

**GCE**

**Mathematics (MEI)**

Unit **4751**: Introduction to Advanced Mathematics (C1)

Advanced Subsidiary GCE

**Mark Scheme for June 2018**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## Annotations and abbreviations

<b>Annotation in scoris</b>	<b>Meaning</b>
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
<b>Other abbreviations in mark scheme</b>	<b>Meaning</b>
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

**Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

- a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.





Question		Answer	Marks	Guidance
5		$3x + 2(4x + 3) = 9$  $11x = 3$  $(3/11, 45/11 \text{ oe})$	M1  M1  A2  <b>[4]</b>	for subst to eliminate one variable; condone one error; or for multn or divn of one or both eqns to get a pair of coeffs the same, condoning one error  for collecting terms and simplifying; condoning one error ft or for appropriate addn or subtn to eliminate a variable, condoning an error in one term;  or $x = 3/11, y = 45/11$ oe isw  allow <b>A1</b> for each coordinate  each M1 is for a correct, constructive step  for this M mark, ft for equiv difficulty
6		selecting the term in $\left(\frac{1}{x}\right)^3 (-3x)^3$ ${}^6C_3 = 20$ soi  -540	M1  M1  A1  <b>[3]</b>	condone wrong or omitted brackets; may be implied by $(-3)^3$ or by -27 selected may be part of a fully correct row in Pascal's triangle  allow <b>B2</b> for -540 included as part of expansion but not identified as required term – ignore errors in other terms  allow <b>B3</b> for -540 obtained and identified by multiplying out brackets, ignoring errors in other terms, otherwise M0 for this method  mark to advantage of candidate if choice eg if correct row seen but wrong element selected
7	(i)	$17\sqrt{7}$	2  <b>[2]</b>	<b>M1</b> for $\sqrt{28} = 2\sqrt{7}$ soi or for $\sqrt{175} = 5\sqrt{7}$



Question		Answer	Marks	Guidance
9		$-a - c = 3$ $64a + 4c = 108$ Correct method for eliminating one variable, condoning one further error $a = 2, c = -5$	B1 B1 M1 A2  <b>[5]</b>	accept $(-1)^3 a$ instead of $-a$ accept $(4)^3 a$ instead of $64a$ dep on two equations in a and c and at least B1 earned <b>A1</b> for one correct if M0 but $a$ and $c$ both correct, allow <b>SC1</b>  may also be obtained after long division etc
10	(i)	$3(x - 1.5)^2 - 1.75$ oe in fractions, www  Line of symmetry is $x = -$ their $b$  min $y =$ their $c$	B4  B1  B1  <b>[6]</b>	<b>B1</b> for each of $a = 3, b = -1.5$  and <b>B2</b> for $c = -1.75$ or <b>M1</b> for $5 - 3 \times 1.5^2$ or ft soi or for $5/3 - 1.5^2$ or ft soi  must ft; if correct, $x = 1.5$  must ft; if correct, $y = -1.75$  <b>B0</b> for just min pt = $(1.5, -1.75)$ oe statement needed not just sketch with $-1.75$ marked on y-axis  ignore ' $= 0$ ' if brackets are there, condone missing power of 2  for last two B marks, do not allow those starting again since not 'hence'

Question		Answer	Marks	Guidance
10	(ii)	<p>intersects y-axis at (0, 5)</p> <p>intersects x-axis at <math>\left(\frac{9 \pm \sqrt{21}}{6}, 0\right)</math> or</p> <p><math>\left(\frac{3}{2} \pm \sqrt{\frac{7}{12}}, 0\right)</math></p> <p>or <math>x = \frac{9 \pm \sqrt{21}}{6}</math> or <math>x = \frac{3}{2} \pm \sqrt{\frac{7}{12}}</math></p> <p><math>\frac{9 - \sqrt{21}}{6} &lt; x &lt; \frac{9 + \sqrt{21}}{6}</math> or ft</p>	<p>B1</p> <p>B2</p> <p>B1</p> <p>[4]</p>	<p><b>M1</b> for quadratic formula used or ft from their completing the square in (i), with at most one error;</p> <p>condone answers not in coordinate form; isw after correct <math>x</math> values obtained</p> <p>ft only for soln using surds; allow ft from wrongly simplified surds;</p> <p>do not accept two separate inequalities</p>
11	(i)	<p>graph of cubic correct way up</p> <p>crossing x-axis at <math>-5/2, 1</math> and <math>4</math></p> <p>crossing y-axis at <math>20</math></p>	<p>B1</p> <p>B2</p> <p>B1</p> <p>[4]</p>	<p><b>B0</b> if stops at <math>x</math>-axis</p> <p>on graph or nearby; may be in coordinate form;</p> <p><b>M1</b> for <math>x^2 - 5x + 4 = (x - 4)(x - 1)</math> or for roots <math>4</math> and <math>1</math> found</p> <p>mark intent for intersections with both axes</p> <p>or <math>x = 0, y = 20</math> seen if consistent with graph drawn</p> <p>must not have any ruled sections; no curving back; condone slight 'flicking out' at ends but not approaching another turning point; allow max on <math>y</math>-axis or in 1st or 2nd quadrants; condone some 'doubling' or 'feathering' (deleted work still may show in scans)</p> <p>allow if no graph, but marked on <math>x</math>-axis</p> <p>condone intercepts for <math>x</math> and / or <math>y</math> given as reversed coordinates</p> <p>allow if no graph, but eg B0 for graph with intn on <math>y</math>-axis nowhere near their indicated <math>20</math></p>





Question		Answer	Marks	Guidance	
12	(iii)	$(x - 2)^2 + (2x + k + 1)^2 = 50$	M1	condone one error, eg omission of +1, but $k$ must be included	eg allow M1 for $5x^2 + 4kx + k^2 - 45 [= 0]$  0 for just 'discriminant = 0' unless implied by later work  can be earned in formula (ignore rest of formula)
		$5x^2 + 4kx + k^2 + 2k - 45 [= 0]$	M1	condone one error; accept constant term $(k + 1)^2 - 46$ ; must be rearranged to '=0' stage unless they go on to complete the square  <b>M0</b> if wrong eqn used – no ft from original error, only condone one error from working with correct eqn	
		$b^2 - 4ac = 0$ oe soi	M1	may be earned near end allow for this condition quoted, even if then applied to wrong equation. It is sometimes earned at beginning	
		$(4k)^2 - 4 \times 5 \times (k^2 + 2k - 45)$	M1	for correct substitution ft into $b^2 - 4ac$ , dep on first M1 earned; brackets / signs must be correct	
		correct simplification to given answer $k^2 + 10k - 225 = 0.$	A1	NB mark working not answer	
			[5]		

Question		Answer	Marks	Guidance	
12	(iii)	<p><b>method 2</b> line perp to <math>y = 2x + k</math> through centre is <math>y = -\frac{1}{2}x</math> oe</p> <p>finding intersection with <math>y = 2x + k</math> [if correct, <math>x = -\frac{2}{5}k, y = \frac{1}{5}k</math> ]</p> <p><math>\left(2 + \frac{2}{5}k\right)^2 + \left(-1 - \frac{1}{5}k\right)^2 = 50</math> oe</p> <p>correct simplification to given answer <math>k^2 + 10k - 225 = 0</math>.</p>	<p><b>or</b> M1</p> <p>M1</p> <p>M1</p> <p>A2</p> <p>[5]</p>	<p>condone attempt <math>y = -\frac{1}{2}x + n</math>, with <math>n \neq 0</math></p> <p>allow for finding intn of <math>y = 2x + k</math> and line with grad <math>-\frac{1}{2}</math> but error in constant</p> <p>for correct substitution ft into circle equation, dep on first M1 earned; brackets / signs must be correct</p> <p>NB mark working not answer; <b>A1</b> for correct expansion of brackets or correctly eliminating fractions as first step, working with correct equation only</p>	<p>M0 for just <math>y = -\frac{1}{2}x + c</math> with no attempt to subst <math>(2, -1)</math> to find <math>c</math></p> <p>using distance from centre = radius, or point of intersection being on circle</p> <p>A1 for <math>4 + \frac{8k}{5} + \frac{4k^2}{25} + 1 + \frac{2k}{5} + \frac{k^2}{25} = 50</math> oe or <math>(10 + 2k)^2 + (5 + k)^2 = 1250</math> oe</p>

Question		Answer	Marks	Guidance	
12	(iii)	<p><b>method 3</b> line perp to <math>y = 2x + k</math> through centre is <math>y = -\frac{1}{2}x</math> oe</p> <p>finding intn of their perp line with circle: <math>(x - 2)^2 + (-\frac{1}{2}x + 1)^2 = 50</math> and simplifying to solvable form</p> $x = \frac{4 \pm \sqrt{160}}{2}, y = \frac{-4 \pm \sqrt{160}}{4}$ <p>oe</p> <p>using <math>y = 2x + k</math> to obtain <math>k</math> <math>k = -5 \pm 5\sqrt{10}</math> oe if correct</p> <p><math>(k + 5)^2 = 250</math> and correct working to obtain given answer <math>k^2 + 10k - 225 = 0</math>.</p>	<p>or M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>condone attempt <math>y = -\frac{1}{2}x + n</math>, with <math>n \neq 0</math></p> <p>allow using line with grad <math>-\frac{1}{2}</math> but error in constant</p> <p>dep on previous Ms; or may use eqn of line gradient 2 through each of these points and compare results with <math>y = 2x + k</math></p> <p>NB mark working not answer;</p> <p>or allow subst of <math>k = -5 \pm 5\sqrt{10}</math> oe into <math>k^2 + 10k - 225 = 0</math> and showing consistent</p>	<p>M0 for just <math>y = -\frac{1}{2}x + c</math> with no attempt to subst <math>(2, -1)</math> to find <math>c</math></p>

Question		Answer	Marks	Guidance	
12	(iii)	<p><b>method 4</b> using calculus:  <math>2x - 4 + 2y \frac{dy}{dx} + \frac{dy}{dx} = 0</math> and subst <math>\frac{dy}{dx} = 2</math>            [if correct, <math>2x - 4 + 4y + 2 = 0</math>]            using <math>y = 2x + k</math>, subst and solving:            [if correct, <math>2x - 4 + 4(2k + 1) + 2 = 0</math> and  <math>x = -\frac{2k}{5}, y = \frac{k}{5}</math>]</p> <p><math>\left(2 + \frac{2}{5}k\right)^2 + \left(-1 - \frac{1}{5}k\right)^2 = 50</math> oe</p> <p>correct simplification to given answer  <math>k^2 + 10k - 225 = 0</math>.</p>	<p><b>or</b> M1</p> <p>M1</p> <p>M1</p> <p>A2</p> <p>[5]</p>	<p>condone one error</p> <p>condone one error</p> <p>for correct substitution ft into circle equation, dep on first M1 earned; brackets / signs must be correct</p> <p>NB mark working not answer;  <b>A1</b> for correct expansion of brackets or correctly eliminating fractions as first step, working with correct equation only</p>	<p>[cf method 2: more work to be done by method 4 to get to the stage of finding the point of contact in terms of <math>k</math>]</p> <p>using distance from centre = radius, or point of contact being on circle</p> <p>A1 for  <math>4 + \frac{8k}{5} + \frac{4k^2}{25} + 1 + \frac{2k}{5} + \frac{k^2}{25} = 50</math> oe or  <math>(10 + 2k)^2 + (5 + k)^2 = 1250</math> oe</p>

**Appendix – sample explanations for 8(i)**

Explanation	Mark
Even if a number is negative, its square will be positive.	1
$n^2 + 6$ will always be positive (because of the $n^2$ ). This being positive does not imply that $n$ is positive.	1
any negative number squared is positive, therefore can be positive or negative	1
B can still be correct even if $n$ was a negative number	1 bod
A can be a negative value and B will still be a positive number	1 bod
B will always be positive but A can be a negative number	1 bod
eg $(-5)^2 = 25$ , so B = 31 is positive	1 bod
eg $-3^2 + 6 = 9 + 6 = 15$ , so the number can be negative	1 bod
any number squared is positive, so $n$ can be positive or negative	0

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