Tuesday 19 June 2018 – Afternoon

FSMQ ADVANCED LEVEL

6993/01 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
• Printed Answer Book 6993/01

Other materials required:
• Scientific or graphical calculator

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

• The Question Paper will be found inside the Printed Answer Book.
• Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
• Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
• Use black ink. HB pencil may be used for graphs and diagrams only.
• Answer all the questions.
• Read each question carefully. Make sure you know what you have to do before starting your answer.
• Do not write in the barcodes.
• You are permitted to use a scientific or graphical calculator in this paper.
• Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

• The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
• You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
• The total number of marks for this paper is 100.
• The Printed Answer Book consists of 20 pages. The Question Paper consists of 8 pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

• Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.
Formulae Sheet: 6993 Additional Mathematics

In any triangle $ABC$

Cosine rule \[ a^2 = b^2 + c^2 - 2bc \cos A \]

Binomial expansion

When $n$ is a positive integer

\[(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \]

where \[ \binom{n}{r} = \frac{n!}{r!(n-r)!} \]
Answer all the questions.

Section A

1 Solve the inequality $2 - x < 1 + 3(x - 2)$. [3]

2 The gradient function of a curve is given by $\frac{dy}{dx} = 2 + 2x - 3x^2$.

Find the equation of the curve given that it passes through the point (2, 3). [4]

3 The graph of $y = x^2 + 2x - 3$ is given on the grid in the Printed Answer Book.

(i) Write down the solution to the equation $x^2 + 2x - 3 = 0$. [1]

(ii) By plotting the appropriate straight line on the grid, find the solution to the equation $x^2 - x - 6 = 0$. [3]

4 You are given that the acute angle $\theta$ is such that $\sin \theta = \frac{1}{5}$.

Find the exact value of each of the following.

• $\cos \theta$
• $\tan \theta$ [4]
Fig. 5 shows part of the graph of the curve with equation $y = 6x^2 - 2x^3$.

Find the area of the shaded region enclosed by the curve and the $x$-axis. [4]

6 (i) Solve these simultaneous equations.

\[
\begin{align*}
3x + 4y &= 18 \\
7x - 3y &= 5
\end{align*}
\]

[4]

(ii) Draw a rough sketch of the lines to demonstrate graphically the solution to part (i). [2]

7 (i) Find the coordinates of the points where the line $y = 7x - 9$ cuts the curve $y = x^2 + 2x - 5$. [4]

(ii) Determine whether the line is a normal to the curve at either of the points of intersection. [3]

8 (i) Simplify the equation \( \frac{x + a}{x} + \frac{x - 2}{4} = 0 \), leaving your answer in the form \((x + p)^2 = q\)

where \( p \) is an integer and \( q \) is given in terms of the constant \( a \). [3]

(ii) Hence write down the range of values of \( a \) for which the equation has real roots. [2]

(iii) Using your answer to part (i), solve the equation when \( a = -1 \), giving your answers exactly. [2]
9 The proportion of people who are left-handed is 20%.

(a) For a group of 10 students chosen at random, use the binomial distribution to find the probability that

(i) no student is left-handed,

(ii) exactly 4 students are left-handed.

(b) State the conditions necessary for the binomial distribution to be valid.

10 Fig. 10 shows an “up and over” garage door, XY, that is 200 cm long. There is a small wheel at the point G on the door. The wheel runs freely up a groove in a fixed vertical door frame, AB. A metal rod AP is fixed to the top of the door frame, A, and is also fixed to the point P on the door. The rod is hinged at both ends.

GP = PX = AP = 60 cm and YG = 80 cm.

When the door is closed, Y is at B and X is at A. When the door is fully open, G is at A and the door is horizontal, 200 cm above the horizontal ground.

(i) Explain why P is the centre of the circle through A, G and X.

(ii) Hence show that AX is horizontal whatever the position of the garage door.

(iii) Find the height of Y above the ground when angle AGP = 40°.
Section B

11 A circle has centre (0, 3) and radius 3.
   (i) Show that the equation of the circle is \( x^2 + y^2 - ky = 0 \) where \( k \) is to be determined. [2]
   The line \( y = mx - 2 \) passes through the point P (0, –2) and is a tangent to the circle.
   (ii) Find the two possible values of \( m \). [6]
   The two tangents from P meet the circle at the points A and B respectively.
   (iii) Find the lengths PA and PB. [4]

12 The shape shown in Fig. 12 is made of metal rods. ABCD is a rectangle.
   \( AB = CD = y \text{ cm} \) and \( BC = DA = 6x \text{ cm} \).
   AED is an isosceles triangle with height \( 4x \text{ cm} \) and \( AE = ED \).
   ![Fig 12]
   (i) Show that the perimeter, \( p \text{ cm} \), can be written as \( p = 16x + 2y \). [3]
   You are given that \( p = 96 \).
   (ii) Show that the area of the shape, \( A \text{ cm}^2 \), can be written as \( A = 288x - 36x^2 \). [3]
   (iii) Find the maximum area of the shape as \( x \) and \( y \) vary and find the values of \( x \) and \( y \) for this area. [6]
Jessie walks at 3 km per hour in a straight line from a point B to a point C, a distance of 5 km. C is on a bearing 050° from B, as shown in Fig. 13.1. Brandon sets out at the same time as Jessie. He starts from a point A which is 2 km due East of B. He walks at 2 km per hour directly to C.

(i) Calculate the distance AC, correct to 3 significant figures. [4]

(ii) Show that Brandon arrives at C approximately 11 minutes after Jessie arrives. [3]

Charlie also sets out at the same time as Jessie. He walks in a straight line from A at 2 km per hour to meet Jessie at a point X on BC, as shown in Fig. 13.2. He arrives at the point X at the same time as Jessie.

(iii) Show that there are two possible positions for X and find the bearing on which Charlie must walk in each case. [5]
14 Two cars, P and Q, accelerate from rest from a point O at the same time.

(a) P accelerates uniformly at 2 m s\(^{-2}\).

(i) Write down the formula for the displacement, \(s\) metres, of P at time \(t\) seconds after leaving O. \([1]\)

(ii) Using appropriate units, find the time taken for P to reach a speed of 90 km h\(^{-1}\). \([3]\)

(b) Q accelerates from rest with variable acceleration \(a\) m s\(^{-2}\) where, at time \(t\) seconds, \(a = 1 + kt\), where \(k\) is a positive constant. Q passes P when \(t = 10\).

(i) Find the value of \(k\). \([5]\)

(ii) Show that at the time when P reaches 90 km h\(^{-1}\), Q is travelling at a speed just less than 130 km h\(^{-1}\). \([3]\)