OCR
Oxford Cambridge and RSA

Wednesday 6 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 The equations of two planes, P and Q, are as follows.

$$\begin{aligned} P: &\quad x + 2y - 2z = 11 \\ Q: &\quad 2x - y + z = 7 \end{aligned}$$

The planes intersect in the line L.

(i) Find a cartesian equation for L. **[5]**

(ii) The point with coordinates $(a, 1, 1)$ is equidistant from P and Q. Find the two possible values of a. **[4]**

The points B and C have coordinates (1, 2, 7) and (1, 0, −5) respectively.

(iii) Show that B lies on Q but not P and that C lies on P but not Q. Explain why this means that the lines BC and L are skew. **[2]**

(iv) Find the shortest distance between the lines BC and L. **[5]**

The point E is the mirror image of C in the plane Q, and O is the origin.

(v) Find the volume of the tetrahedron OBCE. **[8]**

Option 2: Multi-variable calculus

2 The surface S has equation $z = x + 4x^2y - 2y^2 + 2$.

(i) Show that the tangent plane to the surface at the point (1, 1, 5) has equation $z = 9x - 4$. **[5]**

(ii) Show also that if a tangent plane to S has equation $z = 9x + k$ then the only possible value for k is −4. **[3]**

(iii) A point on the surface has coordinates $(1 + a, 1 + a, 5 + b)$ where a and b are small. Show that $b \approx \lambda a$, where λ is a constant to be determined. **[3]**

(iv) Find the coordinates of the points on the surface at which the normal line is parallel to the vector $\mathbf{i} + 16\mathbf{j} - \mathbf{k}$. **[4]**

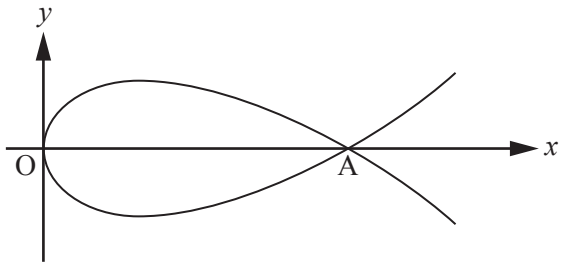
(v) Show that the only stationary point, A, on S has coordinates $\left(-\frac{1}{2}, \frac{1}{4}, \frac{13}{8}\right)$.

By finding the cross-sections through A parallel to $x = 0$ and $y = 0$ respectively, determine the nature of this stationary point. **[9]**

Option 3: Differential Geometry

3 **(a)** Prove by integration that the surface area of a sphere with radius a is given by $S = 4\pi a^2$. **[6]**

(b) A curve has parametric equations $x = 6t^2$, $y = 4t - 3t^3$. The curve crosses the x-axis at the origin O and at the point A, as shown in the diagram.



Find

(i) the values of t at the point A, **[2]**

(ii) the length of the arc OA for which t is positive, **[6]**

(iii) the radius and centre of curvature at the point where $t = \frac{1}{3}$. **[10]**

Option 4: Groups

4 You are given that the set {1, 2, 4, 7, 8, 11, 13, 14} together with the binary operation of multiplication modulo 15 forms a group G.

(i) Find the order of each element of G. **[4]**

(ii) (*A*) A subgroup of G has order n. Write down the possible values of n. **[2]**

(*B*) State all the proper cyclic subgroups of G. **[4]**

(iii) For each of the following three cases, determine whether the set together with the binary operation forms a group. If the set does form a group, state whether or not it is isomorphic to G, justifying your answer. (You may assume that each of the binary operations is associative.)

(*A*) The set {0, 1, 2, 3, 4, 5, 6, 7} together with the binary operation of addition modulo 8.

(*B*) The set {1, 2, 3, 4, 5, 6, 7, 8} together with the binary operation of multiplication modulo 9.

(*C*) The set of matrices

$$\left\{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}, \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}, \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}, \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\right\}$$

together with the binary operation of matrix multiplication. (You may assume that the set is closed under matrix multiplication.) **[14]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 At a factory there are four security lights, A, B, C and D, only one of which is on at any time; which light is on at any time can be randomised by control equipment. A number of programs are devised so that the lights switch from one to another every minute with certain probabilities. For example, if A is on then at the next minute one of B, C or D will come on, with probabilities determined by the particular program being used.

The time after the start of a program is denoted by t minutes. Light A comes on when $t = 0$.

For program 1 the transition matrix is as follows.

$$\mathbf{P}_1 = \begin{pmatrix} 0 & 0.2 & 0.1 & 0.4 \\ 0.2 & 0 & 0.6 & 0.4 \\ 0.3 & 0.4 & 0 & 0.2 \\ 0.5 & 0.4 & 0.3 & 0 \end{pmatrix}$$

The four rows and columns correspond to lights A, B, C, D in that order.

(i) Interpret the values in the leading diagonal, stating the run length for each light. **[2]**

(ii) Find the probabilities that A comes on at $t = 1$, 2, 3 and 4. **[4]**

(iii) The equilibrium probability for A is a. From your working in part **(ii)**, write down a range within which a lies. **[2]**

(iv) Find the probability that the light that comes on at $t = 5$ is different from the light that comes on at $t = 1$. **[5]**

For program 2 the following rules apply.

- The light following A is always B.
- The light following B is never D and is equally likely to be A or C.
- The light following C is never A and is equally likely to be B or D.
- The light following D is always C.

(v) Write down the transition matrix, $\mathbf{P}_2$, for program 2. **[2]**

(vi) For program 2 identify any absorbing states and reflecting barriers. **[3]**

(vii) Find the proportions of times that each light is on over a long period. Give your answers as exact fractions. **[6]**

END OF QUESTION PAPER