Section 2: Matrices and transformations

Exercise level 2

1. Plot the object and image for each of the following on the same diagram and describe each as a single transformation.

Object	Matrix
(i) P(4, 2) Q(4, 4) R(0, 4)	$\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}$
(ii) P(-6, 8) Q(-2, 8) R(-2, 6)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

2. Draw a quadrilateral with vertices A(3, 4) B(4, 0) C(3, 1) D (0, 0) and find its image under the transformation $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$. Describe the transformation, and find the ratio of the image area to object area.

3. Find the images of A(3, 1) B(3, 3) C(6, 3) D(6, 1) under the transformation $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$. Show ABCD and its image on a diagram, and describe the transformation.

4. The transformation R is an anticlockwise rotation about the origin through an angle of 60^0. Find the matrix **R** using exact values only.

5. Find the matrices which represent the following transformations in three dimensions.
   (i) rotation of 90° about the *z*-axis
   (ii) reflection in $y = 0$.

6. The following matrices represent a rotation about the origin. Find the angle and direction of rotation in each case

   (i) $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$ (ii) $\begin{pmatrix} -0.8 & -0.6 \\ 0.6 & -0.8 \end{pmatrix}$

7. Find 2×2 matrices to represent the transformation P, which is a reflection in the *y*-axis, and the transformation Q, which is a rotation of 90° clockwise about the origin. Hence find a single matrix to represent a reflection in the *y*-axis followed by a rotation of 90° clockwise about the origin. Describe this as a single

transformation.

8. Find the effect on the square with vertices O(0, 0) A(1, 0) B(1, 1) C(0, 1) of the matrices **R**, **S**, **RS** and **SR** given that **R** = $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and **S** = $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$.

9. (i) Draw triangle ABC such that A(2, 1) B(7, 1) and C(2, 4)
   (ii) Find the image of ABC under the matrix $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ and plot the image on the same graph.
   (iii) The transformation is a rotation followed by an enlargement. Calculate the angle of rotation and the scale factor of the enlargement.

10. (i) Write down a matrix **S** which represents a stretch, scale factor 3, in the *x*-direction.
    (ii) The matrix **T** is given by $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **T**.
    (iii) The matrix **M** represents the combined effect of the transformation represented by **S** followed by the transformation represented by **T**. Find **M**.