#### Arithmetic series

\[ S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\} \]

#### Geometric series

\[ S_n = \frac{a(1 - r^n)}{1 - r} \]
\[ S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1 \]

#### Binomial series

\[ (a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N}), \]

where \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \)

\[ (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \]
\[ (|x| < 1, \ n \in \mathbb{R}) \]

#### Differentiation

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tan kx )</td>
<td>( k \sec^2 kx )</td>
</tr>
<tr>
<td>( \sec x )</td>
<td>( \sec x \tan x )</td>
</tr>
<tr>
<td>( \cot x )</td>
<td>( -\cosec^2 x )</td>
</tr>
<tr>
<td>( \cosec x )</td>
<td>( -\cosec x \cot x )</td>
</tr>
</tbody>
</table>

#### Quotient Rule

\[ y = \frac{u}{v} \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \]

#### Differentiation from first principles

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

#### Integration

\[ \int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + c \]
\[ \int f'(x)(f(x))^n \, dx = \frac{1}{n+1}(f(x))^{n+1} + c \]

Integration by parts

\[ \int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx \]
Small angle approximations
\[ \sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2} \theta^2, \tan \theta \approx \theta \] where \( \theta \) is measured in radians

Trigonometric identities
\[ \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \]
\[ \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \]
\[ \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi) \]

Numerical methods
Trapezium rule:
\[ \int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}) \} \]
where \( h = \frac{b-a}{n} \)

The Newton-Raphson iteration for solving \( f(x) = 0 \): \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \)

Probability
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ P(A \cap B) = P(A)P(B | A) = P(B)P(A | B) \quad \text{or} \quad P(A | B) = \frac{P(A \cap B)}{P(B)} \]

Sample variance
\[ s^2 = \frac{1}{n-1} S_{xx} \quad \text{where} \quad S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - nx^2 \]

Standard deviation, \( s = \sqrt{\text{variance}} \)

The binomial distribution
If \( X \sim B(n, p) \) then \( P(X = r) = \binom{n}{r} p^r q^{n-r} \) where \( q = 1 - p \)

Mean of \( X \) is \( np \)

Hypothesis testing for the mean of a Normal distribution
If \( X \sim N(\mu, \sigma^2) \) then \( \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \) and \( \frac{X-\mu}{\sigma/\sqrt{n}} \sim N(0,1) \)

Percentage points of the Normal distribution
\[
\begin{array}{c|ccccc}
 p & 10 & 5 & 2 & 1 \\
 z & 1.645 & 1.960 & 2.326 & 2.576 \\
\end{array}
\]

Kinematics
Motion in a straight line
\[ v = u + at \]
\[ s = ut + \frac{1}{2}at^2 \]
\[ s = \frac{1}{2}(u + v)t \]
\[ v^2 = u^2 + 2as \]
\[ s = vt - \frac{1}{2}at^2 \]

Motion in two dimensions
\[ v = u + at \]
\[ s = ut + \frac{1}{2}at^2 \]
\[ s = \frac{1}{2}(u + v)t \]
\[ v^2 = u^2 + 2as \]
\[ s = vt - \frac{1}{2}at^2 \]
Answer all the questions

Section A (23 marks)

1  Show that \((x - 2)\) is a factor of \(3x^3 - 8x^2 + 3x + 2\). \([3]\)

2  By considering a change of sign, show that the equation \(e^x - 5x^3 = 0\) has a root between 0 and 1. \([2]\)

3  In this question you must show detailed reasoning.

   Solve the equation \(\sec^2 \theta + 2 \tan \theta = 4\) for \(0^\circ \leq \theta < 360^\circ\). \([4]\)

4  Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to 1.2 m s\(^{-1}\) as it travels 1.8 m.

   (i) Calculate the acceleration of the box. \([2]\)

   (ii) Find the magnitude of the force that Rory applies. \([2]\)

5  The position vector \(\mathbf{r}\) metres of a particle at time \(t\) seconds is given by

   \[ \mathbf{r} = (1 + 12t - 2t^2)i + (t^2 - 6t)j. \]

   (i) Find an expression for the velocity of the particle at time \(t\). \([2]\)

   (ii) Determine whether the particle is ever stationary. \([2]\)

6  Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

   (i) Calculate how much she saves in two years. \([2]\)

   Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

   (ii) Explain why the amounts Baraka saves each month form a geometric sequence. \([1]\)

   (iii) Determine whether Baraka saves more in two years than Aleela. \([3]\)
Answer all the questions

Section B (77 marks)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force $F$ N applied $x$ m below the top of the rod as shown in Fig. 7.

![Fig. 7](image)

(i) Find the value of $F$. [1]
(ii) Find the value of $x$. [2]

8 (i) Show that $8 \sin^2 x \cos^2 x$ can be written as $1 - \cos 4x$. [3]
(ii) Hence find $\int \sin^2 x \cos^2 x \, dx$. [3]
A pebble is thrown horizontally at 14 m s\(^{-1}\) from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high \(d\) m away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the \(x\)-axis horizontal in the direction in which the pebble is thrown and the \(y\)-axis vertically upwards.

(i) Find the time the pebble takes to reach the ground. [3]

(ii) Find the cartesian equation of the trajectory of the pebble. [4]

(iii) Find the range of possible values for \(d\). [3]

Fig. 10 shows the graph of \(y = (k-x)\ln x\) where \(k\) is a constant \((k > 1)\).

Find, in terms of \(k\), the area of the finite region between the curve and the \(x\)-axis. [8]
11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

![Fig. 11](image)

(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. [2]

(ii) Find the coefficient of friction between the rough plane and Block B. [5]

12 Fig. 12 shows the circle \((x - 1)^2 + (y + 1)^2 = 25\), the line \(4y = 3x - 32\) and the tangent to the circle at the point A \((5, 2)\). D is the point of intersection of the line \(4y = 3x - 32\) and the tangent at A.

![Fig. 12](image)

(i) Write down the coordinates of C, the centre of the circle. [1]

(ii) \((A)\) Show that the line \(4y = 3x - 32\) is a tangent to the circle. [4]

\((B)\) Find the coordinates of B, the point where the line \(4y = 3x - 32\) touches the circle. [1]

(iii) Prove that ADBC is a square. [3]

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. [5]
The function $f(x)$ is defined by $f(x) = \sqrt[3]{27 - 8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0.25</td>
<td>2.9954</td>
<td>-0.056</td>
</tr>
<tr>
<td>0.5</td>
<td>2.9625</td>
<td>-0.228</td>
</tr>
<tr>
<td>0.75</td>
<td>2.8694</td>
<td>-0.547</td>
</tr>
<tr>
<td>1</td>
<td>2.6684</td>
<td>-1.124</td>
</tr>
<tr>
<td>1.25</td>
<td>2.2490</td>
<td>-1.977</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
<td>ERROR</td>
</tr>
</tbody>
</table>

(i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. [3]

(ii) Find the first three terms of the binomial expansion of $f(x)$. [3]

(iii) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of $\int_{0}^{1} \sqrt[3]{27 - 8x^3} \, dx$. Explain why Jenny’s method is valid in this case. (You do not need to evaluate Jenny’s approximation.) [2]

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_{0}^{1} \sqrt[3]{27 - 8x^3} \, dx$. [3]

The calculator gives 2.921 174 38 for $\int_{0}^{1} \sqrt[3]{27 - 8x^3} \, dx$. The graph of $y = f(x)$ is shown in Fig. 13.

(v) Explain why the trapezium rule gives an underestimate. [1]
The velocity of a car, \( v \) m s\(^{-1} \) at time \( t \) seconds, is being modelled. Initially the car has velocity 5 m s\(^{-1} \) and it accelerates to 11.4 m s\(^{-1} \) in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time \( t \) using this model. [3]

(ii) Explain why this model is not appropriate in the long term. [1]

Model A is refined so that the velocity remains constant once the car reaches 17.8 m s\(^{-1} \).

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. [3]

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. [3]

In model B, the velocity of the car is given by

\[
v = \begin{cases} 
5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leq t \leq 8, \\
17.8 & \text{for } 8 < t \leq 20.
\end{cases}
\]

(v) Show that this model gives an appropriate value for \( v \) when \( t = 4 \). [1]

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. [3]

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. [3]

END OF QUESTION PAPER
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