OCR
Oxford Cambridge and RSA

Tuesday 26 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

* 7 3 8 2 7 4 3 1 3 4 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 This question concerns curves with parametric equations

$$x = \frac{2t}{1+t^n}, \; y = \frac{2t^2}{1+t^n},$$

where n is a positive integer.

(i) Sketch the curves for the cases $n = 1$, $n = 2$, $n = 3$ and $n = 4$.

State one feature of the curves present when $n = 2$ and when $n = 4$ but not present when $n = 1$ or when $n = 3$. **[6]**

(ii) For the case $n = 2$ verify algebraically that the curve is a circle. **[3]**

(iii) For the case $n = 1$ find a cartesian equation of the curve and hence find the equations of the asymptotes. **[6]**

(iv) For the case $n = 3$ show that $x^3 + y^3 = kxy$ is a cartesian equation of the curve and find the value of k.

Show that there is no point on the curve corresponding to $t = -1$ but that $\frac{\mathrm{d}y}{\mathrm{d}x} \to -1$ as $t \to -1$.

Find the value of c such that $x + y = c$ is the asymptote to the curve. **[10]**

2 **(i)** Find the roots of the equation $\cosh 2z = 1.05 + 0.1\mathrm{i}$ for which $-\frac{1}{2}\pi < \mathrm{Im}(z) < \frac{1}{2}\pi$. **[3]**

(ii) Give the first three non-zero terms of the Maclaurin expansion for $\cosh 2z$. Hence show that, when z is small, the equation $\cosh 2z = w$ can be written as $z^2 \approx \frac{w-1}{2}$.

Find the errors in the real and imaginary parts of z when using $z^2 = \frac{w-1}{2}$ to find approximations to the roots of the equation in part **(i)**. **[6]**

(iii) The function $1 + 2z^2$ is used to approximate $\cosh 2z$ for $z = a + 0.2\mathrm{i}$ where $a > 0$. Construct a spreadsheet to calculate the error in the real part of this approximation for values of a from 0.01 to 0.5 in steps of 0.01. State the formulae you have used in your spreadsheet.

Use your spreadsheet to find for which of these values of a the real part of $1 + 2z^2$ exceeds the real part of $\cosh 2z$ by the greatest amount. **[6]**

(iv) Find the real and imaginary parts of $\cosh^2(x + \mathrm{i}y)$ and $\sinh^2(x + \mathrm{i}y)$, where $x, y \in \mathbb{R}$. Hence show that $\cosh(2x + 2\mathrm{i}y) = \cosh^2(x + \mathrm{i}y) + \sinh^2(x + \mathrm{i}y)$.

On an Argand diagram sketch the locus of points where $\cosh(2x + 2\mathrm{i}y)$ is real. **[9]**

3 **(i)** Create a program to find all the positive integer solutions of $x^2 - 17y^2 = 1$ with $x \leqslant 500$, $y \leqslant 500$.

Write out your program in full and list any solutions it gives. **[6]**

(ii) Edit your program from part **(i)** so that it will find all the positive integer solutions of $x^2 - 17y^2 = -1$ with $x \leqslant 500$, $y \leqslant 500$.

State the changes to your program and the solutions it gives. **[3]**

(iii) Show algebraically that if $x^2 - ny^2 = -1$ then $(x^2 + ny^2)^2 - n(2xy)^2 = 1$.

Hence, using output from part **(ii)**, find a solution of $x^2 - 17y^2 = 1$ for which x and y are both greater than 500. **[7]**

(iv) By considering the possible values of $m^2 \pmod 4$, show that the equation $x^2 - ny^2 = -1$ has no integer solutions when $n \equiv 0 \pmod 4$ or when $n \equiv 3 \pmod 4$. **[7]**

END OF QUESTION PAPER