OCR
Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

DC (LK/FC) 150693/3

Section A (36 marks)

1 Express $\sin\theta - 2.4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the maximum value of the function $f(\theta) = 1 - \sin\theta + 2.4\cos\theta$, where $0 \leqslant \theta \leqslant 2\pi$. **[5]**

2 The finite region bounded by the curve $y = \ln x$, the x-axis, the y-axis and the line $y = 1$ is rotated through 360° about the y-axis. Find the exact volume of the solid of revolution generated. **[4]**

3 Find the first three terms of the binomial expansion of $\dfrac{1+2x}{(2-x)^3}$ in ascending powers of x.

State the set of values of x for which the expansion is valid. **[7]**

4 A curve has parametric equations $x = \sin 2\theta$, $y = 1 + 2\cos\theta - \cos 2\theta$, where $0 < \theta < \pi$.

(i) Find $\dfrac{dy}{dx}$ in terms of θ. **[3]**

(ii) Find the exact coordinates of the point on the curve where the gradient is zero. **[4]**

5 Fig. 5 shows the curve with equation $y = \sqrt{1+x^3}$.

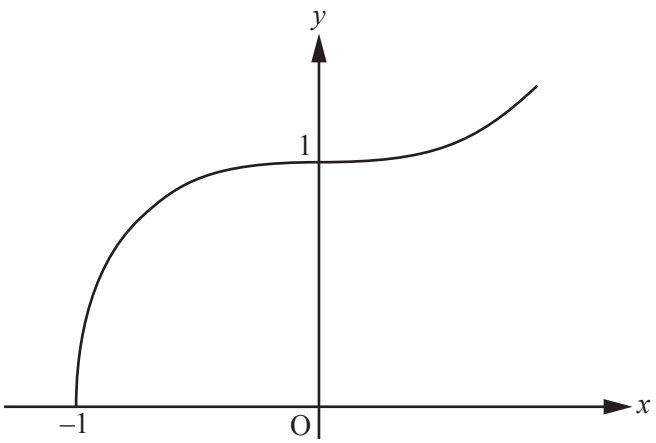

Fig. 5

(i) Use the trapezium rule with 4 strips to estimate the finite area enclosed by the curve and the x- and y-axes, giving your answer correct to 3 significant figures. **[3]**

(ii) Use a quarter circle of radius 1 to estimate this area, giving your answer correct to 3 significant figures. **[1]**

(iii) State, with a reason, which of these estimates is closer to the true area. **[1]**

6 In Fig. 6, triangle ADC is right-angled at C, with CD = h. The point B on AC is such that AB = x, angle ADB = α and angle BDC = β.

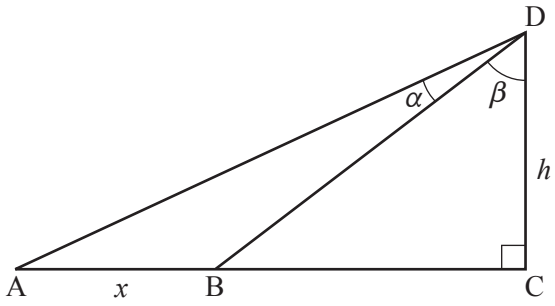

Fig. 6

(i) Find BC and AC in terms of h, α and β.

Hence show that $x = \dfrac{h\tan\alpha\sec^2\beta}{1-\tan\alpha\tan\beta}$. **[5]**

(ii) Given that $x = h$ and $\beta = 30°$, find α, giving your answer correct to 1 decimal place. **[3]**

Section B (36 marks)

7 Three points A, B and C have coordinates A (2, 1, 1), B (1, –3, –1) and C (–4, –1, 0).

(i) Find the lengths AB and AC, and use a scalar product to calculate the angle BAC.

Hence find the area of triangle ABC. **[7]**

The lines with vector equations

$$\mathbf{r} = 2\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}-\mathbf{k}), \quad \mathbf{r} = \mathbf{i}-3\mathbf{j}-\mathbf{k}+\mu(-\mathbf{i}+3\mathbf{j}+3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -4\mathbf{i}-\mathbf{j}+\nu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$$

pass through the points A, B and C respectively.

(ii) Show that these three lines meet at a point D. **[6]**

You are given that the plane ABC has equation $\mathbf{r}.(\mathbf{j}-2\mathbf{k}) = -1$. The normal through D to the plane ABC meets the plane at E.

(iii) Find the coordinates of E. **[3]**

The volume of a tetrahedron is $\frac{1}{3}\times$ area of base $\times$ height.

(iv) Find the volume of the tetrahedron ABCD. **[3]**

8 The speed $v\,\text{m}\,\text{s}^{-1}$ of an object at time t seconds is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -kv(4+v^2),$$

where k is a positive constant. Initially, $v = 4$.

(i) Find constants A, B and C such that $\dfrac{1}{v(4+v^2)} = \dfrac{A}{v} + \dfrac{Bv+C}{4+v^2}$. **[5]**

(ii) Hence show by integration that $v = \dfrac{4}{\sqrt{5\mathrm{e}^{8kt}-4}}$. **[9]**

(iii) After 1 second the speed of the object is $2\,\text{m}\,\text{s}^{-1}$. Find the value of k. **[3]**

END OF QUESTION PAPER