Oxford Cambridge and RSA

Monday 14 May 2018 – Afternoon

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 2k & -k \\ 0 & 1 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 3 & -3 \\ -2 & 4 \end{pmatrix}$, where k is a constant.

(i) Find, in terms of k, the matrix **AB**. **[2]**

(ii) Find the value of k for which matrix **AB** is singular. **[2]**

2 The quadratic equation $x^2 + px + q = 0$ has roots α and β, where

$$\alpha^2 + \beta^2 = -16,$$
$$\alpha - \beta = 6\mathrm{j}.$$

By considering $(\alpha - \beta)^2$, find the value of $\alpha\beta$. Hence state the value of q and find the possible values of p. **[5]**

3 **(i)** Sketch on an Argand diagram the set of points representing complex numbers z for which

$$|z - (3 + 3\mathrm{j})| = 3.$$ **[2]**

(ii) Find the greatest possible value of $|z|$ for this set of points. **[2]**

(iii) Mark on your Argand diagram the particular point for which $\arg\left(z - (3 + 3\mathrm{j})\right) = \frac{2}{3}\pi$. Find this value of z in the form $a + \mathrm{j}b$. **[3]**

4 **(i)** Use standard series formulae to show that

$$\sum_{r=1}^{n} r(2 + 3r) = \frac{1}{2}n(n+1)(2n+3).$$ **[4]**

(ii) Hence find the value of n such that

$$\sum_{r=1}^{4n} r(2 + 3r) = 198n(4n+1).$$ **[3]**

5 You are given that $z = 2 + 5\mathrm{j}$ is a root of the cubic equation $2z^3 - 5z^2 + pz + q = 0$, where p and q are real constants. Find the values of p and q. **[6]**

6 Prove by induction that, for all positive integers n, $\sum_{r=1}^{n} r2^r = 2\left[1 + (n-1)2^n\right]$. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{2x^2 - 5x - 3}{x^2 + x - 2}$.

(i) Find the values of x for which $y = 0$. **[2]**

(ii) Find the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. **[2]**

(iv) Sketch the curve. **[3]**

(v) Solve the inequality $\dfrac{2x^2 - 5x - 3}{x^2 + x - 2} \geqslant 0$. **[3]**

8 You are given that $\dfrac{1}{2r-1} - \dfrac{1}{2r+3} = \dfrac{4}{(2r-1)(2r+3)}$ for all integers r.

(i) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+3)} = k - \frac{n+1}{(2n+1)(2n+3)},$$

stating the value of k. **[6]**

(ii) The sum of the infinite series

$$\frac{1}{(2(n+1)-1)(2(n+1)+3)} + \frac{1}{(2(n+2)-1)(2(n+2)+3)} + \frac{1}{(2(n+3)-1)(2(n+3)+3)} + \dots$$

is $\frac{7}{195}$. Show that n satisfies $28n^2 - 139n - 174 = 0$ and hence find the value of n. **[5]**

9 You are given that $\mathbf{M} = \begin{pmatrix} 4 & a \\ -6 & -2 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} -2 & 6 \\ -4a & -14 \end{pmatrix}$, where a is a real constant. Find the possible value(s) of a in each of the following cases.

(i) The point (1, –2) is invariant under the transformation represented by matrix **M**. **[2]**

(ii) $(\mathbf{NM}^{-1})^{-1}\mathbf{NM} = \mathbf{N}$. **[4]**

(iii) A triangle T_1 has an area of 9 square units. The triangle T_1 is transformed to triangle T_2 by the transformation represented by matrix **M**. The area of triangle T_2 is 144 square units. **[6]**

END OF QUESTION PAPER