GCE

Mathematics B (MEI)

Unit H630/01: Pure Mathematics and Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2018
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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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### Annotations and abbreviations

<table>
<thead>
<tr>
<th>Annotation in scoris</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ and ✗</td>
<td></td>
</tr>
<tr>
<td>BOD</td>
<td>Benefit of doubt</td>
</tr>
<tr>
<td>FT</td>
<td>Follow through</td>
</tr>
<tr>
<td>ISW</td>
<td>Ignore subsequent working</td>
</tr>
<tr>
<td>M0, M1</td>
<td>Method mark awarded 0, 1</td>
</tr>
<tr>
<td>A0, A1</td>
<td>Accuracy mark awarded 0, 1</td>
</tr>
<tr>
<td>B0, B1</td>
<td>Independent mark awarded 0, 1</td>
</tr>
<tr>
<td>SC</td>
<td>Special case</td>
</tr>
<tr>
<td>^</td>
<td>Omission sign</td>
</tr>
<tr>
<td>MR</td>
<td>Misread</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other abbreviations in mark scheme</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>E1</td>
<td>Mark for explaining a result or establishing a given result</td>
</tr>
<tr>
<td>dep*</td>
<td>Mark dependent on a previous mark, indicated by *</td>
</tr>
<tr>
<td>cao</td>
<td>Correct answer only</td>
</tr>
<tr>
<td>oe</td>
<td>Or equivalent</td>
</tr>
<tr>
<td>rot</td>
<td>Rounded or truncated</td>
</tr>
<tr>
<td>soi</td>
<td>Seen or implied</td>
</tr>
<tr>
<td>www</td>
<td>Without wrong working</td>
</tr>
<tr>
<td>AG</td>
<td>Answer given</td>
</tr>
<tr>
<td>awrt</td>
<td>Anything which rounds to</td>
</tr>
<tr>
<td>BC</td>
<td>By Calculator</td>
</tr>
<tr>
<td>DR</td>
<td>This indicates that the instruction <strong>In this question you must show detailed reasoning</strong> appears in the question.</td>
</tr>
</tbody>
</table>
Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a  Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b  An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c  The following types of marks are available.

M  A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A  Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B  Mark for a correct result or statement independent of Method marks.

E  A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.
When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

If in any case the scheme operates with considerable unfairness consult your Team Leader.
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>AOs</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ \frac{8}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = 6 + 2\sqrt{5} ]</td>
<td>M1</td>
<td>1.1a</td>
<td>Attempt to rationalize the denominator</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Must be in correct notation</td>
</tr>
<tr>
<td>2</td>
<td>EITHER [ (3)^3 + 3(3)^2(-2x) + 3(3)(-2x)^2 + (-2x)^3 ] = 27 - 54x + 36x^2 - 8x^3</td>
<td>M1</td>
<td>1.1a</td>
<td>Use of Binomial coefficients</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Powers of 3 and (-2x)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
<td>1.1b</td>
<td>Condone no brackets or (2x) used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td></td>
<td>At least 3 simplified terms correct</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>1.1b</td>
<td>All correct and simplified</td>
</tr>
<tr>
<td>OR</td>
<td>[ (3 - 2x)^2 = (9 - 12x + 4x^2) ] [ (3 - 2x)(9 - 12x + 4x^2) ] = 27 - 54x + 36x^2 - 8x^3</td>
<td>M1</td>
<td></td>
<td>Attempting to square</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
<td></td>
<td>Multiplying their answer by third bracket</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td></td>
<td>At least 3 simplified terms correct</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td></td>
<td>All correct and simplified</td>
</tr>
</tbody>
</table>
### Question 3

\[
\begin{array}{c}
\begin{aligned}
\mathbf{a} + \mathbf{b} &= \mathbf{0} \\
\begin{bmatrix} 2a \\ -3a \\ b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\end{aligned}
\end{array}
\]

\[a = -4, \quad b = -12\]

**Marks**: M1 2.5
**AOs**: A1 1.1b
**Guidance**: Setting up a correct vector equilibrium equation or two separate equations.

### Question 4

\[T - \frac{1}{2}mg = 4ma \]

\[mg - T = ma \]

\[a = \frac{g}{10} \text{ m s}^{-2}, \quad T = \frac{9mg}{10} \text{ N} \]

**Marks**: B1 3.1b
**AOs**: M1dep 1.1a
**Guidance**: Setting up a correct vector equilibrium equation or two separate equations. Award for first of these two equations Signs consistent with first equation Attempting to solve the simultaneous equations leading to either \(T\) or \(a\) soi Both \(T\) and \(a\) needed simplified

### Question 5

#### i

\[y = 4 \cos x \]

\[y = 2 \sin x \]

**Marks**: B1 1.2
**AOs**: B1 1.2
**Guidance**: Correct shapes and relative sizes Graphs labelled with correct amplitudes clear Ignore any graph beyond the given interval

#### ii

\[4 \cos x = 2 \sin x \]

\[\tan x = 2 \]

\[x = \arctan 2 \]

\[4 \cos x = 4 \times \frac{1}{\sqrt{5}} = \frac{4}{5} \sqrt{5} \]

Point of intersection is \((\arctan 2, \frac{4}{5} \sqrt{5})\)

**Marks**: M1 3.1a
**AOs**: A1 1.1b
**Guidance**: Forming equation leading to \(\tan x = k\) oe Award for 63.4° for this first A mark Attempting to find exact value for \(\cos x\) or \(\sin x\) Must be a coordinate in this form

#### iii

The student’s argument is invalid as the period of \(\tan x\) is 180°.

**Marks**: E1 2.3
**AOs**: E1 1.1b
**Guidance**: Must state or imply that the argument is invalid and give a correct reason Allow for “roots occur every 180°” oe
<table>
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<tbody>
<tr>
<td>6 i</td>
<td>(f(-1) = 4 \times (-1)^3 - 3(-1) + 1 = -4 + 3 + 1 = 0)</td>
<td>M1</td>
<td>2.1</td>
<td>DR Use of (f(-1)) must be seen. Do not allow for algebraic division.</td>
</tr>
<tr>
<td></td>
<td>Therefore ((x + 1)) is a factor</td>
<td>A1 [2]</td>
<td>2.2a Clear conclusion must be made</td>
<td></td>
</tr>
<tr>
<td>ii</td>
<td>(f(x) = (x + 1)(4x^2 - 4x + 1) = 0)</td>
<td>M1</td>
<td>1.1a</td>
<td>DR Attempt to divide or to factorise by inspection with (4x^2) correct quadratic factor seen or implied by correct linear factors</td>
</tr>
<tr>
<td></td>
<td>(= (x + 1)(2x - 1)^2 = 0)</td>
<td>A1</td>
<td>1.1b Both roots seen derived from 3 correct linear factors or use of quadratic formula</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(x = -1, \frac{1}{2}) [repeated]</td>
<td>A1 [3]</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>EITHER acceleration phase</td>
<td>M1</td>
<td>3.1b Use of (suvat) equation(s) to find velocity. Do not allow if (s = 10) used</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(v = 0 + 2.5 \times 2 = 5\ \text{m s}^{-1})</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>slowing phase (v^2 = u^2 + 2as)</td>
<td>M1</td>
<td>3.1b Use of (suvat) equation(s) with (s = 10) to find acceleration</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0 = 5^2 + 2a \times 10)</td>
<td>A1</td>
<td>1.1b FT their velocity. Must be correct sign.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a = -1.25\ \text{m s}^{-2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>([-R] = 1.5 \times (-1.25) = -1.875)</td>
<td>M1</td>
<td>1.1a Use of Newton’s second law.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Magnitude of (R = 1.875\ \text{N}) (1.88 to 3sf)</td>
<td>A1 [6]</td>
<td>1.1b FT their (a) (a \neq 2) Must be positive</td>
<td></td>
</tr>
</tbody>
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<td>7</td>
<td>EITHER acceleration phase</td>
<td>M1</td>
<td>3.1b Use of (suvat) equation(s) to find velocity. Do not allow if (s = 10) used</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(v = 0 + 2.5 \times 2 = 5\ \text{m s}^{-1})</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
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<tr>
<td></td>
<td>slowing phase (v^2 = u^2 + 2as)</td>
<td>M1</td>
<td>3.1b Use of (suvat) equation(s) with (s = 10) to find acceleration</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0 = 5^2 + 2a \times 10)</td>
<td>A1</td>
<td>1.1b FT their velocity. Must be correct sign.</td>
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<tr>
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<td>1.1b FT their (a) (a \neq 2) Must be positive</td>
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</tr>
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<td>Answer</td>
<td>Marks</td>
<td>AOs</td>
<td>Guidance</td>
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<td>----------</td>
<td>--------</td>
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</tr>
<tr>
<td>OR</td>
<td>acceleration phase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v = 0 + 2.5 \times 2 = 5 \text{ m s}^{-1} )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|          | [Diagram of velocity-time graph]
|          | using the distance to find the time it takes to stop using areas (second triangle): |
|          | \( \frac{1}{2} (T - 2.5) \times 5 = 10 \) |
|          | \( T = 6.5 \) so time to stop is 4 s. |
|          | So \( 0 = 5 + 4a \) |
|          | Giving \( a = -1.25 \text{ m s}^{-2} \) |
|          | \([-R] = 1.5 \times (-1.25) = -1.875 \) |
|          | Magnitude of \( R = 1.875 \text{ N (1.88 to 3sf)} \) |

**Guidance**

- Use of \( suvat \) equation(s) to find velocity. Do not allow if \( s = 10 \) used.
- Use of area and \( suvat \) equation(s) to find acceleration.
- Must be correct sign.
- Use of Newton’s second law.
- FT their \( a \ a \neq 2 \) Must be positive.

- Must recognise two phases of motion for first 4 marks.
- Consistent sign convention needed for full credit.
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<tbody>
<tr>
<td>8</td>
<td>EITHER</td>
<td>M1 1.1a</td>
<td>DR</td>
<td>Allow with $k = 1$ and without $y =$ Ignore $0$ if seen</td>
</tr>
<tr>
<td></td>
<td>Equation of the form $y = k(x + 1)(x - 2)$</td>
<td>M1 3.1a</td>
<td></td>
<td>Attempt to find $k \neq 1$</td>
</tr>
<tr>
<td></td>
<td>(0, -4) on curve so $k = 2$</td>
<td>A1 1.1b</td>
<td></td>
<td>All correct</td>
</tr>
<tr>
<td></td>
<td>OR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equation of the form $y = ax^2 + bx + c$</td>
<td>(M1)</td>
<td></td>
<td>Uses one point to form an equation Allow for $c = -4$ seen</td>
</tr>
<tr>
<td></td>
<td>(0, -4) on curve $c = -4$</td>
<td>(M1)</td>
<td></td>
<td>Uses both other points and attempts to solve simultaneous equations</td>
</tr>
<tr>
<td></td>
<td>(-1, 0) on the curve $0 = a - b - 4$</td>
<td></td>
<td>All correct</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2, 0) on the curve $0 = 4a - 2b - 4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solving simultaneous equations $a = 2$, $b = -2$</td>
<td>(A1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BOTH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area $= \int_{-1}^{2} (2x^2 - 2x - 4) , dx$</td>
<td>M1 1.1a</td>
<td>Integration – allow without limits – condone one error</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\left[ \frac{2x^3}{3} - x^2 - 4x \right]_{-1}$</td>
<td>A1 1.1b</td>
<td>FT their quadratic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \left( \frac{2 \times 2^3}{3} - 2^2 - 4 \times 2 \right) - \left( \frac{2 \times (-1)^3}{3} - (-1)^2 - 4 \times (-1) \right)$</td>
<td>M1 1.1a</td>
<td>Substitution of limits clearly seen</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Complete argument leading to exact answer.</td>
<td>A1 2.1</td>
<td>Allow for 9 if there is an argument to explain the change of sign even if -9 not seen.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{20}{3} - \frac{7}{3} = -9$</td>
<td>E1 2.4</td>
<td>Must give modulus and explain the change of sign. FT if their definite integral is negative.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area is 9 below the $x$-axis.</td>
<td>[8]</td>
<td></td>
<td>“Area must be positive” is not sufficient explanation.</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
<td>Marks</td>
<td>AOs</td>
<td>Guidance</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
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</tr>
<tr>
<td>9 (i)</td>
<td>Using $y = f\left(\frac{x}{a}\right)$ $y = \left(\frac{x}{\frac{1}{2}} - 1\right)^2 = (2x-1)^2$ $= 4x^2 - 4x + 1$</td>
<td>M1</td>
<td>1.1a</td>
<td>Allow for 2 instead of $\frac{1}{2}$ used for method mark or attempt to write equation of quadratic that touches axis at $(0.5, 0)$ $(2x - 1)^2$ seen is sufficient for M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1 [2]</td>
<td>2.1</td>
<td>AG Must be a convincing argument that references either stretch or $f(2x)$ or similar</td>
</tr>
<tr>
<td>9 (ii)</td>
<td>EITHER $C_2$ is $y = 4.25x - x^2 - 3$</td>
<td>B1</td>
<td>3.1a</td>
<td>Finding the equation of $C_2$. Any form</td>
</tr>
<tr>
<td></td>
<td>Normal to $y = 4x^2 - 4x + 1$</td>
<td>M1</td>
<td>1.1a</td>
<td>Finding the derivative</td>
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<tr>
<td></td>
<td>$\frac{dy}{dx} = 8x - 4$</td>
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<td></td>
<td>At $(0.1)$ $\frac{dy}{dx} = -4$</td>
<td>M1</td>
<td>1.1b</td>
<td>Finding negative reciprocal of their gradient</td>
</tr>
<tr>
<td></td>
<td>Gradient of normal is $\frac{1}{4}$</td>
<td>A1</td>
<td>1.1a</td>
<td>FT their value for derivative</td>
</tr>
<tr>
<td></td>
<td>$(0, 1)$ on line so equation of normal is $y = \frac{1}{4}x + 1$</td>
<td></td>
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<tr>
<td></td>
<td>Intersection of normal and $C_2$ $\frac{1}{4}x + 1 = 4.25x - x^2 - 3$</td>
<td>M1</td>
<td>3.1a</td>
<td>Attempt to solve simultaneous equations</td>
</tr>
<tr>
<td></td>
<td>$4x^2 - 16x + 16 = 0$</td>
<td>A1</td>
<td>1.1b</td>
<td>Repeated factor or root, or zero discriminant seen.</td>
</tr>
<tr>
<td></td>
<td>EITHER $(x - 2)^2 = 0$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>OR discriminant $16^2 - 4 \times 4 \times 16 = 0$</td>
<td>E1 [7]</td>
<td>3.2a</td>
<td>Must interpret their solution in the context.</td>
</tr>
<tr>
<td></td>
<td>Repeated root so the normal is a tangent to $C_2$</td>
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<tr>
<td>Question</td>
<td>Answer</td>
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<tr>
<td>OR</td>
<td>C₂ is ( y = 4.25x - x^2 - 3 )</td>
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<td></td>
</tr>
<tr>
<td>Normal to</td>
<td>( y = 4x^2 - 4x + 1 )</td>
<td></td>
<td></td>
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<tr>
<td>( \frac{dy}{dx} = 8x - 4 )</td>
<td>M1 Finding the derivative</td>
<td></td>
<td></td>
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<tr>
<td>At (0.1) ( \frac{dy}{dx} = -4 )</td>
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<tr>
<td>Gradient of normal is ( \frac{1}{4} )</td>
<td>M1 Finding negative reciprocal of their gradient</td>
<td></td>
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</tr>
<tr>
<td>Equation of normal is ( y = \frac{1}{4}x + 1 )</td>
<td>A1 FT their value for derivative</td>
<td></td>
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</tr>
<tr>
<td>Point on C₁ where gradient is ( \frac{1}{4} )</td>
<td>M1 Attempting to find the point on C₁ where tangent parallel to the normal found.</td>
<td></td>
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</tr>
<tr>
<td>( \frac{dy}{dx} = 4.25 - 2x = \frac{1}{4} )</td>
<td>A1 Both coordinates required</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>giving ( x = 2 ) ( y = 1.5 )</td>
<td></td>
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<tr>
<td>EITHER So the equation of the tangent is</td>
<td>E1 Correct equation for the tangent in form that makes it clear it is the same line as the normal.</td>
<td></td>
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</tr>
<tr>
<td>( y - \frac{3}{2} = \frac{1}{4}(x - 2) )</td>
<td></td>
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<tr>
<td>Which is the same equation as the normal to C₁</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OR show that point (2, 1.5) lies on normal</td>
<td>(E1)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>So the normal to C₁ is a tangent to C₂</td>
<td>[7]</td>
<td></td>
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<tr>
<td>Question</td>
<td>Answer</td>
<td>Marks</td>
<td>AOs</td>
<td>Guidance</td>
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<tr>
<td>SPECIAL CASE when the candidate tries to show that the normal to $C_2$ is a tangent to $C_1$</td>
<td>$C_2$ is $y = 4.25x - x^2 - 3$</td>
<td></td>
<td></td>
<td>Finding the equation of $C_2$. Any form</td>
</tr>
<tr>
<td>Normal to $y = 4.25x - x^2 - 3$</td>
<td>$\frac{dy}{dx} = 4.25 - 2x$</td>
<td>M1</td>
<td></td>
<td>Finding the derivative</td>
</tr>
<tr>
<td>At $(0, 1)$ $\frac{dy}{dx} = 4.25$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gradient of normal is $-\frac{4}{17}$</td>
<td></td>
<td>A1</td>
<td></td>
<td>Finding negative reciprocal of their gradient</td>
</tr>
<tr>
<td>Equation of normal is $y = -\frac{4}{17}x + 1$</td>
<td></td>
<td>A0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EITHER point of intersection with $C_1$</td>
<td>$4x^2 - 4x + 1 = -\frac{4}{17}x + 1$</td>
<td>M1</td>
<td></td>
<td>Attempt to solve simultaneous equations</td>
</tr>
<tr>
<td>OR Attempt to find both coordinates of the point on $C_1$ with gradient $-\frac{4}{17}$</td>
<td>$\frac{dy}{dx} = 8x - 4 = -\frac{4}{17}$</td>
<td>(M1)</td>
<td></td>
<td>Attempting to find the point on $C_1$ where tangent parallel to the normal found.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>No further marks are available</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>$4/7$ maximum</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
<td>Marks</td>
<td>AOs</td>
<td>Guidance</td>
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</tr>
<tr>
<td>10 i</td>
<td><img src="image" alt="Graph" /></td>
<td>B1</td>
<td>1.1a</td>
<td>Two line segments with one horizontal $(T, 4)$ and $(12.5, 4)$ labelled or indicated on scales. Allow their 2.5 marked instead of $T$. On axes labelled $v$ and $t$ oe</td>
</tr>
<tr>
<td>ii</td>
<td>$\frac{1}{2} \times 4 \times (12.5 + (12.5 - T)) = 45$</td>
<td>M1</td>
<td>3.1a</td>
<td>Attempt to find area of trapezium or both the triangle $\left(\frac{1}{2}T \times 4\right)$ and the rectangle $(12.5 - T) \times 4$. cao</td>
</tr>
<tr>
<td></td>
<td>$T = 2.5$</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>iii</td>
<td>EITHER</td>
<td>M1</td>
<td>1.1a</td>
<td>Soi</td>
</tr>
<tr>
<td></td>
<td>$a = \frac{4}{2.5} = 1.6 \text{ m s}^{-2}$</td>
<td>A1</td>
<td>3.3</td>
<td>FT their $T$</td>
</tr>
<tr>
<td>OR</td>
<td>$a = \frac{4}{2.5} = 1.6 \text{ m s}^{-2}$</td>
<td>M1</td>
<td>Soi</td>
<td>FT their $T$</td>
</tr>
<tr>
<td></td>
<td>$v = \int a , dt = 1.6t + c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>When $t = 0$, $v = 0$ so $c = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s = \int v , dt = 0.8t^2 + c$</td>
<td>A1</td>
<td>1.1a</td>
<td>Must be complete solution – do not award without consideration of $+c$ at least once</td>
</tr>
<tr>
<td></td>
<td>When $t = 0$, $s = 0$ so $c = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Giving $s = 0.8t^2$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>iv</td>
<td>$0.8t^2 = 4$</td>
<td>B1</td>
<td>3.4</td>
<td>FT their quadratic model in (iii)</td>
</tr>
<tr>
<td></td>
<td>$t = \sqrt{5} = 2.24 \text{ s}$</td>
<td>FT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
<td>Marks</td>
<td>AOs</td>
<td>Guidance</td>
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<tr>
<td>v</td>
<td><img src="image1.png" alt="Graph" /></td>
<td>B1</td>
<td>1.1a</td>
<td>Must have curved section of the graph decreasing gradient. S must be labelled.</td>
</tr>
</tbody>
</table>
| vi | Total distance (area under the graph) can only be equal if $S > T$
If $S > T$
If $S = T$
If $S < T$ | E1 | 3.5c | Needs to give reason relating to the refinement of the model. Graphs not required |
<p>|       | | | | “It takes longer to reach 4 ms$^{-1}$” is not sufficient reason |</p>
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>AOs</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 i</td>
<td>$y = kx^n$</td>
<td>B1 [1]</td>
<td>3.3</td>
<td>Allow any letters used for constant of proportionality and power.</td>
</tr>
<tr>
<td>ii</td>
<td>$\ln y = \ln (kx^n)$, $\ln y = \ln k + \ln (x^n) = \ln k + n \ln x$</td>
<td>M1 [1]</td>
<td>2.1</td>
<td>Taking natural logs of both sides and one correct use of laws of logs used. Convincing argument. AG</td>
</tr>
<tr>
<td>iii</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\begin{array}{</td>
<td>c</td>
<td>c</td>
<td>} \hline \text{Mercury} &amp; -1.179 \ \text{Jupiter} &amp; 1.599 \ \text{ln} x &amp; 9.575 \ \ln y &amp; 4.022 \hline \end{array}$</td>
</tr>
<tr>
<td>iv</td>
<td>EITHER</td>
<td>M1 [1]</td>
<td>1.1a</td>
<td>using gradient formula</td>
</tr>
<tr>
<td></td>
<td>$b = \frac{9.575 - 4.022}{-1.179 - 1.599} = -1.999$ (-2.00 to 3sf)</td>
<td>A1 [1]</td>
<td>3.1a</td>
<td>Allow -2</td>
</tr>
<tr>
<td></td>
<td>$a = 7.218$</td>
<td>A1 [3]</td>
<td>1.1b</td>
<td>$a$ correct to at least 2 sf</td>
</tr>
<tr>
<td></td>
<td>OR</td>
<td>M1 [1]</td>
<td></td>
<td>Setting up pair of equations by substitution of their values. Allow one slip.</td>
</tr>
<tr>
<td></td>
<td>$9.575 = a - 1.179b$</td>
<td>A1 [1]</td>
<td></td>
<td>$a$ correct to at least 2 sf</td>
</tr>
<tr>
<td></td>
<td>$4.022 = a + 1.599b$</td>
<td>A1 [3]</td>
<td></td>
<td>$b$ correct to at least 2 sf</td>
</tr>
<tr>
<td></td>
<td>Giving $a = 7.218$</td>
<td></td>
<td></td>
<td>Simultaneous equations can be solved using calculator</td>
</tr>
<tr>
<td></td>
<td>$b = -1.999$ (-2.00 to 3sf)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>v</td>
<td>$y = 1363x^{-2.00}$</td>
<td>B1 [1]</td>
<td>2.2a</td>
<td>FT their equation in (i) awrt 1300 or 1400, or $c^{2.0}$ or better. FT their $a$ Allow for $x^2$ or better. FT their $b$</td>
</tr>
<tr>
<td>vi</td>
<td></td>
<td>B1 [1]</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>vii</td>
<td>Earth $x=1, y = 1363 \times 1^{-2} = 1360 \text{ W m}^{-2}$ (3sf)</td>
<td>B1 [1]</td>
<td>3.4</td>
<td>FT their (v)</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
<td>Marks</td>
<td>AOs</td>
<td>Guidance</td>
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<tr>
<td>ii</td>
<td>SC1</td>
<td></td>
<td></td>
<td>Correct use of log instead of ln and no other error</td>
</tr>
<tr>
<td>iii</td>
<td>ln x</td>
<td>B0</td>
<td>B1</td>
<td>All correct. Must be 4 sf</td>
</tr>
<tr>
<td></td>
<td>ln y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mercury</td>
<td>-0.5122</td>
<td>4.158</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Jupiter</td>
<td>0.6946</td>
<td>1.747</td>
<td></td>
</tr>
<tr>
<td>iv</td>
<td>As in main scheme</td>
<td>M1</td>
<td>A1</td>
<td>FT their equation in (i) awrt 1300 or 1400, or e^3.1 (= 22.2), 10^3.1 or better.</td>
</tr>
<tr>
<td></td>
<td>a = 3.1336</td>
<td></td>
<td></td>
<td>FT their a allow for x^2 or better. FT their b</td>
</tr>
<tr>
<td></td>
<td>b = -1.999</td>
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<tr>
<td>v</td>
<td>y = 1360x^{-2}</td>
<td>B1</td>
<td></td>
<td>FT their (v)</td>
</tr>
<tr>
<td>vii</td>
<td>Earth x=1, y=1363×1^{-2} = 1360 W m^{-2} (3sf)</td>
<td>B1</td>
<td></td>
<td></td>
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</tbody>
</table>