OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4776**: Numerical Methods

Advanced Subsidiary GCE

Mark Scheme for June 2018

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a   Annotations should be used whenever appropriate during your marking.

         The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

         For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

         Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

         M
         A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

         A
         Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

         B
         Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

       If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			use of forward difference method	**M1**	eg $\dfrac{4.113250 - 2.718282}{2 - 1}$
			h *est*		
			1 1.394968	**A1**	accept to 4, 5 or 6 d.p.
			0.5 1.370032	**A1**	
			0.25 1.362212	**A1**	
			0.125 1.359960	**A1**	
			f'(1) ≈ 1.36 cao	**A1**	
				[6]	
2	**(i)**		$2^{10} \times 2^{-10} = 1$	**B1**	
			$2^{10} \div 2^{-10} = 1048576$	**B1**	
				[2]	
2	**(ii)**		0.999999488 to 0.9999995	**B1**	
			1048576.5 to 1048576.537	**B1**	
				[2]	
2	**(iii)**		0.000976562 is not an exact value (and the error introduced in multiplication & division (1024) is large enough to affect the seventh significant figure)	**E1**	
				[1]	

Question			Answer	Marks	Guidance
2	(iv)		loss of accuracy from manual entry only affects the tenth decimal place, so the displayed values are the same	**E1** **[1]**	
2	(v)		$\dfrac{0.000976562 - 2^{-10}}{2^{-10}} = -0.00000051$ relative error in $A \times B$ = rel err in A + rel err in B there is no error in 2^{10} so relative errors are the same	**B1** **E1** **[2]**	accept signs reversed
3	(i)		M_1 = 1.22 to 1.2247449 $T_1 = \frac{1}{2} \times \{1 + 1.5\}$ oe 1.25	**B1** **M1** **A1** **[3]**	
3	(ii)		1.2373724 to 1.24	**B1** **[1]**	
3	(iii)		1.2 cao	**B1** **[1]**	
3	(iv)		$h = 0.5$ S = 1.23 to 1.23316325 $h = 0.25$ S = 1.233 to 1.23315245 $I \approx 1.2332$	**B1** **B1** **B1** **[3]**	 accept 1.23315

Question	Answer	Marks	Guidance
4 (i)	$(x+5000)^2 - 5000^2 + 1[=0]$ $(x+5000)^2 = 25000000 - 1$ $x = -5000 + \sqrt{24\ 999\ 999}$	M1 A1 [2]	completing the square at least one correct intermediate step needed **NB AG**	
4 (ii)	$\dfrac{-10000 + \sqrt{10000^2 - 4 \times 1 \times 1}}{2 \times 1}$ oe seen and completion to given result $\dfrac{-10000 + \sqrt{99999996}}{2}$	B1 [1]	 **NB AG**	
4 (iii)	both calculations involve the difference of nearly equal numbers: accuracy is lost due to the error in the square root in each case	E1 [1]		
4 (iv)	$\dfrac{-10000 + \sqrt{99999996}}{2} \times \dfrac{-10000 - \sqrt{99999996}}{-10000 - \sqrt{99999996}}$ and completion to $\dfrac{2}{\left(-10000 - \sqrt{99999996}\right)}$ -0.0001	 B1 B1 [2]	 **NB** -0.000100000001	or product of roots $= 1$

	Question		Answer	Marks	Guidance	
5	**(i)**		use of correct formula e.g. $\dfrac{0.5f(0.2)-0.2f(0.5)}{f(0.2)-f(0.5)}$ -3.17 to -3.17257 $\log_{10}(-3.17)$ is undefined so it is not possible to continue the iteration **[3]**	**M1** **A1** **E1**	NB f(0.5) = $-$ 1.19897 and f(0.2) = $-$ 1.10103	
5	**(ii)**		use of correct formula 0.010344, 0.010193, 0.010239, [0.010239] 0.01024 cao **[3]**	**M1** **A1** **A1**	e.g. $\dfrac{0.01f(0.02)-0.02f(0.01)}{f(0.02)-f(0.01)}$ At least two iterations	**NB** f(0.01) = 0.01 and f(0.02) = $-$ 0.28103
5	**(iii)**		e.g. it is generally faster to converge e.g. it is easier to automate as there is no sign check at each stage **[2]**	**B1** **B1**		

6	**(i)**		-2
			   3
			 1 10
			   13 48
			 14 58
			   71
			 85
			
			[3]
				M1
				
				A2
					A1 if one error, **A0** if two or more errors	

Question			Answer	Marks	Guidance
6	(ii)		$$-2+\dfrac{x\times 3}{1}+\dfrac{x(x-1)\times 10}{2!}+\dfrac{x(x-1)(x-2)\times 48}{3!}$$	**M2**	**M1** for first three terms
			$f(x)=8x^3-19x^2+14x-2$	**A3**	**A2** for three terms correct, **A1** for two terms correct
				[5]	
6	(iii)	**A**	differentiation of their cubic f(x)	**M1**	**NB** $24x^2-38x+14$
			evaluation of f'(2.5)	**M1**	**FT** their f(x)
			69 cao	**A1**	
				[3]	
6	(iii)	**B**	integration of their cubic f(x)	**M1**	**NB** $F[x]=2x^4-\dfrac{19}{3}x^3+7x^2-2x$	ignore "+ c"
			evaluation of F[3]	**M1**	**FT** their f(x)
			48 cao	**A1**	
				[3]	
6	(iv)		Evaluating f(4) 262	**B1**	**FT** their f(x)
			new data used to find another third difference	**M1**	
			(third difference is 144)		
			so relationship not cubic for all data (although it may be approximately cubic over first interval)	**E1**	
			estimates therefore (probably) unreliable	**E1**	
				[4]	

Question			Answer	Marks	Guidance
7	(i)		f(–2) = –16 and f(–1) = 9	B1	
			sign change so there is a root in this interval	E1	
				[2]	
7	(ii)		f'(x) = 5x⁴ – 6 seen	B1	
			– 1, 8, 6.401680180,5.1245293, 4.1056074, 3.2955892,…	M1	evidence of N-R iteration used to obtain the first three iterates
			converges to 1.310(2349)	A1	
				[3]	
7	(iii)		$\begin{array}{r} -1 \\ -1.5848932 \\ -1.683166 \\ -1.6976089 \\ -1.6996906 \\ -1.6999898 \\ -1.7000328 \\ -1.7000389 \\ -1.7000398 \end{array}$	M1 A1	use of correct formula first four iterates correct to 3 d.p. or more
			–1.700040 cao	A1	
				[3]	
7	(iv)		convergence of the iteration is first order	B1	
			because the ratio of differences is (more or less) constant	E1	
				[2]	

Question			Answer			Marks	Guidance	
7	(v)		x_r	differences	ratio	B1	first estimate	
			0					
			0.6666666667	0.6666666667	0.03940887	M1	N-R iteration. At least 4 values.	
			0.6929392447	0.02627258	0.01670515			
			0.6933781321	0.00043889	0.0003019			
			0.6933782645	0.00000013				
			0.6933782645					
			0.69337826 cao			A1		
			differences			M1		
			ratios			M1		
			all correct to 3 or more sf			A1		
						[6]		
7	(vi)		the ratio is decreasing rapidly,			B1		
			which shows that the convergence is faster than 1st order			E1		
						[2]		

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