GCE

Mathematics (MEI)

Unit **4755**: Further Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

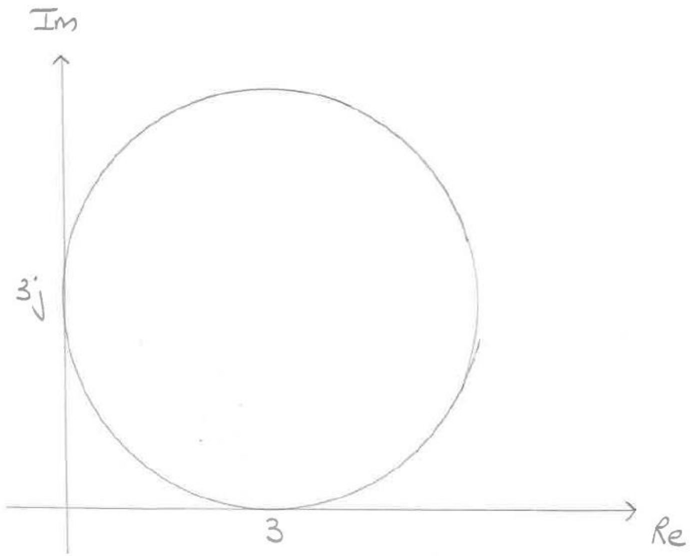
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

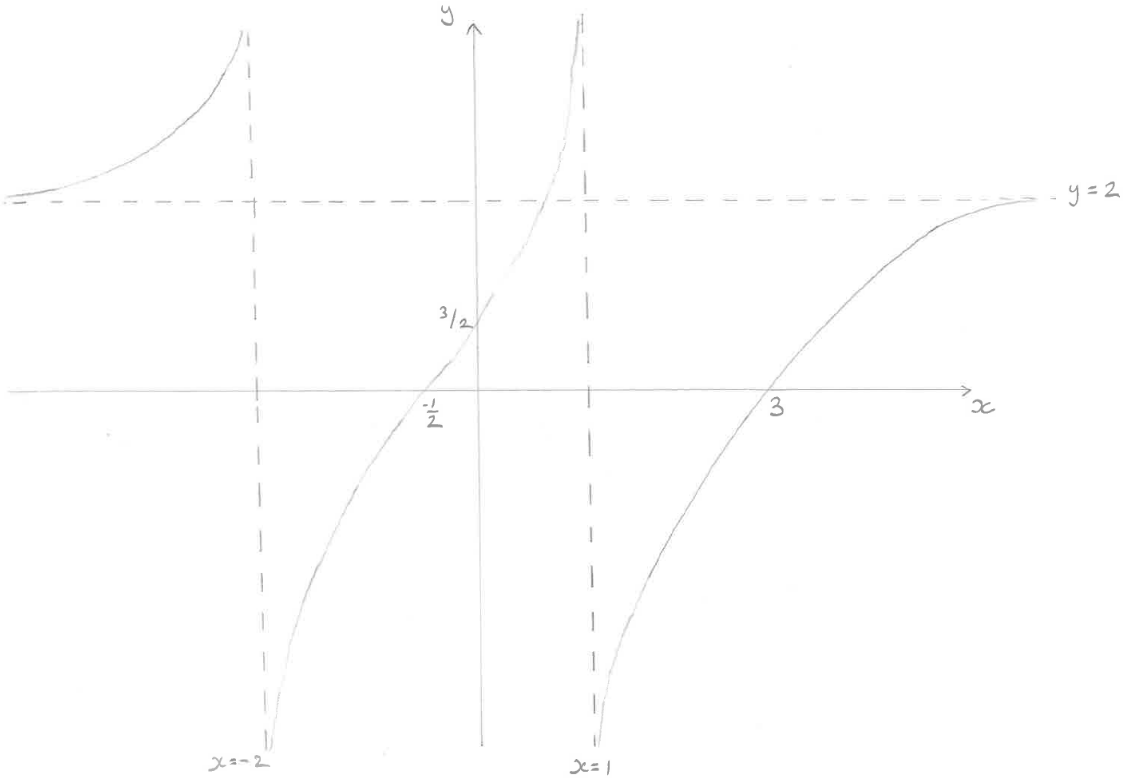
Question		Answer	Marks	Guidance
1	(i)	$\mathbf{AB}=\begin{pmatrix}2 & 2k & -k\\ 0 & 1 & -1\end{pmatrix}\begin{pmatrix}1 & 2\\ 3 & -3\\ -2 & 4\end{pmatrix}=\begin{pmatrix}2+8k & 4-10k\\ 5 & -7\end{pmatrix}$	B1 B1 **[2]**	B1 any two elements correct
1	(ii)	$(2+8k)(-7)-(5)(4-10k)=0$ $k=-\frac{17}{3}$ o.e.	M1 A1 **[2]**	Attempt at determinant and equate to zero FT their matrix
2		$(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2 \Rightarrow (6j)^2=-16-2\alpha\beta$ $\alpha\beta=10$ $q=10$ $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$ $(\alpha+\beta)^2=-16+20 \Rightarrow \alpha+\beta=\text{L}$ $p=\pm 2$	M1 A1 A1 M1 A1 **[5]**	Expand $(\alpha-\beta)^2$ and substitute given values FT their αβ Considers expansion of $(\alpha+\beta)^2$, substitutes values and solves for p (or other valid method) Both answers for p, FT their αβ

Question		Answer	Marks	Guidance
3	(i)		B1 B1 **[2]**	Circle Centre $3+3\mathrm{j}$ and radius 3 touching the axes
3	(ii)	$\lvert z\rvert_{\max} = 3+\lvert 3+3\mathrm{j}\rvert$ $=3+3\sqrt{2}$	M1 A1 **[2]**	3 + attempt at modulus of $\lvert 3+3\mathrm{j}\rvert$
3	(iii)	$\operatorname{Re} z = 3-3\sin\frac{\pi}{6}$ or $\operatorname{Im} z = 3+3\cos\frac{\pi}{6}$ $z=\frac{3}{2}+\left(\frac{3\sqrt{3}}{2}+3\right)\mathrm{j}$	B1 M1 A1 **[3]**	Correctly marking z on Argand diagram, $2\pi/3$ soi on diagram or implied in subsequent working M1 for complete method to find either the real or imaginary part of z

Question		Answer	Marks	Guidance
4	(i)	$\sum_{r=1}^{n} r(2+3r) = 2\sum_{r=1}^{n} r + 3\sum_{r=1}^{n} r^2$	M1	Separate into two sums (may be implied by later working)
		$= 2\left[\frac{n}{2}(n+1)\right] + 3\left[\frac{1}{6}n(n+1)(2n+1)\right]$	M1*	Use of standard results
		$= \frac{1}{2}n(n+1)[2+2n+1]$	M1dep*	Attempt to factorise with either ½ or $n(n+1)$
		$= \frac{1}{2}n(n+1)(2n+3)$ **(AG)**	E1	cao
			[4]	
4	(ii)	$2n(4n+1)(8n+3)$	B1	Correctly substitutes $4n$ for n in (i)
		$2n(4n+1)(8n+3) = 198n(4n+1) \Rightarrow 8n+3 = 99$	M1	Equate and attempt to solve for n
		$n = 12$	A1	Cao, extra solutions is A0
			[3]	
5		$\sum \text{roots} = (2+5\text{j}) + (2-5\text{j}) + \alpha = \frac{5}{2}$	B1	For using the root $2-5j$
			M1	Attempt at using sum of roots to find real root
		$\alpha = -\frac{3}{2}$	A1	
		$-\frac{q}{2} = (2-5\text{j})(2+5\text{j})\left(-\frac{3}{2}\right)$	M1	Attempt at using product of roots to find q
		$q = 87$		
		$\frac{p}{2} = (2-5\text{j})(2+5\text{j}) + (2+5\text{j})\left(-\frac{3}{2}\right) + \left(-\frac{3}{2}\right)(2-5\text{j})$	M1	Attempt at using $\sum \alpha\beta$ to find p
		$p = 46$	A1	For both p and q cao
		[OR After finding one of p, q substituting a root and solving to find the other]	[M1]	
			[6]	

Question		Answer	Marks	Guidance
		OR		
		$(z-2-5j)(z-2+5j)=z^2-4z+29$	B1 B1	For using 2 – 5j For quadratic factor
		$(z^2-4z+29)(az+b)$, $a\neq 1$ or division of cubic by quadratic $a=2, -8+b=-5 \Rightarrow b=3$ or $2z+3$	M1 A1	Quadratic × linear Correct linear factor
		$p=-4b+58, q=29b$ or by inspection of zero remainder $p=46, q=87$	M1 A1 **[6]**	Attempt to equate coefficients and solve for either p or q Both cao
		OR		
			M1	Attempt to expand both $(2+5\mathrm{j})^2$ and $(2+5\mathrm{j})^3$
		$(2+5\mathrm{j})^2=-21+20\mathrm{j}$	A1	soi
		$(2+5\mathrm{j})^3=-142-65\mathrm{j}$	A1	soi
		$-284-130\mathrm{j}+105-100\mathrm{j}+2p+5p\mathrm{j}+q=0 \Rightarrow 5p=230$ or $2p+q=179$	M1	Substitute and equate either real or imaginary parts
		$p=46$, $q=179-2(46)=87$	M1 A1 **[6]**	Use of other equation Both cao

Question		Answer	Marks	Guidance
6		When $n=1, \sum_{r=1}^{1} r2^r = 1(2^1) = 2$ and $2\left[1+(1-1)2^1\right] = 2(1) = 2$ so true for $n=1$	B1	
		Assume true for $n=k$ that is $\sum_{r=1}^{k} r2^r = 2\left[1+(k-1)2^k\right]$	E1	Assume true for $n=k$
		$(\sum_{r=1}^{k+1} r2^r =)2\left[1+(k-1)2^k\right]+(k+1)2^{k+1}$	M1	Add correct term
		$=2\left[1+(k-1)2^k+(k+1)2^k\right]$	M1	Attempt to obtain a factor of 2
		$=2\left[1+(2k)2^k\right]$ $=2\left[1+k2^{k+1}\right]$		
		$=2\left[1+((k+1)-1)2^{k+1}\right]$	A1	cao with correct simplification
		But this is the given result with $k+1$ replacing k. Therefore if it is true for $n=k$, it is true for $n=k+1$.	E1	Dependent on A1 and first E1
		Since it is true for $n=1$, it is true for $n=1,2,3,\ldots$ and so is true for all positive integers.	E1	Dependent on B1 and second E1
			[7]	
7	(i)	$2x^2-5x-3=0 \Rightarrow (2x+1)(x-3)=0$	M1	Sets numerator equal to zero and attempt to solve
		$x=3$ and $x=-\frac{1}{2}$	A1	
			[2]	
7	(ii)	horizontal asymptote: $y=2$	B1	Allow $y \neq 2$
		$x^2+x-2=0 \Rightarrow (x-1)(x+2)=0$	M1	Sets denominator equal to zero and attempt to solve
		vertical asymptotes: $x=1$ and $x=-2$	A1	A0 if $\neq$ has been used anywhere
			[3]	

Question		Answer	Marks	Guidance
7	(iii)	Some evidence of method needed e.g. substitute in 'large' values or argument involving signs Large positive x, $y \to 2^-$ and large negative x, $y \to 2^+$	M1 A1 **[2]**	
7	(iv)		B1 B1 B1 **[3]**	3 branches correct Asymptotes correct and labelled Intercepts correct and labelled
7	(v)	$x < -2, -\frac{1}{2} \le x < 1, x \ge 3$	B3 **[3]**	One mark for each. Correct inequality signs (B3 then –1 if more than 3 inequalities) SC B1 y used instead of x

Question		Answer	Marks	Guidance
8	(i)	$\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+3)}=\frac{1}{4}\sum_{r=1}^{n}\frac{1}{2r-1}-\frac{1}{2r+3}$	M1	Use of given result, at least three consecutive terms of the series, ignore ¼
		$=\frac{1}{4}\left[\left(1-\frac{1}{5}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{5}-\frac{1}{9}\right)+\ldots+\left(\frac{1}{2n-3}-\frac{1}{2n+1}\right)+\left(\frac{1}{2n-1}-\frac{1}{2n+3}\right)\right]$	A1 A1	First three terms correct Last two terms correct
		$\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+3)}=\frac{1}{4}\left(1+\frac{1}{3}-\frac{1}{2n+1}-\frac{1}{2n+3}\right)$	A1	For reduced terms after cancelling, ignore 1/4
		$=\frac{1}{3}-\frac{1}{4}\left(\frac{(2n+3)+(2n+1)}{(2n+1)(2n+3)}\right)=\frac{1}{3}-\frac{1}{4}\left(\frac{4n+4}{(2n+1)(2n+3)}\right)$	M1	Attempt to combine their algebraic fractions
		$=\frac{1}{3}-\frac{(n+1)}{(2n+1)(2n+3)}$ so $k=\frac{1}{3}$	E1 **[6]**	All correctly done
8	(ii)	Infinite series, $S_\infty=k$	B1	Sum to infinity equals their k soi
		$\frac{n+1}{(2n+1)(2n+3)}=\frac{7}{195}$	M1	Difference of S_∞ and S_n
		$28n^2-139n-174=0$ **AG**	E1	www
		$(28n+29)(n-6)=0$	M1	Attempt to solve quadratic
		$n=6$	A1 **[5]**	Withhold if $n=-29/28$ stated as final answer

Question		Answer	Marks	Guidance
9	(i)	$\begin{pmatrix} 4 & a \\ -6 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$	M1	$\mathbf{Mx} = \mathbf{x}$ with $\mathbf{x} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
		$4 - 2a = 1 \Rightarrow a = \frac{3}{2}$	A1 **[2]**	
9	(ii)	$\left(\mathbf{NM}^{-1}\right)^{-1} = \mathbf{MN}^{-1}$	M1*	Use of $\left(\mathbf{AB}\right)^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ or premultiply both sides by $\mathbf{NM}^{-1}$
		$\mathbf{M}^2 = \mathbf{N}$ or $\mathbf{M} = \mathbf{M}^{-1}\mathbf{N}$	A1	
		$\mathbf{M}^2 = \begin{pmatrix} 16-6a & 2a \\ -12 & 4-6a \end{pmatrix} \Rightarrow \begin{pmatrix} 16-6a & 2a \\ -12 & 4-6a \end{pmatrix} = \begin{pmatrix} -2 & 6 \\ -4a & -14 \end{pmatrix}$	M1dep*	Forming an equation in a
		$\mathbf{M}^{-1}\mathbf{N} = \frac{1}{6a-8}\begin{pmatrix} 4+4a^2 & 14a-12 \\ -12-16a & -20 \end{pmatrix} \Rightarrow \frac{1}{6a-8}\begin{pmatrix} 4+4a^2 & 14a-12 \\ -12-16a & -20 \end{pmatrix} = \begin{pmatrix} 4 & a \\ -6 & -2 \end{pmatrix}$		SC B1 Following M0 M0 for $\mathbf{NM}^{-1}$ or $\mathbf{MN}^{-1}$ correct
		$a = 3$	A1 **[4]**	
9	(iii)	$\det \mathbf{M} = 4(-2) - a(-6)$	M1	Attempt at determinant of $\mathbf{M}$ allow $ad \pm bc$
		$9(6a-8) = 144$ o.e.	M1	Sets up one equation with their determinant and correct use of 9 and 144
		$6a - 8 = 16 \Rightarrow a = 4$	A1 A1	A correct equation $a = 4$
		$9(6a-8) = -144$	M1	Sets up a second equation with their determinant and correct use of 9 and –144
		$6a - 8 = -16 \Rightarrow a = -\frac{4}{3}$ o.e.	A1 **[6]**	

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