three functions $f(x)$, $g(x)$ and $h(x)$ are defined as follows:

$$f(x) = \frac{x}{1-2x^2}, \quad g(x) = 1 + \sin 2x \quad \text{and} \quad h(x) = 3e^{-2x^2}.$$

In the table in the Answer Book, write Yes or No in each space to indicate whether the function is odd, whether it is even, and whether it is periodic. If a function is periodic, state its period. **[4]**

3 The mass of a radioactive material decreases exponentially. Its *half-life* is the time required for the mass of the material to reduce to half its initial value. The half-life of plutonium 241 is 14.4 years.

(i) Write down the percentage of the initial mass of plutonium 241 remaining after 28.8 years. **[1]**

(ii) The mass M grams of plutonium 241 at time t years is given by the equation

$$M = M_0 e^{-kt},$$

where M_0 grams is the initial mass and k is a constant. Find k, giving your answer correct to two significant figures. **[3]**

4 Fig. 4 shows part of the curve with equation $y = \mathrm{e}^{-x}\sin 2x$.

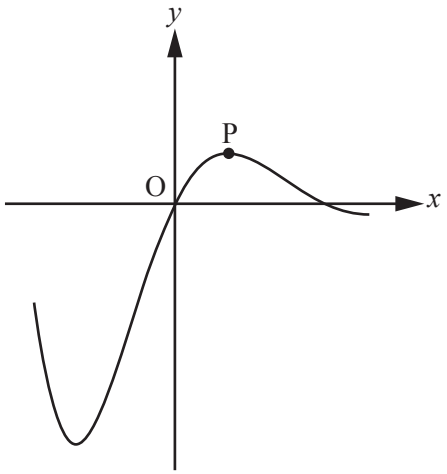

Fig. 4

Find the coordinates of the maximum point P. **[6]**

5 **(i)** On the same axes, sketch the graphs of $y = -|x+1|$ and $y = 2x$. **[3]**

(ii) Solve the equation $-|x+1| = 2x$. **[2]**

6 The function $\mathrm{h}(x)$ is such that $\mathrm{h}(x) = \mathrm{fg}(x)$, where $\mathrm{f}(x) = 2x + \frac{1}{2}\pi$ for $x \in \mathbb{R}$ and $\mathrm{g}(x) = \arcsin x$ for $-1 \leqslant x \leqslant 1$.

(i) Find $\mathrm{h}\left(\frac{1}{2}\right)$, giving your answer as a multiple of π. **[2]**

(ii) Find $\mathrm{h}^{-1}(x)$. **[4]**

7 Prove that $n^3 - 3n^2 + 2n$ is divisible by 6 for all positive integers n. **[5]**

Section B (36 marks)

8 Fig. 8 shows the curve with equation $y = \dfrac{2\sqrt{x}}{1+\sqrt{x}}$. The tangent to the curve at P (1, 1) intersects the y-axis at Q.

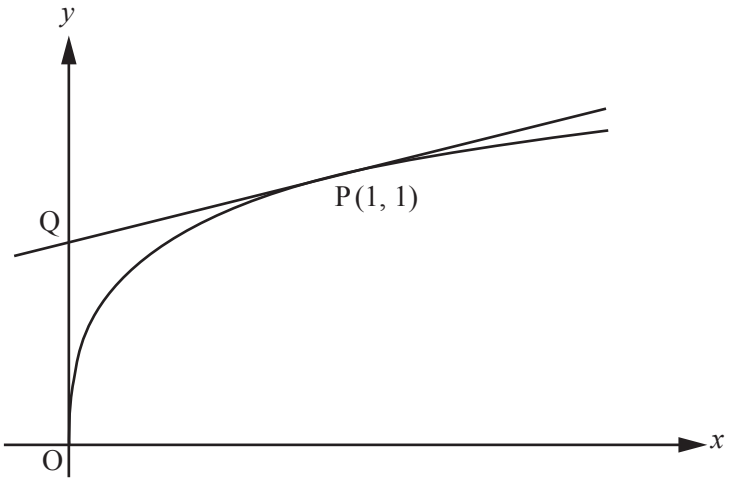

Fig. 8

(i) Show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x}(1+\sqrt{x})^2}$.

Hence find the equation of PQ, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

(ii) Show that the substitution $u = 1 + \sqrt{x}$ transforms $\displaystyle\int \frac{2\sqrt{x}}{1+\sqrt{x}}\,dx$ to $\displaystyle\int \frac{4(u-1)^2}{u}\,du$. **[3]**

(iii) Hence find the exact area of the region enclosed by the curve, the y-axis and the line PQ. **[8]**

9 Fig. 9 shows the curves with equations $y = \ln x$ and $y = 2\ln(x-2)$ which intersect at Q. The curve $y = 2\ln(x-2)$ crosses the x-axis at P.

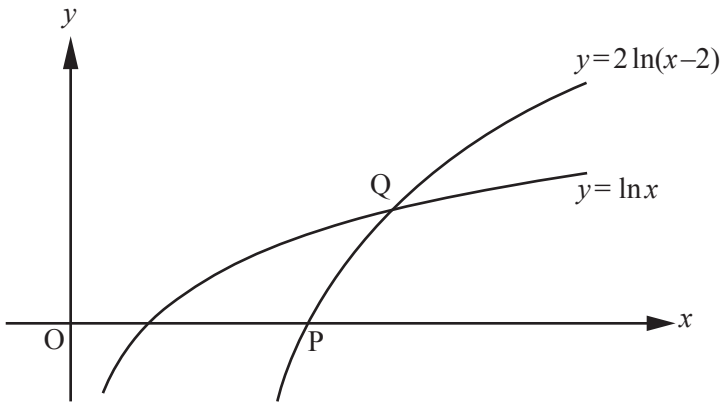

Fig. 9

(i) Describe a sequence of two transformations which maps the curve $y = \ln x$ onto the curve $y = 2\ln(x-2)$. **[3]**

(ii) Find the exact coordinates of P and Q. **[5]**

(iii) Using integration by parts, show that $\int \ln x \, dx = x\ln x - x + c$, where c is an arbitrary constant. **[3]**

(iv) Hence show that the area of the finite region enclosed by the curve $y = \ln x$, the curve $y = 2\ln(x-2)$ and the x-axis is $m\ln 2 + n$, where m and n are integers to be determined. **[7]**

END OF QUESTION PAPER

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