

Tuesday 26 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4773/01 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:

- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the barcodes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

• Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.



1 A "rugby sevens" team has to be chosen from 9 rugby players. The players have differing physiques and skills, which fit them for different positions, as shown in the table.

player	А	В	C	D	Е	F	G	Н	Ι
1 – tight head prop	1	1			1				
2 – hooker		1						1	
3 – loose head prop	1	1			1				
4 – scrum half			1	1			1		
5 – fly half			1				1		
6 – centre			1			1		1	1
7 – wing			1						1

(i) Represent this information in a bipartite graph.

The team manager starts to pick a team by choosing A to be tight head prop, B to be loose head prop, C to be wing, G to be scrum half and H to be centre.

(ii) Show this incomplete matching as a bipartite graph, and apply the matching algorithm to produce a complete matching. [7]

The team manager holds training sessions during which he assesses each player on a scale from 1 to 5 for each potential position, with 5 being the top score and 1 the lowest score. These scores are shown below.

player	А	В	C	D	Е	F	G	Н	Ι
1 – tight head prop	4	4			3				
2 – hooker		3						2	
3 – loose head prop	4	4			3				
4 – scrum half			5	4			4		
5 – fly half			5				4		
6 – centre			5			4		3	5
7 – wing			5						4

(iii) Formulate as a linear programming problem the problem of choosing the best team.	[5]
(iv) Run your LP and interpret the solution.	[4]

[2]

- 2 A mathematical model of an epidemic has the following features:
 - A Individuals are either susceptible, infective or recovered.
 - B A susceptible individual can be infected when in contact with an infective individual. The susceptible then becomes an infective.
 - C Infectives can recover. Recovered individuals are no longer infective or susceptible.

A simulation follows the following additional rules:

- D In any hour at most one susceptible individual can become infective, and at most one infective individual can recover.
- E The probability of an infection taking place during an hour is $\lambda \times$ the number of susceptible individuals at the beginning of the hour \times the number of infective individuals at the beginning of the hour, where λ is a constant.
- F The probability of a recovery taking place during an hour is $\beta \times$ the number of infective individuals at the beginning of the hour, where β is a constant.

Let the number of infective individuals at time *t* (hours) be f(t) and the number of susceptible individuals at time *t* (hours) be s(t). So, in the simulation, if a random number between 0 and 1 is less than $\lambda \times f(t) \times s(t)$, then there is a new infection in the hour beginning at time *t* hours.

The number of recovered individuals is modelled in a similar way, but following the rule given in F.

Ten people share a hut in a remote area, cut off by bad weather. Two are infective and the other eight are susceptible.

- (i) Build an Excel simulation model for the hour-by-hour progress of the epidemic, allowing the input of specific values for λ and β . [8]
- (ii) Run your simulation with $\lambda = 0.04$ and $\beta = 0.2$ until f(t) = 0, noting the time at which this happens and the value of s(t) at that time. [2]
- (iii) Repeat part (ii) 19 more times. Summarise and interpret your 20 results. [5]
- (iv) Compare and contrast what happens if a medicine is available which reduces λ to 0.01 and increases β to 0.3. [2]
- (v) Describe how to improve the simulation so that it more closely approximates to reality. [1]

- 3 Soil temperatures are measured at 10 cm beneath the surface at a number of locations at an agricultural research institute. It is suggested that the daily mean soil temperature across these sites can be modelled by a recurrence relation of the form $T_n = aT_{n-1} + bT_{n-2} + c$, where T_n is the mean soil temperature at the institute on day *n*.
 - (i) Solve the recurrence relation in the case a = 0.8, b = 0.2 and c = 0, i.e. find an expression in terms of *n* which will give the value of T_n . Start with $T_1 = 7$ and $T_2 = 8$. [8]
 - (ii) Construct a spreadsheet to show that your formula is correct for the first 20 terms. [3]

[1]

(iii) Give the exact value to which the model temperature converges.

The "c" term allows for a daily increase in soil temperature if the weather is warm, and a daily decrease if the weather is cold.

- (iv) Use your spreadsheet from part (ii) with a = 0.8 and b = 0.2, to investigate what happens to the modelled soil temperature if c is constant at a value of 0.1 over a period of 20 days. Describe what happens to the temperature differences. [2]
- (v) Adapt your answer to part (i) to give a formula in terms of *n* which will give the value of T_n when c = 0.1 and when *n* is greater than 15, correct to the accuracy of your spreadsheet. [1]
- (vi) Find values of a, b, and c which lead to modelled values which are close to the following observed values:[3]

п	1	2	3	4	5	6	7	8	9	10
T _n	7	8	7.8	8.08	8.17	8.33	8.47	8.61	8.75	8.90

n	11	12	13	14	15	16	17	18	19	20
T _n	9.04	9.18	9.33	9.47	9.61	9.76	9.90	10.04	10.18	10.33

4 A water supply network is shown in the diagram. The arcs represent pipes and the weights on the arcs represent pipe capacities. The pipe connecting C and F is undirected.



Pumps at A and D deliver water into the network. Pump A has capacity 40 and pump D has capacity 40. Water is extracted at B and E. B has a demand of 50 and E has a demand of 30.

- (i) Add a super source and a super sink with associated arcs to model this situation. [2]
- (ii) Find a set of feasible flows which give a total flow of 70 through the network, and prove that this is a maximal flow. Say which demand cannot be satisfied. [3]
- (iii) Construct a linear program to model flows through the network. [5]
- (iv) Run your program, interpret your results, and check that the maximal total flow is as per your answer to part (ii).
- (v) Change your program to model the effect of decreasing the demand at B to 30 and increasing the demand at E to 50, and report on the result. [2]

New pumps are installed at A and D, each with capacity 50. Demands at B and E are now each 50.

(vi) Give 3 separate modifications to individual pipes, any one of which will allow the demands to be met. In each case give a set of feasible flows which satisfy the demands.[3]

END OF QUESTION PAPER

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