OCR
Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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Answer **all** the questions.

1 **(i)** Express $\frac{3}{2x+1}-\frac{2}{x+1}$ as a single algebraic fraction in its simplest form. **[2]**

(ii) Hence express $\left(\frac{3}{2x+1}-\frac{2}{x+1}\right)\left(\frac{6x+3}{x^2+x-2}\right)$ as a single algebraic fraction in its lowest terms. **[4]**

2 The equations of two lines are

$$\mathbf{r}=\begin{pmatrix}2\\-3\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\1\end{pmatrix} \quad \text{and} \quad \mathbf{r}=\begin{pmatrix}-4\\6\\-2\end{pmatrix}+\mu\begin{pmatrix}4\\-11\\3\end{pmatrix}.$$

(i) Explain why these lines are not parallel. **[1]**

(ii) Determine whether these lines are skew or whether they intersect. **[4]**

3 **(i)** Find the quotient and the remainder when $2x^3-3x^2+3x+2$ is divided by x^2-2x+1. **[3]**

(ii) Hence show that, if x is small,

$$\frac{2x^3-3x^2+3x+2}{1-2x+x^2}\approx a+bx+cx^2,$$

where a, b and c are constants to be determined. **[4]**

4 The parametric equations of a curve are

$$x=2\tan 2t \quad \text{and} \quad y=1+\tan t, \quad \text{where } -\tfrac{1}{2}\pi<t<\tfrac{1}{2}\pi.$$

(i) Show that a cartesian equation of the curve is $4y+xy^2-2xy=4$. **[3]**

(ii) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **[4]**

(iii) Find the gradient of the curve at the point where the curve intersects the y-axis. **[2]**

5 **(i)** Given that $u=\ln(\cos x)$, find $\frac{\mathrm{d}u}{\mathrm{d}x}$. **[2]**

(ii) Hence find $\int 2\cos 2x\ln(\cos x)\,\mathrm{d}x$. **[5]**

6 Use the substitution $u=1+2\sqrt{x}$ to find $\int\frac{\sqrt{x}}{1+2\sqrt{x}}\,\mathrm{d}x$. **[7]**

7 **(i)** Express $\dfrac{12-6x}{(1+x)(1-2x)^2}$ in partial fractions. **[5]**

(ii) Find $\displaystyle\int \frac{12-6x}{(1+x)(1-2x)^2}\,\mathrm{d}x$. Hence evaluate $\displaystyle\int_1^2 \frac{12-6x}{(1+x)(1-2x)^2}\,\mathrm{d}x$, giving your answer in the form $A-\ln B$, where A and B are integers to be determined. **[5]**

8 A is the point (1, 3, 1), B is the point (3, 2, 4) and P is the point (15, 4, 6). The point Q is on the line through A and B such that angle $AQP = 90°$.

(i) Write down a vector equation of the line through A and B. **[2]**

(ii) Find the coordinates of Q. **[5]**

(iii) Find the area of triangle AQP. **[3]**

9 When a container is partially filled with liquid to a depth of x centimetres, the volume $V\,\mathrm{cm}^3$ of liquid in the container is given by the formula

$$V = (x+1)^3 - 1.$$

Initially the container is empty. Liquid is poured into the container so that the rate at which V increases is directly proportional to e^{-t}, where t is the time in seconds since the addition of liquid began. When $t = 2$, the rate at which V is increasing is $10\,\mathrm{cm}^3\,\mathrm{s}^{-1}$.

(i) Show that $V = 10\mathrm{e}^2(1-\mathrm{e}^{-t})$. Hence find how long it takes for the depth of liquid in the container to reach 3 cm. **[8]**

(ii) Find the value that the depth of liquid approaches as t increases, giving your answer correct to 3 significant figures. **[3]**

END OF QUESTION PAPER