OCR
Oxford Cambridge and RSA

Tuesday 26 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4777/01 Numerical Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the barcodes.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** x_0, x_1, x_2 are three terms in a first order iteration converging to α. You are given that the error in x_0 is ε and the error in x_1 is $k\varepsilon$ (where ε is small). State the approximate error in x_2.

Show that

$$\alpha \approx \frac{kx_0 - x_1}{k-1}, \qquad \text{where} \qquad k \approx \frac{x_2 - x_1}{x_1 - x_0}.$$ **[6]**

Solutions of the equation $\sin x + \mathrm{e}^x = c$ are required for values of $c > 1$ and $x > 0$.

(ii) Use calculus to show that there is exactly one positive root for any $c > 1$. Illustrate this result graphically. **[5]**

(iii) Show numerically that the iteration $x_{r+1} = \ln(c - \sin x_r)$ converges slowly when $c = 1.1$. Use the method in part (i) to find the solution correct to 4 significant figures. **[9]**

(iv) Show by means of several examples that, as c increases, there is progressively less need for the method in part (i) when solving the equation. **[4]**

2 **(i)** Obtain from first principles the Gaussian two-point rule for numerical integration:

$$\int_{-h}^{h} \mathrm{f}(x)\,\mathrm{d}x \approx h\left(\mathrm{f}\left(-\frac{h}{\sqrt{3}}\right) + \mathrm{f}\left(\frac{h}{\sqrt{3}}\right)\right).$$

This rule is exact up to polynomials of degree n. What is the value of n? **[6]**

The Gaussian two-point rule is to be used to find values of the integral I where

$$I = \int_0^k \frac{1}{\sqrt{1+x^4}}\,\mathrm{d}x\,.$$

(ii) For the case $k = 1$, find the value of I correct to 4 decimal places.

By considering differences and ratios of differences for successive estimates of I, discuss briefly the order of the Gaussian two-point rule. **[14]**

(iii) Find, correct to 2 decimal places, the value of k for which $I = 1$. **[4]**

3 The differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = (x-y)(x^2+y^2)$ with initial conditions $x=0,\ y=0$ is to be solved numerically. A solution is required as far as $x = 1$.

(i) Obtain solutions using Euler's method with $h = 0.1, 0.05, 0.025$. Hence show that Euler's method is first order. **[8]**

(ii) Obtain solutions using the modified Euler method with $h = 0.1, 0.05, 0.025$. Hence determine the order of this method. **[8]**

(iii) Suppose now that the initial conditions are modified to $x = 0,\ y = 1$. Obtain a sketch of the solution curve as far as $x = 1$.

By reducing h appropriately, find the coordinates of the minimum point on the solution curve correct to 2 decimal places. Comment on your answer in relation to the differential equation. **[8]**

4 In the table, the x values may be regarded as exact, but the y values are subject to experimental error. It is thought that y is a polynomial function of x.

x	1.0	1.5	1.8	2.1	2.3	2.5	3.0
y	–3.542	–0.224	0.226	0.241	0.312	0.647	3.867

(i) Use your spreadsheet to obtain a sketch of the data. What degree of polynomial looks to be a good fit to the data? **[3]**

(ii) Explain why Newton's divided difference method is better suited to fitting a polynomial to these data than either the ordinary difference method or Lagrange's method. **[3]**

(iii) Construct a divided difference table for the data and explain what this suggests about the fit of a polynomial. **[4]**

(iv) Obtain a series of estimates of increasing degree for the value of y

(A) when $x = 2$,

(B) when $x = 2.7$.

In each case, give your answer to the accuracy that is justified. **[10]**

(v) Find an estimate of the value of x for which $y = 0$. Give this value of x correct to 2 decimal places. **[4]**

END OF QUESTION PAPER